ADVANCED
LIFE INSURANCE

WITH ILLUSTRATIONS OF THE PRINCIPLES AND PRACTICES OF ACTUARIAL SCIENCE

BY

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PREFACE

Although the title of this book indicates that it treats of life insurance in an advanced manner, much of it is not advanced in the sense that it is difficult to understand.

Most of those engaged in life insurance have acquired a considerable knowledge of it through experience and through the reading of books and other publications on that subject. Many of these persons wish to extend their knowledge of this vast and rapidly expanding business. Hitherto, they have found progress toward a more advanced knowledge of it very difficult, because of the technical manner in which treatises on actuarial science have been written. Special training in higher mathematics is necessary in order to gain an understanding of such treatises. The purpose of this book is to bridge the chasm between insurance text books and these more profound works, so that the reader may pass over from the one to the other with a minimum of training and mental effort.

With this purpose in view, the author has illustrated the principles with an abundance of examples, arranged in a manner somewhat different from that heretofore employed. The object of this arrangement is to present the illustrations so as to conform to the principles of pedagogy, thus rendering the principles and calculations of life insurance more easily understood by those without technical training. To this end, also, the presentation of each subject is started in an elementary way, thus making it unnecessary for the reader to review his previous studies. Each subject is then developed gradually, by stages easily understood, until the information concerning it reaches a point considerably beyond that contained in the books, hitherto available, on life insurance. Because of this method of treatment, even the most technical portions will not appear difficult to a person with a knowledge of arithmetic and high school algebra.

Many portions of the book are not technical in any sense of the word. These are designed to be of practical value to purchasers of life insurance, and to agents and brokers.

Among the subjects not treated previously in any text book are the following: the determination of the net cost to the policyholder;

the determination of the rate of return on the "investment" element in life insurance policies; an analysis of this investment element so that it may be compared with other investments as regards safety and the rate of return; re-insurance in life insurance; and the organization and management of new companies. The more practical uses of compound interest tables, also, are explained and clearly illustrated by examples.

Most of the information concerning the practices of the business is presented in such a manner that knowledge of any particular practice may be gained without reading large portions of the book dealing with other subjects.

The book is also designed to serve as a text book for the more advanced insurance courses in the universities, and as an aid to those preparing for actuarial careers.

The author is deeply indebted to Professor S. S. Huebner, Ph.D., Professor of Insurance and Commerce, University of Pennsylvania, and to Mr. Valentine Howell, F.A.S., Associate Actuary, Guardian Life Insurance Company, New York City. Both of these gentlemen read the manuscript most carefully and made many corrections and valuable suggestions. Neither of them is responsible for any errors now existing in the book.

C. K. KNIGHT.

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ADVANCED LIFE INSURANCE

CHAPTER I

NATURE, DEVELOPMENT, AND RECENT EXPERIENCES

Definition and Fundamental Requirements.—Life insurance is a business device for meeting financial losses due to death, by transferring to a group of persons the risk of death to which each individual member of the group is exposed. Modern life insurance, as will be seen later, also helps to provide against the contingency of dependent old age. The group to which this risk is transferred may be incorporated as a life insurance company, a fraternal benefit society, or a business assessment association; or the business may be conducted by governmental agencies, such, for instance, as the United States Government War-Risk Insurance Division. It is essential that many risks should be combined in one group in order to assure the operation of averages.[1] The element of risk may then be practically eliminated. This is sometimes referred to as "changing uncertainty into certainty." It is accomplished when an insured person eliminates his individual risk by transferring it to an insurance company, and when a company in turn eliminates its risk by insuring enough persons to enable it to estimate with a fair degree of accuracy the number of claims it will be expected to pay in a given period of time. Experience has taught the necessity of accumulating a fund for the prompt payment of claims, and of applying to this end scientific principles and a practical method. These scientific principles are discussed later. The practical method of operation consists in the making of a contract "whereby for a stipulated compensation, called the premium, one party (the insurer) agrees to pay the other (the insured), or his

[1] For a discussion of this subject see Chapter III.

beneficiary, a sum upon the happening of death or of some other specified event.''[2]

The Origin and Development of the Business.—In 1762, after a long period of experiment during which many plans for protection were devised, there was organized in London a society which conducted a life insurance business in such a manner as to make it comformable to the definition set forth above. It was known as "The Society for Equitable Assurances on Lives and Survivorships." Joint-stock companies soon followed, so that by 1800, commercial life insurance may be said to have become quite firmly established in England.[3] In the United States, the first corporations to grant life protection were the Presbyterian Corporation, chartered by Pennsylvania in 1759, and the Episcopal Corporation, established ten years later; but they were for the benefit of clergymen only. The first commercial life insurance company in this country was the Pennsylvania Company for Insurances on Lives and Granting Annuities, established in 1812. It was followed by the Massachusetts Hospital Life Insurance Company in 1818, the New York Life Insurance and Trust Company, and the Baltimore Life Insurance Company, in 1830. All of these commercial companies were joint-stock, or "Proprietary" companies. The New England Mutual Life Insurance Company was chartered by Massachusetts in 1835, but did not begin business until 1844. The first of the so-called mixed companies was the Girard Life Insurance and Trust Company, chartered by Pennsylvania in 1836. In 1843,[4] the setting in operation of the mutual plan by the Mutual Life Insurance Company of New York marked the end of the proprietary or joint-stock period and inaugurated the period of participation. From this time down to the close of the Civil War, practically all of the life insurance written was on a participating basis. The period of the Civil War, which extends from 1861 to 1870, as regards life insurance business, witnessed no great progress during the first year, but an unprecedented growth took place during each of the last three years of the conflict, and this continued during the four years immediately following its close. The problems confronting companies at this time were similar in many respects to those which they had to solve during the recent war. Following 1870,

[2] Huebner, S. S., Life Insurance, p. 3.

[3] For a detailed account of the origin and development of life insurance abroad, see the author's History of Life Insurance in the United States to 1870, with an introduction to its Development Abroad, pp. 7-48.

[4] For a discussion of stock, mutual, and mixed companies, see Ch. II.

a number of failures occurred, due in part to inexperience in the business, and in large measure, also, to the industrial depression and to the severity of valuation laws.[5]

The stronger companies survived, however, and the business progressed quite steadily, the tendency down to 1905 being to concentrate in a few of the larger companies. The way for the formation of a large number of new companies in the smaller centers of population throughout the country was opened in the latter year by the New York Investigation, and its discussion in the press. While many of these companies have disappeared, a number have succeeded and are now on a firm basis. Meanwhile the older companies have been growing rapidly until the larger "legal-reserve" or "old-line" companies now rank among our largest financial institutions.

Fraternal Insurance.—Though a more detailed account of the origin and development of fraternal life insurance is given below,[6] it may be noted here that this form of life protection arose with the establishment of the Ancient Order of United Workmen in 1868. This organization was followed by others of a similar nature whose business grew rapidly until checked by failure on the part of some, and on the part of others, by the necessity of readjusting their business to a more scientific basis than that originally adopted. Thus there has been a slight decrease in the amount of fraternal insurance in force during the past ten years.

Recent Events.—In order to understand certain problems connected with surplus limitations, the valuation of assets, and government insurance, it is necessary to study the more important events which have recently influenced the life insurance business.

The Epidemic of Influenza.—For years preceding the outbreak of this epidemic, many persons believed that the catastrophe hazard in life insurance no longer existed—that the day of plagues which take off a large number of lives in a short period of time was over.[7] Not since life insurance had got under way had such an event occurred in the more advanced countries, nor, in fact, for centuries before that time. One prominent insurance authority has stated that the effects of the epidemic may be shown with a fair degree of accuracy by comparing amounts paid as death claims during 1918 with those paid in 1917, the larger amount of business on the books in 1918 being compensated by an unusually favorable mortality rate up to the fall

[5] Testimony taken by the New York Legislative Insurance Investigating Committee, Vol. III., pp. 2229, et seq. Testimony of Emory McClintock.

[6] Chapter XX, below.

[7] See Ch. XI. for further discussion of the mortality experienced by companies.

of that year. The comparison shows that companies reporting to the New York Insurance Department paid during 1918 about $272,000,000 in death claims, not including industrial claims, as compared with approximately $189,000,000 in 1917.

The real effects on the companies were still more severe, in many instances, than even these figures indicate, for a large proportion of the deaths from influenza were at the younger ages where policies had been in force but a short time and had in consequence accumulated only small reserves.[8]

The maximum of claims from the epidemic was reached about the beginning of winter, 1918, but very heavy losses continued during the early months of 1919. As a result, some companies reduced their dividends in 1919; others, in 1920; a few raised their gross premium rates, and others suffered substantial declines in surplus in order to maintain their dividend schedules.

The experience of the epidemic was valuable in that it forcibly brought the attention of the public to bear upon the benefits of life insurance, thus creating, in conjunction with other things mentioned later, an unprecedented demand for life insurance which resulted in an enormous increase in business. Confidence in the companies was developed by the fact that they survived such an unfavorable experience without a single important failure. The catastrophe also served as a warning to those who, particularly since the Armstrong Investigation in 1905, have criticized, and urged legislation against, the maintenance of high contingency reserves by life insurance companies.

The Decline in Bond Prices.—At this time, too, a severe blow was dealt the life insurance companies by the deflation of the prices of many of their securities.

It has long been the practice of life companies to invest a considerable proportion of their funds in long-time bonds bearing a moderate rate of interest.[9] Safety, in investment, as regards the regularity of interest payments and the ultimate payment of the principal, has been their main consideration. For present purposes it is sufficient to remark that life companies usually select long-time bonds because, in the first place, a large proportion of their funds is set aside as legal reserves which for the most part will not be needed for many years; and secondly, because they wish to avoid the expenses necessarily involved in frequently reinvesting their funds.

[8] The significance of this statement will be appreciated if the reader understands the principles set forth in Chapter IX.

[9] See Ch. XV. for further discussion of investments.

Twenty years, approximately, is the average length of time their bonds have to run to maturity. As the interest rate rises, the price of a security bearing a fixed rate of return and having several years to run to maturity, necessarily declines. During the war, from the time of its outbreak in Europe, the interest rate steadily advanced and the prices of long-time low-interest-bearing bonds declined. So great was the depreciation of bonds held by the companies that if the insurance commissioners had considered only their market value as assets, many of the companies would have been in difficulty. Fluctuations in market prices caused by changes in current interest rates, however, do not materially interfere with the purposes of the companies, since they may hold the bonds to maturity and their objects will be accomplished if the interest and principal are paid promptly as they fall due. Moreover, the companies' liabilities are valued upon interest bases, which do not vary with market conditions. In determining the amount which should be admitted as assets, therefore, the insurance commissioners may use the amortized or bond-table values of the bonds, calculated on the basis of the same net rate of return as that on which they were originally purchased.[10] According to the law of some states, companies need not adopt the amortized method unless they so desire, but most of them have used it as the basis of valuation, with the result that they were quite properly permitted to report legal values considerably in excess of actual market prices. The amortized method was applicable to most of the bonds owned by life companies. Bonds in default, however, perpetual bonds, and stock, must, under the law, be valued at their market prices, and there were quite a number of defaults, especially in street railway bonds and the bonds issued by foreign governments.

Furthermore, the companies subscribed to the various issues of Liberty Bonds at times when they could have purchased other issues at a higher rate of return. One of the social advantages urged for life insurance, however, is that it collects from many individuals a huge reservoir of funds which may be drawn upon for needful purposes when proper assurance of their return at a moderate rate of interest is given.

Results of the Decline in Bond Prices.—There are few financial institutions that could withstand such serious declines in the value of their investments; yet the life companies were able to do so and speedily recovered as well their former position of unparalleled strength. A large increase in new business was placed on their books,

[10] Further explanation of the amortized method is made later.

and funds that came in to the companies, in excess of disbursements, were invested at the high rates of return that prevailed for some time following the close of the war. Thus the high rates earned on the investments soon compensated for the losses occasioned by the defaults and declines mentioned above. It should also be noted that at the present time the bonds owned by the companies have recovered from the temporary decline in market prices.

War Mortality.—War mortality is another element in the business that at first caused fears on the part of some persons regarding its possible effects upon the companies. War deaths may be divided into two classes: (1) those of persons living in countries that entered the war early, and (2) those among persons residing in the United States. Heavy losses were expected, especially in the first of the two classes, but these were only partially realized. Thus one company had nearly $200,000,000 of insurance in force in countries that entered the war early, including Canada, yet at the close of 1919 it had lost only $2,000,000, or about 1 per cent of the amount in force, by way of claims directly traceable to the war. In considering the above figures, due allowance must be made, however, for the fact that most of the lives covered by this company were beyond military age since the company discontinued writing new business in Prussia in 1895, the remainder of Germany in 1904, and France and Italy in 1906. The claims traceable to the war among residents of the United States amounted to approximately $2,200,000 from approximately $1,800,000,000 in force, or about 0.12 per cent, and of these nearly two-thirds were caused by influenza and pneumonia.[11] Since the average age at which most of the insurance is written is about 37, the reason why war mortality did not affect our companies seriously is apparent. It caused a few extra deaths among lives insured in commercial companies, but its effects were slight; the companies' mortality rate as a whole having increased only a very little, as was also the case at the time of the Civil War.

Soldiers' and Sailors' Insurance.—The soldiers' and sailors' insurance law was passed by Congress on October 6, 1917, for the specific purpose of providing protection at low rates for the dependents of those entering the military or naval service. One part of the act passed at this time provided for "allotments and family allowances." This had nothing to do with insurance but served in place of a pension system to care for dependents while the breadwinner was away. A second part provided for compensation for

[11] Strong, Wendell M., in American Year Book, 1919, pp. 389, 390.

death or disability in case either or both should occur. It was modeled after our workmen's compensation laws, and the funds to pay the claims were appropriated by Congress so that those in the service were not asked to contribute anything in the way of a premium. The third part provided that those in the service might voluntarily secure a policy of life insurance not exceeding $10,000 in amount, on the yearly-renewable term plan, by paying for it at the net premium rate for the attained age, calculated according to the American Experience table of mortality with interest at 3½ per cent. Since these low rates were sufficient only for death claims anticipated when accepting lives engaged in ordinary civilian pursuits, the law stipulated that Congress should appropriate the money to meet office and other administrative expenses as well as extra mortality costs. The law further provided that the insurance might be continued at the yearly-renewable term rates for a period of five years after the termination of the war, and that at any time within that period the insured might convert his policy into one of the regular forms of life insurance at the net *level* premium rate for his attained age, calculated upon the same mortality and interest basis. The term insurance had to be converted some time before the end of the five-year period, otherwise it was subject to discontinuance.

To carry out the provisions of the act, the Bureau of War-Risk Insurance was charged with the responsibility of receiving premiums, issuing policies in the name of the United States Government, paying claims, and attending to other necessary administrative work. No commercial company had anything to do with either the term or the converted insurance, and it was not the intention of the companies nor of the Government that they should have. A converted policy may be ordinary life, twenty-payment life, thirty-payment life, twenty-year endowment, thirty-year endowment, or endowment maturing at age 62 (the army retirement age) at the option of the insured. The converted policies now contain provisions for cash surrender values, policy loans, settlement options, disability benefits, and other privileges similar to those provided by commercial companies, except that the provisions of the converted Government policies are more liberal toward the insured in several respects than policies which it is practicable for private companies to offer. For instance, the total and permanent disability clause of the converted Government policies contains no age limit, which is an important and valuable consideration, especially in ordinary and limited-payment life policies. As compared with this, commercial life insurance

provides that the disability benefits will not be paid unless the insured becomes disabled before some specified age, usually 60 or 65. The converted policies are participating, and dividends were begun in 1921. The annual cost of the insurance to the government policyholder is thus reduced below the net level premium on the basis of the American Experience table and 3½ per cent. It must be emphasized that the soldiers' and sailors' insurance was offered to those in the service at low rates to compensate in part for the great sacrifices made by them.

With the hazards of war removed, hundreds of thousands of discharged soldiers and sailors allowed their Government insurance to lapse. Most of them were below the ages when men begin to provide an adequate amount of insurance; many had no one depending upon them, and felt that they did not need or could not afford the protection. Some of them were somewhat displeased and dissatisfied on account of the delays and mistakes necessarily attendant upon undertaking to create in so short a time the administrative machinery required to handle a business of such vast proportions.

The primary purpose of the Government, however, was to furnish cheap protection against the hazards of war, and this was accomplished at a time when commercial companies were asking from those who insured during the war and entered the service an extra premium of approximately $37.50 per $1000 of insurance, in addition to the regular premium.[12] Commercial policies issued before the war did not require an extra premium, nor the consent of the companies issuing them, to entrance into the service, and war restrictions, for the most part, are no longer found in the provisions of life policies. Some companies still retain military restrictions as regards the disability clause.

Increasing Prices.—As the price-level rose during this period, and heavier taxes were imposed, the home office expenses of the companies advanced. However, since insured persons were obliged to seek larger coverage in order to furnish adequate protection to their families or business associates, it was easy for agents to increase their earnings without an increase in commission rates, and the amount of insurance applied for correspondingly increased.

Summary of Gains and Causes of Growth.—The amount of life insurance in force in the United States at the close of 1922 had increased to vast proportions. More than half of this growth had taken place during this period of the great World War, and during the

[12] Some companies charged $100 per $1000 for service abroad, with no extra charge for service in this country.

years immediately following its close. Thus at the close of 1913, the last insurance year before the outbreak of the war in Europe, there were 259 old-line or legal reserve companies operating in the United States, with approximately $21,000,000,000 of insurance in force. The fraternals had nearly $10,000,000,000, or a total of over $30,000,000,000 in all. At that time, these figures were considered almost incomprehensible.[13] Yet at the close of 1922, the amount of legal-reserve or old-line insurance, including group and industrial business, had increased to two and one-half times what it was in 1913, or to a total of over $50,000,000,000. Of this, over $41,000,000,000 was ordinary business, over $8,650,000,000 was industrial, nearly $2,000,000,000 group, and the remainder unclassified. To this amount of old-line life insurance must be added $8,687,939,447 of fraternal insurance, and at least $3,000,000,000 of United States Government War-Risk insurance, thus making a grand total of about $62,000,000,000. In 1926 this total is over $80,000,000,000.

There was, then, a great increase in the amount of insurance in force during these years, yet the rate of this increase was not uniform; and many events affecting life insurance transpired during this time, some of the more important of which demand special attention. Thus in 1917 there was a considerable slowing up in the rate at which commercial life insurance was increasing. Toward the close of 1918, however, there was a marked increase in the demand for insurance, due in part to the fact that the epidemic of influenza caused people to realize to a greater extent than previously the importance and practical benefits of life insurance. So far as the new business of old-line companies is concerned, the year 1919 was the most remarkable in our history down to that date, new business being nearly double that of 1918. The increase continued throughout the greater part of 1920. In the latter part of 1920, people were lapsing and surrendering more, and purchasing less than during the first nine months. The slowing down was not nearly so marked in this line of business, however, as in others, the amount written during 1921 being about 85 per cent of that written during 1920, with steady gains since.

Substantial gains in industrial insurance were also made during the war, and the amount of fraternal insurance in force remained about stationary. For the fraternals, the epidemic of influenza was a serious matter, since their organization and method of assessing premiums were peculiarly subject to adverse results when heavy mortality was experienced. In general, fraternals plan to levy extra

[13] Huebner, S S., Life Insurance, p 5.

assessments when necessary to pay claims, but when assessments are not paid the amounts payable to beneficiaries must be reduced below the face values of their certificates. The difficulties are great, therefore, when claims are increased by an epidemic. Healthy members withdraw, and new members are difficult to secure. Adjustments other than those occasioned by the epidemic also caused lapses in fraternal insurance.

The causes of the unprecedented growth in ordinary and industrial business may be summarized as follows:

1. The decline in the purchasing power of money. Because of the increase in prices, insured persons were obliged to increase their insurance to provide adequate protection, as much as $1000 being required at that time to purchase what could have been secured for less than $500 before the war began. This is the most important reason for the great increase in insurance during this period.

2. The emphasis placed by the epidemic and the war upon the need of protection.

3. The notice given the business by Government insurance and the high standard set by it as regards the amount of insurance a comparatively poor man should carry.

4. The increasing use of life insurance for business purposes during the period. Those accumulating funds in times of prosperity often choose life insurance as a safe means of saving and investment to provide for periods of financial adversity. Policies to provide for the payment of inheritance taxes, to protect valuable employees of corporations, and group policies to protect the main body of employees became more and more popular throughout the period.

5. The great progress made in promoting and extending insurance education.

Size and Strength.—A few comparisons may serve to show the size of the life insurance business in this country toward the close of the period of inflation following the Great War. When the public debt of the United States had increased until it amounted to $23,000,000,000 in 1922, the amount of life insurance in force was approximately two and one-half times this figure. It was over three times as great as the total value of railroad property as found by the Bureau of Valuation of the Interstate Commerce Commission. The assets of legal-reserve companies alone amounted to almost half of the total assets of all national banks.

It will be recognized that safety is one of the fundamental requirements for carrying out the purposes of this gigantic financial institu-

tion. As to its stability, the first test to which it was subjected was the panic of 1857. The life companies survived this panic with few failures, and thus gained the confidence of the business world at that time. The panic of 1873 witnessed a number of failures among the weaker companies, but much of their business was reinsured, so that the total loss was not large. Much was then learned about successful management and state supervision, so that the panics of 1893, 1907 and 1913 were without appreciable effect. The events during the World War, however, furnished more severe tests than any of these. The epidemic of influenza, which in the first instance drew heavily upon surplus funds, defaults of bonds, the depreciation of bond prices, which threatened to undermine the legal reserves, the fear of adverse war mortality, the extra premium charged those who wished to insure and enter the military or naval service, increased tax burdens and expenses of operation, the action of the Government which resulted in removing until the close of the war so many of those just attaining the insuring age from the realm of possible prospects, all combined to subject life companies to the severest test in their history. It is doubtful if any other class of financial institutions could have withstood so many adversities. The fact that the life companies were able to do so without a single noticeable failure, and indeed were able to turn some of the unfortunate events to their advantage, demonstrates beyond question that they are among our strongest financial institutions. In fact, for nearly fifty years there has not been a single failure of importance in which the policyholders were not promptly reinsured in other companies with but slight losses. No other class of institutions can boast of such a record.[14]

[14] It must be noted that at this point the writer refers to the old-line or legal-reserve companies, and not to fraternal or assessment societies. For an account of their record, see Chapter XX.

CHAPTER II

Companies and Contracts

"Legal-reserve," or "old-line" companies are those which maintain reserves to provide for the payment of future claims according to the standards established by the insurance laws of the various states. They also comply with the requirements of the insurance laws regarding incorporation, license to operate, deposits with state officials, annual reports, periodical examinations, etc. They differ thus from fraternal benefit societies, which do not come under the requirements of the insurance laws proper, but which are supervised according to the provisions of special fraternal insurance legislation.[1] There are three types of legal-reserve companies—stock, mutual, and mixed. A stock company is owned and controlled by the holders of its capital stock in that the stockholdiers elect directors and thus indirectly control the management of the company, as well as determine what shall be done with any surplus that may accumulate. As a rule, stock companies issue non-participating policies, under the terms of which the insured persons are charged a relatively low premium, but are not entitled to share in the surplus. Thus, any profits that arise from such policies belong to the stockholders. Mutual companies, however, have no capital stock, and are owned and theoretically controlled by their policyholders who elect trustees to appoint officers and otherwise serve in the capacity of directors. Nearly all of their policies are participating, which means that the policyholder is charged a relatively high initial premium but is entitled to share in the distribution of the surplus. In this country, the policyholder's allotted share of the surplus is called a "dividend." This may be used to reduce his premium or used in other ways. In New York, domestic companies are not now permitted to issue both participating and non-participating policies, but must confine themselves to one or the other of the two plans. Mixed companies are those having capital stock but providing in their charters that the

[1] See Chapter XX for the technical definition of a fraternal benefit society.

policyholders shall share in the control, and in the distribution of the surplus, according to some definite arrangement. For instance, it may be arranged that upon the distribution of any portion of the surplus, 7 per cent shall first be paid on the capital stock, and the remainder divided, one-eighth to the stockholders and seven-eighths to the policyholders. In some mixed companies the election of a minority of the board of directors is in the hands of the policyholders, but the actual control of the company rests with the stockholders. It is customary to refer to such a company as a stock company, even though the policyholders share in the distribution of the surplus. So, although there are many of the so-called mixed companies in operation, it is seldom, except in academic discussions, that one is referred to as such in insurance literature.

Comparison of Participating and Non-participating Plans.—*Historical.*—Unfortunately, history sheds little light on the relative merits of the two plans, since what seems best in one period of development must be accorded only second place in another. For instance, from 1813 to 1843, life insurance was carried on in this country primarily by stock companies and on the non-participating basis. Since the business was new and untried, for the sake of safety, a large capital stock and high premiums were desirable. By 1843, however, it had become evident that the early premiums charged were unnecessarily high, if the policyholder were to be given no share in the surplus. Premiums had been reduced somewhat during the latter part of this period,[2] yet it was evident that they were still higher than was made necessary by the death rate and the expenses of carrying on the business. At the same time, there still existed uncertainties to render doubtful the feasibility of further reduction. The participating plan, whereby the policyholder could be charged enough to assure the safety of the company, the overcharge to be returned to him after the uncertainties of a period were safely passed, was especially adapted to meet the conditions. There were other reasons for the movement toward participation,[3] but the above explains why this plan seems to have been best fitted to the conditions existing from 1843 down to the Civil War.

The experience gained during the latter period made it possible to revive the non-participating plan, and to issue policies at rates much lower than the initial participating rates. The non-participating company claimed to discount the dividends that were being paid by

[2] See the author's, "History of Life Insurance," pp. 99-100.
[3] *Ibid.*, pp. 90-94, 102.

the participating companies and to declare them in advance by offering policies at lower rates in the first place. From the Civil War to the New York investigation of 1905, both plans existed side by side, the advocates of each maintaining its superiority over the other. Nearly all new companies were started on the stock plan to provide safety, to meet state deposit requirements, etc., but many of these arranged for mutualization after the hazards of the formative period were safely passed. Following the investigation of 1905, many small stock companies were incorporated in different parts of the country, and several large stock companies began to change to the mutual plan. Events during the World War, especially the lowering of the dividends of the mutuals, demonstrated the advantages to the insured of a definite annual cost that could not be increased by lowering dividend schedules, but it also demonstrated the strength of the mutual plan in that dividends could be cut and depleted surplus funds immediately restored. Some have argued that since no strictly mutual company operating on a legal-reserve basis has failed within the past fifty years, whereas many stock companies have been obliged to reinsure their business during this period, the mutual plan is the safer one of the two. Most of the stock companies that were obliged to close, however, and turn over their affairs to some stronger and better managed company, were small, new companies that never really got started. They were operating on the stock rather than the mutual plan because, as previously explained, owing to legislative requirements concerning deposits, etc., it is much easier to organize a company on the stock plan.[4]

Comparison as Regards Cost of Protection to the Policyholder.—It is generally conceded that the participating plan costs the policyholder a higher initial premium, and that his payments continue to be greater during the first few years, but that as dividends increase, his payments will be reduced so that in the later years they will be lower than under the non-participating plan.[5] It might be urged that the entire cost and expenses of the business as actually experienced by the company, including a reasonable return on capital invested, should be borne by the policyholders, and that this cannot be exactly determined in advance. As opposed to this, it might be stated that under the non-participating plan, the policyholder knows definitely what his policy will cost each year, and that it will not change with the varying experiences of the company.

[4] See Chapter XV.
[5] For a detailed discussion see Chapter XII

Comparison as Regards Control and Efficiency of Management.—It has been urged that stock companies are compelled by competition to issue policies at low rates, and tend through the self-interest of the stockholders to provide efficient management, and to exercise care in the preservation of the company's funds. Against mutual companies the objection has been made that the policyholders manifest little interest in the company, that the great majority of them do not vote when they have an opportunity, that many of them do not understand the business well enough to vote intelligently, and that they are so scattered and so completely out of touch with one another that concerted action on the part of large numbers of them is virtually impossible. Control, therefore, remains in the hands of a few individuals whose interests may or may not coincide with those of the main body of policyholders. On the other hand, it has been argued that mutuals are forced by competition to maintain efficiency of management in order to pay dividends regularly and thus reduce the policyholder's average annual payment to, or below, the level of stock rates; that mere fear of the exercise of the power of the policyholders, should they be aroused to action by the development of a crisis in the company's affairs, will restrain managements from extremes to which they might be tempted to go if control were secured through ownership of a majority of the company's stock. It is also said to be easier for unscrupulous persons to gain control through the purchase of a majority of a company's stock than to attain the same end through the votes of policyholders. Mixed companies have been said to offer peculiar temptations in this respect, since the control is usually lodged with the stockholders, yet the greater portion of the profits is ordinarily supposed to revert to the policyholders. Again, there might be a temptation to so manage the company's investments or expenditures that some other corporation, in which those in control of the life company are also interested, will profit at the expense of the policyholders. At this point it should be stated that much has been done by state legislation and supervision toward removing the possibilities of this and other irregularities.[6]

Policies.—*Term Insurance.*—The usual forms of contracts issued by legal-reserve companies are term, ordinary life, limited-payment life, and endowment insurance policies. A term insurance policy is a contract that furnishes protection against death for a limited period of time, such as five, ten, or twenty years. The face value of the policy is payable only if death occurs within the term of years. Noth-

[6] See Chapter XV.

ing is payable if the insured person survives the stipulated period. They are issued on the annual level-premium plan, whereby the premium remains the same for each year of the term. They often contain a renewable clause, or a convertible clause, or both. The renewable feature means that at the end of the term the insured person may renew the policy for a similar term by paying the premium for such a contract at the age then attained by him. The convertible feature means that the insured may, within a stated number of years, exchange the contract for an ordinary life policy or some other higher-premium form, by paying the premium for the converted policy at the age then attained by him, or as of the original date if back premiums are paid according to the contract. No medical examination is required, either for renewal or for conversion. What makes these features particularly valuable is the provision against the danger that the insured may under-estimate the period during which he needs protection, and may be unable to pass the medical examination at the expiration of his term policy. Term insurance is particularly useful in that it furnishes the maximum of protection over a limited period of time for the minimum outlay of cash on the part of the policyholder. Young professional or business men who need cheap protection for their families when just starting their careers; authors, students, and others who wish to protect relatives who are assisting them to get a start in life; promoters or experts who need, or whose firms need, protection on their lives during the time of some temporary business hazard; persons making regular deposits with a savings bank or building and loan association and who wish to hedge against the possibility of the saving period's being cut short by death,—these are some of the persons whose circumstances are such that term insurance may prove advantageous.

The disadvantages of the plan are that the insured may outlive the term and be dissatisfied with life insurance because he has nothing to show for what he has paid to the company. It is true that he has had the protection, but this is often not understood, or if understood it is not appreciated, especially when the insured learns that his neighbor was able to surrender his policy, which was on some other plan, for a cash consideration. It is also easy for an insured person to forget or to neglect to take advantage of the privilege of renewing or converting the policy. The company cannot be criticized in this case if it fails to make a strenuous effort to have him exercise his conversion or renewal privilege, especially if a considerable period has elapsed since his original medical examination, because he may

no longer be a good risk. Furthermore, the older a person is when he takes a policy providing permanent protection, the higher will be his annual premium. Also, term insurance carries no savings feature. And finally, term insurance to extend beyond age 60, or 65, is ordinarily not granted. Except as a means of providing temporary protection at low initial cost, therefore, term insurance should not be recommended.

From the point of view of the company and the agent, term insurance presents several disadvantages. The amount of money that the agent receives as a commission on the sale of a term policy is less, ordinarily, than that on the sale of a higher-premium policy of the same amount. Of course, he may receive another commission if the insured exercises the conversion privilege. Both agent and company suffer when persons are dissatisfied at the expiration of their term policies, and for want of understanding of the matter, refuse to become interested in further insurance. Also, the mortality experience on term policies has often been less favorable than on other forms.

Ordinary Life Insurance.—Ordinary life insurance is often referred to as "whole-life," or "straight-life," insurance. An ordinary life policy provides, in exchange for uniform premiums throughout life, for payment of the face amount upon the death of the insured, whenever that event may occur. A limited-payment life policy also covers the whole term of life, but premiums are limited to a definite period. Both insure against an event that is certain to occur at some time. Ordinary life policies are issued on the annual level-premium plan, whereby the premium remains the same throughout life. The principal advantage of this form of contract is that it furnishes permanent protection at the lowest annual premium rate. Persons whose incomes are small and who wish a considerable amount of permanent protection, or wealthy persons who desire low-premium protection so that they may use the greater portion of their funds in their own business enterprises, find the ordinary life policy a very attractive contract. It also combines a considerable amount of saving with insurance, as a considerable reserve fund must be accumulated on this type of policy, the greater part of which fund, after the policy has been in force a few years, is returned to the policyholder should he wish to discontinue the insurance. The principal objection to this form of contract is that the premium payments must continue throughout life if the full face amount is to be paid upon death. Several options granted the insured should he desire to stop paying

the premiums, however, such as allowing dividends to mature the policy as an endowment, tend to overcome, for the most part, the force of this objection. This contract is the most popular one offered by American life companies.

Limited-payment Life Policies.—The limited-payment life policy serves to meet the needs of the person whose income is sufficient to pay higher premiums than are required to provide an adequate amount of protection under the ordinary life plan, and who desires to finish paying for a permanent policy during the income-producing years of his life. It is important to observe that under this contract, the face value will not be paid until death. The principal difference between this form of insurance and that of ordinary life is that in ordinary life the premiums continue throughout the lifetime of the insured, whereas under the limited-payment plan the annual premiums, though higher, continue for a limited number of years only. The "twenty-payment life policy," for instance, is one on which twenty annual premiums are paid, after which the premiums cease and the policy continues in force until the death of the insured, when the face value will be paid by the company. When all of the premiums have been paid, the policy is said to have been "fully paid." It is obvious that the shorter the premium-paying period is, the greater must be each annual premium. The twenty-payment life is the most popular of the limited-payment life policies, although single-premium, five, ten, fifteen, twenty-five, and thirty-payment policies are frequently issued. The primary advantage of the limited-payment form to the insured person is that with a little extra effort and economy he can pay in full for his insurance during the years when money is most easily earned, and thus save himself from premium payments in old age when they would perhaps be more burdensome. The savings feature under this plan is greater than under the ordinary life plan, since the holder of a limited-payment policy may surrender it for a larger sum than could be obtained by surrendering a whole-life policy. Paid-up and extension benefits, which are described later, are also greater under the limited-payment plan.

Endowment Insurance Policies.—Endowment insurance policies, or "endowment policies," as they are often called, provide not only for the payment of the face value in the event of death at any time within a stated number of years, but also for the payment of the full face amount if the insured person survives the stipulated period. "Pure endowments" provide for the payment of the face amount only on survival to the end of some designated period, but these are

rarely issued. Endowment insurance policies are designed to meet an objection often raised against the types of contracts just discussed above. The objection is that under term, whole-life, or limited-payment life policies, the insured person must "die to win." The endowment insurance policy satisfies the desire on the part of the insured to receive the face amount of his insurance contract himself, if he survives a period. Under this type of contract, also, twenty years is the most popular period, although ten, fifteen, twenty-five, thirty, and forty-year endowments, and endowments maturing at some specified age such as 60, 62, 65, or 85, are not uncommon. At the usual insuring ages, the premiums for this type are higher than they would be for a limited-payment life policy of like amount issued at the same age and with a similar premium-paying period designated. Double and semi-endowments may also be issued, whereby the company agrees to pay double, or half, respectively, of the face amount in case of survival, only the face amount being paid in the event of death during the period. These are practically obsolete.

Though ordinary life, limited-payment life, and endowment insurance policies all furnish an incentive to save, and thus tend to inculcate habits of thrift, in the endowment contracts the savings feature is more in evidence. These policies provide not only a safe means of systematic saving, but also protection against the ever-present hazard that the saving period may be cut short by death before the desired amount has been accumulated. Economy is enforced by them through the necessity of meeting the premiums as they fall due. These policies, therefore, furnish an effective means of making provision for old age. An endowment maturing at age 65 or 70, the business retirement age, can be used to make the income during productive years cover the entire span of life. Thus by paying moderate premiums from, say, ages 25 to 65, a person may protect his children against financial loss due to his premature death, and may also provide a fund sufficient to care for his own needs and those of his wife, when he reaches the retirement age. Endowment insurance may also be used to accumulate funds for other purposes. Such a policy on the life of the directing genius of a corporation may be used to create a fund for the retirement of a bond issue. In fact, such an arrangement may be of assistance in placing the bond issue in the first place. The same may be said regarding the liquidation of a mortgage on a home. Endowments may also be used to provide funds for benevolent purposes, as for instance, when each member of a graduating class secures and maintains a small endowment policy

with his Alma Mater named as the beneficiary. To provide funds for sending children through college or starting them in business, and for countless other purposes, this type of policy is especially adapted.

The objections to endowment policies from the insured person's viewpoint are that the premiums are high if the endowment period is short; that the extra amount of each premium paid for the endowment feature will be sacrificed if the insured should die before the end of the period, as he would have received the same benefits in case of death had he selected a lower-premium policy; and that the insured might gain more by taking a lower-premium policy and investing in his own business the difference between its cost and the endowment premium. This last objection is open to question, a question which each person contemplating endowment insurance must, in the light of his own circumstances and experience, decide for himself.

Joint-Life Policies.—A joint-life policy is one payable upon the happening of the first death among two or more lives. Few companies issue such policies in the form of term insurance, but most of the larger companies offer joint-life policies on ordinary life, limited-payment life, and endowment forms. According to the endowment plan, the company agrees to pay the face value of the policy upon the happening of the first death, or at the end of a stated period if all of those insured under the contract survive. Joint-life policies are issued primarily to husband and wife, or to protect partnerships. For instance, if a firm consists of two partners, each may wish protection against the death of the other. Now the premium for a joint-life policy is lower than the sum of two premiums would be for separate insurances, each person insuring under the same type of contract for the same amount of insurance as provided by the joint-life contract. In many cases, however, partners prefer individual policies because each then has his own contract, and can change his beneficiary if he finds it necessary or desirable to withdraw from the firm.

Other Types of Contracts,—Annuities.—An annuity is a contract whose terms require the insurance company to pay a stipulated sum of money each year that one or more persons survive. The payments may be made annually, semi-annually, quarterly or monthly, and may continue throughout the life of the annuitant, or during a fixed term of years. Annuity payments made by the company usually begin one year after the date on which the annuity is purchased by the annuitant, in case the payments are to be made annually. Annuities

payable semi-annually, quarterly, or monthly, usually require the company to make the first payment at the end of the first half-year, quarter-year, or month, respectively. The calculation of values of annuities beginning at once is necessary, however, as explained in Chapter V. If the payments are to continue for the whole of life, the annuity is called a life annuity; if for a term of years, ceasing at prior death, a temporary life annuity. If the payments are to be made for a certain period regardless of whether the annuitant survives or not, the contract is called an annuity certain. If the annuity is to continue beyond this guaranteed period upon the annuitant's survival, it is called an annuity certain and continuous. If the purchaser pays the company for the contract in one lump sum, and if the first payment is to be made by the company at the end of one year, half-year, quarter-year, or month, respectively, it is said to be an immediate annuity. If the purchaser pays for the contract over a number of years, or by a single premium, the company making its first payment at the end of a period of years, it is said to be a deferred annuity. If the company pays upon the death of the annuitant, a proportion of the annuity payment corresponding to the interval which has elapsed since the last payment, it is said to be a complete annuity. In comparing the annuity rates quoted by different companies, it is important to note whether or not the rates are for complete annuities, especially if the annuity is to be paid annually, and is issued at one of the higher ages. Except in the case of annuities certain, payments by the company cease with the death of the annuitant. Some annuity contracts provide for the return of all or a part of the money paid in, without interest, but the return premium feature costs the annuitant what it is worth.

The Life Annuity.—The immediate, whole-life annuity is commonly used in this country. It is attractive primarily in this, that a person who has reached the retirement age with a moderate fortune and who has no one to whom he particularly cares to leave his estate, may avail himself of the annuity to secure a life income larger than could be realized with safety through the ordinary channels of investment. The older the annuitant is when he purchases such a contract, the higher will be his rate of return, because the fewer will be the number of payments the company must make before the contract is terminated by his death. The gain to the company from those annuitants who die soon after entering into the arrangement will, on the average, off-set the losses incurred by it through the continuation of the payments to those who live beyond the average after-lifetime.

Immediate whole-life annuities provide a safe and scientific method of spending one's principal so that the most can be secured from it each year, and at the same time it will assure an income as long as it is needed.

RETURNS ON ANNUITIES [7]

(American Companies)

Age		Female	Male	Age		Female	Male
40	Highest..	6 08%	6.29%	70	Highest...	12.47%	13 85%
	Lowest. .	5 29	5 40		Lowest...	11.39	11 91
	Average*.	5 53	5 83		Average.	11 94	13 27
50	Highest..	7.07	7.58	80	Highest...	20 20	22 47
	Lowest...	6 22	6 57		Lowest..	16.57	17 76
	Average...	6.51	7 03		Average..	17.60	19 35
60	Highest...	8 83	9 87				
	Lowest.. ..	7.89	8 55				
	Average..	8 31	9.21				

*The average is for the number of companies in each group.

It will at once be observed that the return is greater to male than to female annuitants, which means that the prices charged for annuities on females are higher than for males at the same ages. This is made necessary by the superior vitality of females at the higher ages.[8]

From the table of rates given above it is evident that annuities are not highly remunerative purchases at the lower ages. It is at the higher ages, where the return exceeds, say, 8 per cent, that the annuity makes its greatest appeal. Annuities are more popular, also, in countries where there are large accumulations of funds and where interest rates are relatively low. The large accumulations of funds in this country resulting from our prosperity during the war afford a favorable opportunity for the development of the annuity business, and this opportunity improves as the rate on ordinary investments declines.

[7] These figures are taken from a table prepared by M. Albert Linton, F.I.A., F.A.S., and published in the Annals of the American Academy of Political and Social Science, Vol. LXX, p. 23.

[8] NOTE:—A number of insurance manuals are published annually which give the prices of annuities quoted by the various companies. The Unique Manual-Digest, published by the National Underwriter Co., Cincinnati, Ohio, gives them in convenient form.

Miscellaneous Forms of Annuity Contracts.—There are a number of different types of annuities which have not as yet been discussed here, such as joint-life annuities, last-survivor annuities, reversionary annuities, etc. A joint-life annuity is one that is payable so long as two or more lives survive, and ceases upon the death of one of them. A last-survivor annuity is one payable to two or more persons, the payments to continue so long as any one of them is alive. A deferred annuity may be used to make provision for old age, since the annuitant may arrange to pay the company a stipulated sum each year during his income-producing period, the company agreeing to begin paying him an annuity upon his attaining a certain age. This plan of protection, however, is not so popular as endowment insurance. If the latter is used, the insured may or may not decide to use the proceeds to purchase an annuity if he survives the endowment period. It will cost an individual less per annum, however, to provide the same income in old age, in case he survives to need it, by means of a deferred annuity than by an endowment insurance policy whose proceeds are used to purchase an annuity.

The reversionary annuity is a combination of a life insurance policy with an annuity. It provides for the payment to a beneficiary of a life annuity commencing upon the death of the insured. In case the beneficiary dies first, the contract terminates. The purpose is frequently to protect the beneficiary at low cost against the death of the breadwinner. It is particularly applicable when a son desires to protect an aged parent. Much the same purpose is served, however, by the continuous monthly income policy, discussed later.

The annuity principle is rapidly becoming more and more popular in this country as the public becomes more familiar with the fact that it supplies the surest method by which one can provide the maximum guaranteed life income for the outlay of a given sum of money. The public, however, is taking advantage of it by the use of the income policy which is a modified form of annuity contract, rather than by the use of annuities proper.

Income Policies.—The income policy differs from an ordinary insurance or endowment policy in that the annuity principle is applied as a means of settlement rather than the payment of the proceeds to the beneficiary in one lump sum. In other words, under the income policy plan, a stipulated income will be paid to the beneficiary, or, more rarely, to the insured, in the case of an endowment, by the company at regular intervals, beginning at the time of occurrence of an event insured against. Numerous variations of the income

method of settlement exist, such as those provided in installment policies, guaranteed interest bonds, etc. An installment policy is one whose proceeds are paid in installments of a definite amount and covering a definite number of years. Ordinary policies now give the beneficiary an option of either a lump sum payment or settlement in the form of installments, or "continuous installments" in which the payments are certain for a period and continuous for the after-lifetime of the beneficiary. The company may also agree to give a "guaranteed interest bond" in settlement, interest on which will be paid at a specified rate for a definite number of years, or other status, such as the beneficiary's life, at the end of which the principal will be paid. The continuous installment policy, or life income policy, however, is now quite frequently used when settlement by the income method is desired. The payments made by the company under this type of policy are usually guaranteed for a specified period, ordinarily twenty years, and then continue so long as the beneficiary survives that period. Since the income is certain to the estate for twenty years regardless of whether the beneficiary lives or not, children will be protected until they are self-supporting. For the guaranteed benefits, too, the beneficiary may be changed, or a cash surrender value may be demanded in case it is necessary or desirable to discontinue the contract. In case the original beneficiary dies prior to the insured, the premiums will be reduced. Naturally, a new beneficiary cannot, without special arrangements, be substituted for the annuity benefits beyond the guaranteed period. These policies have been further refined to meet the needs of beneficiaries with the result that they are now generally made payable monthly, and are frequently referred to as "monthly income" policies. The most complete form of monthly income policy provides not only for payments certain for a period and continuous to the beneficiary should she outlive the insured, but also contains a deferred annuity provision under which the income commences upon the insured's living a specified number of years, or to some stated age, such as age 60 or 65. In the latter event, the income is paid to the insured during his remaining lifetime. Total and permanent disability features are also frequently added. Thus the contract is adapted to furnish complete protection to a husband and wife, and also protection during minority to any children living at the death of the insured, or at the time his endowment matures, if his policy is an endowment insurance contract.

The Income Option in Ordinary Insurance Policies.—Most in-

surance policies now being issued contain a settlement option whereby the proceeds may be paid by the company in the form of a monthly income, certain for a stipulated period regardless of the survival of the designated beneficiary, and continuous for so long a period as the beneficiary may survive the guaranteed period. The guaranteed period is usually twenty years, although other periods are not infrequently provided. The difference between this policy and the monthly income policy is that this provides for a definite face value which may be paid in cash, when it becomes a claim, or used to purchase an annuity certain and continuous, as may be desired, the size of each of the annuity payments to be made by the company depending upon the age attained by the beneficiary when the payments begin and therefore unknown at the date of issue. The monthly income policy provides at the time the policy is issued for an income to the beneficiary of a fixed amount, regardless of the beneficiary's age when the policy becomes a claim. Under the regular insurance policy the older the beneficiary is when the proceeds fall due the larger will be the income per month, unless the age attained is so great that the guaranteed period will carry the payments out to the extreme length of life indicated by the mortality table in use. Also, the regular insurance policy permits the insured to change the beneficiary at will, if he reserves this right when applying for the policy, whereas the monthly income policy permits this change only as regards the guaranteed payments. The insured may exercise the income option at the time he applies for the insurance, or at any time during the continuance of the policy if he has reserved the right to change the beneficiary. The option may also be exercised by the beneficiary upon the death of the insured.

The Primary Uses of Income Policies.—Enough has been said in describing the main types of income policies to indicate that their general purpose is to enable the insured to reach out into the future and thus take upon himself the burden, which would otherwise rest upon his dependents, of safeguarding the proceeds of his life insurance, and of providing for a proper distribution of it. The income provision enables him to "insure his insurance"; that is, to make certain that the fundamental purpose of his insurance, namely, the protection of his beneficiary after his death, will be realized. Except when life insurance is used for business purposes, or when the amount is small, the payment of the proceeds in one lump sum, sometimes called the "lump-sum payment plan," leaves much to be desired. If the beneficiary succeeds in safeguarding the principal, it must

ordinarily be done at a sacrifice of current income, since safety as regards investments does not usually go hand in hand with a high rate of return. In this case, then, the beneficiary will leave an estate, and this may not have been the insured's intention. In other words, the fruits of the insured's sacrifices may ultimately pass to persons in whom he had little or no interest at the time he negotiated the contract. Furthermore, the beneficiary may have preserved the principal, not because she wished to leave an estate, but because she feared to spend any part of the principal lest the latter fail to last out her lifetime. Beneficiaries may also purchase annuities, but this is not often done unless the insured has arranged for it in advance by exercising the continuous installment or income option, or by taking an income policy.

In the above statement, it was assumed that the beneficiary would succeed in preserving the principal. It is known, however, that in many instances insurance funds left to beneficiaries in lump sums are lost through unfortunate investments or poor management within a short time following the death of the insured. In insurance literature on this subject it is not uncommon to find the beneficiary of a life insurance policy payable in one sum, referred to as "a shining mark for a mining shark." The greatest objection to the lump-sum payment plan when applied to family protection, therefore, is the danger of losing the money through poor investments or through an attempt to maintain social standards more expensive than can be afforded. The fundamental purpose of family insurance, namely, to provide an income to dependents as long as it may be needed, may not be fulfilled, therefore, when the proceeds are made payable in one lump sum. Income policies make the insurance company the administrator of the insurance fund, thus guaranteeing a safe and economical administration of it, and the carrying out of the original intentions of the insured. The beneficiary benefits in still another way in that she is freed from all worry concerning the management of the insurance fund and the necessity of passing judgment on proposed investments. She is also relieved of the court costs, taxes, embarrassing delays, and attorney's fees so often involved in settling estates in the ordinary manner. The beneficiary is also said to profit by the tendency of the income plan to reduce policy loans, since the insured is more likely to view these as being in the nature of a loan against the beneficiary's future income. It is also thought that the insured is likely to provide a greater amount of protection if he translates into terms of income his few thousands of insurance and

sees them as woefully inadequate to provide proper support for the beneficiary.

The income policy, however, is not urged to the entire exclusion of the lump-sum plan. Where policies are taken for business purposes, such as those taken by corporations on the lives of their officers, the income plan would probably not be feasible. Even in family protection, it might be well to have, say, 10 or 20 per cent payable in a lump sum to provide the required amount of ready cash upon the death of the head of the household.

It might also be stated that many persons now arrange to have their insurances paid to a trust company under an agreement whereby the latter will manage the funds for the benefit of designated beneficiaries. Such an arrangement overcomes many of the objections to lump-sum payments. The trust company makes a charge for this service, however, and in case the funds are lost, the assets of the trust company are not available to the losers unless it can be shown that the company failed to use due care and diligence in managing the funds. A claim against an insurance company, on the other hand, is a direct claim on all of its resources.

CHAPTER III

Interest and Probabilities

Interest.—When a company issues insurance policies on a large number of lives, it may be sure that the latter will not all fail immediately. While it cannot forecast the time of death of any individual life, it can tell, with some degree of certainty, the approximate number of the group that will fail in each successive year until all may be expected to have died. From this it follows that the company may estimate about how many premiums it will receive from the survivors of the group each year, and consequently, about how many premiums may be expected before the claims arise. Money paid in as premiums by the group of individuals, therefore, need not all be held as ready cash to pay claims, but may be invested, some in securities that may be quickly liquidated, and some in assets that mature at some future time, which need not be so readily convertible into cash. Now, since the company invests the money paid in as premiums and earns interest thereon, it is only fair that it should estimate the interest which it expects to earn, and discount the funds collected from the policyholder by quoting him a lower premium rate than would otherwise be necessary. This is exactly what the companies do; hence, it is necessary to understand a few of the elementary operations in determining compound interest and discount, before attempting the calculation of insurance premiums. Likewise, when an insurance company receives a sum of money from an individual who has just purchased an annuity, it may immediately invest the same at the prevailing rates of interest. Knowing this, the company is able to quote lower rates on annuities than would be necessary if it were compelled to forego investments and keep the ready cash on hand. The calculation of premiums for annuities, then, also necessitates a knowledge of interest and discount calculations.

The company, in estimating the rate of interest which it expects to earn, must be conservative, for its contracts may remain in force for many years, during which the interest rate that can be safely

earned on investments may fluctuate between wide margins. Three and 3½ per cent for participating premiums, 4¼ or 4½ for non-participating, are the prevailing rates assumed by American companies; we shall use, for illustrative purposes, 3 per cent.

There are four compound interest tables with which the person who wishes to understand this subject must be familiar. They show (1) the sum or amount to which a principal sum of one invested for a number of years or other units of time will increase at a specified rate of interest per unit, (2) the present value which, at a specified rate of interest, will accumulate to a unit at the end of a given number of years (the reciprocal of 1), (3) the accumulated value of an annual payment of one unit, made at the end of each year for a specified period, and (4) the present value of the same series of annual payments. In this volume, compound interest and compound, or "true," discount primarily are considered, because the companies reinvest the money which comes in to them as interest earnings, and thus earn interest upon interest.

I. *Accumulations.*—Taking up the first of these, if $1 is invested at 3 per cent for one year, the amount of both principal and interest at the end of the year will be, 1×1.03, or $1.03. If the $1.03 is now invested for another year, the amount of both principal and interest at the end of the second year will be, $(1.03)^2 = 1.0609$. If the $1.0609 is again invested for another year at 3 per cent, the amount will be, $1.0609 \times 1.03 = 1.092727$, which is the cube of 1.03. It is thus observed that the amount of 1 at the end of one year with interest at 3 per cent is 1.03, at the end of two years if the same rate is compounded annually $(1.03)^2$, and at the end of three years, it is $(1.03)^3$, and so on. Thus a table showing the values of 1.03 to the first, second, third, fourth, etc., power, will show the amount of $1 at the end of one, two, three, four, etc., years, if interest is at 3 per cent. Also, the amount of any principal sum invested for any number of years, or other units of time at 3 per cent compounded annually could be found by multiplying that principal by 1.03 raised to the power equal to the number of years. If S represents the amount, or sum, at the end of the period, i the rate of interest, P the principal invested, and n the number of years, the general formula becomes, $S = P\ (1 + i)^n$.

In the Appendix will be found the amount of 1 at the end of each unit of time for one hundred units at several different rates of interest. If it is desired to find the amount of, say, $75 at the end of eleven years, with interest at 3 per cent compounded annually,

multiply the figure opposite the eleventh year, or 1.38423387 by 75, to get \$103.82, the amount to the nearest cent.

II. *Present Values.*—The second table, showing the present value of a sum due at a designated time in the future, may be formed by reversing the process of constructing the first one. In the first, the principal was multiplied by one plus the rate of interest raised to the power equal to the number of time units in the period, to find the amount at the end of it. Given the amount at the end of the period, the principal may be found by dividing the amount by one plus the rate raised to the same power. Again using 3 per cent, the amount of \$1 at the end of one year was found to be $1 \times 1.03 = \$1.03$. Given the amount, \$1.03, to find the principal, the operation is $1.03 \div 1.03 = \$1$. If the amount due 2 years hence is \$1.0609, the present value of this amount with interest at 3 per cent compounded annually is $1.0609 \div (1.03)^2 = 1.0609 \div 1.0609 = \$1$. Now suppose \$1 is to be received one year hence. To find how much the \$1 is worth at the present moment, $1 \div 1.03 = \$0.97087379$. If \$1 is to be paid 2 years hence, its value at the present moment is $1 \div (1.03)^2 = 1 \div 1.0609 = \$0.94259591$, because \$0.94259591 put at interest for 2 years at 3 per cent compounded annually amounts to $0.94259591 \times (1.03)^2 = \$1$. Continuing to divide 1 by $(1.03)^3$, $(1.03)^4$, etc., for, say, 100 operations and tabulating the results, Table II, in the Appendix may be reproduced; that is, if the values of 1.03 $(1.03)^2$, $(1.03)^3$, etc., are given in Table I, Table II may be derived from Table I by dividing unity by each of the figures in Table I. It is customary to use the symbol v^n to represent these values. Thus at 3 per cent, $v = 1 \div 1.03 = 0.97087379$, $v^2 = 1 \div (1.03)^2 = .094259591$, and $v^n = 1 \div (1.03)^n$.

$$
\begin{aligned}
\text{Now, } 1 \div 1.03 &= 1 \times .97087379; \\
1 \div (1.03)^2 &= 1 \times .94259591; \\
\text{and, } 1 \div (1.03)^{100} &= 1 \times .05203284.
\end{aligned}
$$

If any sum is divided by 1.03, the result will be the same as if it were multiplied by .97087379, which is the value of v if $i = .03$. So any sum divided by $(1 + i)^2$ equals the same sum multiplied by v^2, which is .94259591, according to Table II, if i is .03. To find the present value of any sum due at the end of any number of years, multiply the sum by the ascertained value of v^n. Since Table II shows the values of v, v^2, v^3, etc., for 100 units of time, at various rates of interest, it may be used to find the present value of any

sum due within 100 units, at the interest rates indicated at the heads of the columns. Thus suppose $1000 will fall due 20 years from date. What is it worth to-day if interest be taken at 3 per cent compounded annually? From Table II, the present value of 1 due 20 years hence is 0.55367575. The present value of $1000 would, therefore, be 0.55367575 × 1000 = $553.67575. To check the result, find the amount of $553.67575 invested for 20 years at 3 per cent, compounded annually. It is, of course, 553.67575 × 1.80611123 = $1000.

III. *The Amount of Annual Payments.*—Regarding the third table, the amount of a number of annual payments at the end of any given period may be found by changing slightly the method of procedure followed in the first. Assuming that the payments are made at the beginning of the year, to make them correspond to annual insurance premiums which are paid annually in advance, at 3 per cent, the amount of $1 at the end of one year will be 1 × 1.03, or $1.03.

At the beginning of the second year, however, another payment of $1 will be made, which may be invested, so that a total of $2.03 will be at work earning interest during this year. The amount at the end of the second year would thus be 2.03 × 1.03, or $2 0909. At the beginning of the third year, another payment of $1 will be made, so that $3.0909 is at work during this year, and at the end of this year the amount would be 3.0909 × 1.03, or $3.183627, and so on indefinitely. It is possible to construct a table showing the sum to which $1 *per annum,* paid at the beginning of each year, will increase at compound interest, in any number of years sufficiently great for all ordinary requirements.

The process of forming such a table from a table showing the amount of 1 principal, is merely one of addition. From Table I which was previously explained, take the following figures in the 3 per cent column:

1.0300000, amount of 1 at end of 1 year.
1.0609000, amount of 1 at end of 2 years.
1.0927270, amount of 1 at end of 3 years.
1.1255088, amount of 1 at end of 4 years.
1.1592741, amount of 1 at end of 5 years.

The first figure is the same as the first figure in the table showing the amount of 1 per annum. Now add the first two numbers above, and the result is 2.0909, the second figure in Table III. Then add

the third figure, 1.092727, above, to the 2.0909, and the result is 3.183627. To this, add the fourth number, or 1.1255088, and the result is 4.3091358. Table III is thus easily constructed from Table I.

It is important to note that this table can, with certain modifications, be used to find the amount of annual payments made *at the end* of each year. Thus if an agreement to pay $1 one year hence, instead of at once, $1 two years hence, etc., for a period of years, is made, clearly the person receiving the payments cannot begin earning interest on them until one year has elapsed. Then he may set the first $1 at work earning interest, and at the end of the *second* year it will amount to $1.03, if interest is at 3 per cent. At the end of the second year, however, another $1 is received, so that $2.03 may be put at interest during the third year. So, 1 per annum payable at the end of each contract-year, would amount to only 2.03 at the end of the second year, instead of $2.0909, the amount as shown in this table. This table, therefore, may be used to find the amount of payments made at the end of each year by taking the amount at the end of a period one year shorter, and adding one payment to it. Thus suppose it were desired to find the amount of $25 per annum, payable at the end of each year, for 40 years, at 2½ per cent. Since the first payment is not made until one year hence, interest can be earned for 39 years only. From Table III, the amount of 1 per annum in advance at 2½ per cent for 39 years, is 66.40255354. There are to be 40 payments, however, the last of which is made at the end of the period. This last payment of 1, added to 66.40255354, yields 67.40255354, the amount of 1 per annum payable at the end of each year over the period at 2½ per cent. Therefore, $25 \times 67.40255354 = \$1685.06$, the amount of $25 per annum payable at the end of each year of the 40-year period.

IV. *The Present Value of Annual Payments.*—The fourth table, by which the present value of annual payments may be found, remains to be considered. It is often referred to as the method for finding the present value of an annuity certain. Using 3 per cent for illustrative purposes as before and assuming that annual payments of $1 per annum at the end of each year are to be made, the present value of the payments may be easily derived from Table II. For example, what is the present value of an annuity certain of $1 per annum for 3 years? The present value of the first $1 due one year hence is $1 \div 1.03$, or $0.97087379, as shown in Table II. The present value of the second payment of $1 due 2 years hence is $1 \div (1.03)^2$, or $0.94259591. And the third payment is worth $1 \div (1.03)^3$, or

\$0.91514166, at the present time. The present value of all three payments is, of course, the sum of the present values of each taken separately, or \$2.82861136. To construct Table IV, therefore, take the first figure in Table II as the first one in Table IV, add the first two figures in Table II for the second in Table IV, and so on.

Table IV may be used to find the present value of an annuity certain, of any amount, by multiplying the present value of an annuity of 1 by the annual payment. Thus, referring to Table IV, the present value of an annuity certain of \$1000 for 30 years at 3 per cent, is $19.60044135 \times 1000 = \$19600.44135$.

Tables have been constructed showing the annuity certain whose present value is 1, so that the annuity certain which a given sum will purchase may be readily determined. With but little trouble, however, Table IV may be used for this purpose. For example, suppose we have \$10,000 in cash and wish to pay 20 annual installments with it, the first installment being due one year hence. If we allow interest at 3 per cent compounded annually on the money remaining in our possession, what should be the amount of each installment? From Table IV, the present value of \$1 per annum for 20 years is \$14.87747486. In other words, if we had \$14.877 we could afford to pay 20 annual installments of \$1 each. We have, however, \$10,000. Therefore, each installment should be as much greater than \$1 as 10,000 is greater than 14.87747486. Stated in the form of a proportion, $10{,}000 : 14.87747486 :: x : 1$. Dividing 10,000 by 14.87747486, the annual payment is found to be \$672.1571. Hence, the rule is, divide the sum on hand by the present value of 1 per annum for the period over which the installments are to be paid, and at the stated rate of compound interest.

If the payments are to be made *annually in advance*, that is, if the first payment is due at once instead of at the end of one year, their present value may be found by varying slightly this use of Table IV. Suppose the problem is to find the present value of 20 annual payments of \$1 each in advance (sometimes called an annuity-certain due as distinguished from an immediate annuity certain). Obviously the first payment of \$1 at once is worth \$1. Then there will be 19 more payments, the first of which is due one year hence. The present value of \$1 per annum for 19 years, plus the \$1 paid at once, is the present value of the 20 payments.

Observations.—A few observations concerning the uses of interest tables may be of value. They may be used to find the amount of any sum put at interest for a given period, the amount of periodical

payments, the present value of a sum due in the future, and the present value of periodical payments. Furthermore, the rate of interest on a sum which has been accumulated by periodic payments over a number of years may be estimated from them accurately enough for ordinary purposes. For instance, suppose a man learns that the investment of $19.09 per annum, over a period of 20 years, now amounts to $672.42. Dividing 672.42 by 19.09, he finds that for each $1 per annum invested, he now has approximately $35.20. What rate of interest did he receive on his investment? Turning to Table III, and looking down the first column to the end of 20 years, then out to the right, 34.719 is found in the 5 per cent column to be nearest to the $35.20. His rate of interest has been approximately 5⅛ per cent, compounded annually. The present value of a $1000 bond due 20 years hence may also be determined according to any assumed interest basis. Thus suppose the bond pays 5 per cent, or $50 per annum, and is quoted to yield 3 per cent, what is the quoted price? The corporation issuing the bond promises to pay $50 one year hence, another $50 two years hence, another three years hence, etc., the present values of which at 3 per cent may be found from Table IV, and added to the present value of the $1000 face due 20 years hence, also discounted at 3 per cent (see Table II), to obtain the quoted price. If this operation were continued for many bonds at different yields and rates of interest, and different maturity dates, a table of bond values could be formed. Ordinarily, bond table values are simply the present values of the principal and interest promised by the issuing corporation, at assumed rates of discount. Furthermore, the rate of interest on accumulations may be determined by the use of Tables I, or II. Thus suppose a person is told that he may deposit $553.68 in a savings bank, and that at the end of 20 years the sum will have amounted to $1000. What rate of compound interest does the bank expect to pay? Dividing the deposit by the 1000, the result is 0.55368. From Table II, this figure is found opposite the twentieth year in the 3 per cent column, indicating the rate to be 3 per cent compounded annually. The method, then, is to divide the principal invested by the amount to which it will increase in a given period, and find the figure in Table II nearest to the result, and opposite the year corresponding to the number of years in the period. If in this operation Table I is preferred, divide 1000, the accumulation, by 553.67575, the deposit, and the result, or 1.8061, is found opposite the twentieth year in the 3 per cent column of Table I, indicating 3 per cent, as before.

Clearly, also, the interest tables described above may be used to find the approximate period of time required for a given sum, or a stipulated annual payment, to increase to some other amount at a certain rate of interest. In the same manner the length of time required for money to double itself at a stated rate of interest compounded annually, may be determined. How long, for example, will it take for $1 to double itself at 3 per cent? From Table I, 1 amounts to 2 in a little over 23 years. For those who prefer it, the development of the various algebraic expressions is herewith given:

$$S = P(1 + i)^n. \qquad (1)$$

$$P = \frac{S}{(1 + i)^n}. \qquad (2)$$

$$(1 + i)^n = \frac{S}{P}. \qquad (3)$$

$$1 + i = \sqrt[n]{\frac{S}{P}}, \quad \text{or} \quad \left(\frac{S}{P}\right)^{\frac{1}{n}}. \qquad (4)$$

$$i = \left(\frac{S}{P}\right)^{\frac{1}{n}} - 1. \qquad (5)$$

From (3), $n \log (1 + i) = \log S - \log P. \qquad (6)$

$$n = \frac{\log S - \log P}{\log (1 + i)}. \qquad (7)$$

$$\log n = \log [\log S - \log P] - \log [\log (1 + i)]. \qquad (8)$$

If

$$P = 1, \quad \text{and} \quad S = 2, \text{ from (7);}$$

$$n = \frac{\log 2 - \log 1}{\log (1 + i)}. \qquad (9)$$

But,

$$\log_e 2 = .30103 \times 2.3026 = .69315,$$

$$\log_e 1 = 0,$$

$$\log_e (1 + i) = i - \frac{i^2}{2} + \frac{i^3}{3} \cdots$$

Therefore,

$$n = \frac{.69315}{i}\left(1 + \frac{i}{2} - \frac{i^2}{12}\right)$$

$$= \frac{.69315}{i} + .35, \text{ approximately.} \qquad (10)$$

And if

$$i = .03,\ \frac{.69315}{.03} + .35 = 23.455 \text{ years},$$

as was discovered approximately from the table.

The rough rule commonly used to find the number of years in which money doubles itself is, divide .69 by the rate per cent of interest.

These tables should not be confused with *simple* interest tables. A few expressions of simple interest values are as follows:

$$S = P(1 + ni),$$

$$P = \frac{S}{1 + ni},$$

$$i = \frac{S - P}{nP},$$

$$n = \frac{S - P}{iP},$$

$$v = \frac{1}{1 + i},$$

The discount,

$$\begin{aligned} d &= 1 - v \\ &= 1 - \frac{1}{1 + i} \\ &= \frac{i}{1 + i}. \end{aligned}$$

These values of v and d apply to both simple and compound interest for one unit of time.

The tables may sometimes be used to advantage when payments are made at intervals less than one year and interest is compounded more often than annually. Thus suppose a man invests \$1000 at 5 per cent, payable semi-annually for 20 years, and also reinvests each of the semi-annual interest payments of \$25 on the same basis. What sum will he have accumulated at the end of the 20-year period? At the end of the first six months, his accumulated sum of both principal and interest will be \$1025, found by multiplying 1000 by 1.025, since .025 is one-half the annual rate of .05. At the end of one year, his accumulation will be 1025×1.025, or $1000 \times (1.025)^2$, or \$1050.625. This figure may also be found by consulting the 2½ per cent column of Table I. By multiplying the figure in that column opposite the

second year, or 1.050625, by 1000, the same result will be obtained. Extending this reasoning and taking 2.68506384, the figure opposite the fortieth unit of time in the 2½ per cent column and multiplying by 1000, the answer to the problem, or \$2685.06, at once appears. Again, suppose a company pays \$25 every six months for 20 years, and that the person receiving the payments immediately invests them at 5 per cent compounded semi-annually. What sum will he have accumulated from the payments of \$25 alone at the end of 20 years? From Table III, \$1 per annum at 2½ per cent, payable at the end of each unit of time, amounts to \$67.40255354 at the end of 40 units. $67.40255354 \times 25 = \$1685.06$ the amount of \$25 every six months for 20 years at 5 per cent per annum, compounded semi-annually. Furthermore, suppose a \$100, 5 per cent bond, with interest payable semi-annually and having 20 years to run to maturity, is quoted to yield 4 per cent, what is the quoted price? The present value of \$100 due 40 units of time hence at 2 per cent is \$45.289042, from Table II. And the present value of \$2.50 every six months for 20 years is 27.35547924×2.50, or \$68.3886981, from Table IV, the 2 per cent column at the end of 40 years. The price quoted to yield 4 per cent, therefore, is the sum of these two values, or \$113.677740.

It will be noticed that in Tables I and III, the higher the rate of interest, the greater will be the amount to which a certain principal, or annual payment, will accumulate over a given period of time. Also, in Tables II and IV, it will be observed that the higher the rate of interest, the lower will be the present value of a sum due at a stated future time, or of stipulated annual payments over a period of time. Furthermore, if interest is payable semi-annually or quarterly, the return on an investment of a given principal is slightly greater than if interest at the same annual rate were only payable at the end of each year. This leads to a discussion of the "effective" rate of interest.

Nominal and Effective Rates of Interest.—If the interest rate is 5 per cent per annum, but is payable semi-annually, the interest payment received at the end of the first six months may be immediately reinvested and made to bear further interest during the remaining six months of the year, whereas if the interest is paid only at the end of the year, no interest payment can be reinvested until the beginning of the second year. Thus if \$100 is invested for one year at 5 per cent payable semi-annually, at the end of six months \$2.50 interest will be received. This \$2.50 may be immediately reinvested, and at 5 per cent, compounded semi-annually, it will earn \$0.0625

by the end of the year. At the end of the year another regular semi-annual payment is due, so that the total interest earned on the $100 during the year, compounded at 5 per cent semi-annually is:

$2.50,	earned on the $100	during the first six months,
0.0625,	earned on this $2.50	during the second six months,
2.50,	earned on the $100	during the second six months,

or 5.0625, total amount of interest earned.

It is obvious, therefore, that while the *nominal* rate, that is, the rate named in the note, bond, or other document, is only 5 per cent, the rate actually earned because of the semi-annual feature is 5.0625 per cent. The rate actually earned, in this case 5.0625 per cent, is commonly spoken of as the *effective rate* of interest. Using j to represent the nominal rate, and i the effective rate, if interest is payable m times a year, the amount of 1 at the end of the year may be expressed as $\left(1 + \frac{j}{m}\right)^m$. Deducting 1, the principal, the expression for the effective rate of interest becomes, $i = \left(1 + \frac{j}{m}\right)^m - 1$. Or deducting 1 from the second figure in the $2\frac{1}{2}$ per cent column of Table I, if 5 per cent is the nominal rate, the effective rate on a semi-annual basis is .050625, or 5.0625 per cent. On a quarterly basis, the effective rate would be, 5.094534 per cent (found by deducting 1 from fourth figure in $1\frac{1}{4}$ per cent column of Table I). A few of the values are herewith set down:

CORRESPONDING EFFECTIVE RATES WHEN INTEREST IS PAID

Nominal Rates, Per Cent	Annually, Per Cent	Semi-annually, Per Cent	Quarterly, Per Cent
2 5	2 5	2 515625	2 523523
3 0	3 0	3 022500	3 033868
3 5	3 5	3 530625	3 546206
4 0	4 0	4.040000	4.060401
5 0	5 0	5 062500	5.094534
6 0	6 0	6 090000	6 136355
7 0	7.0	7 122500	7 185903
8 0	8.0	8 160000	8.243216

Probabilities in Life Insurance.—If it is possible for an event to happen in a ways and to fail to happen in b ways, all being equally

likely, the probability that it will happen in any one trial is, $p = \frac{a}{a+b}$; and that it will not happen, $q = \frac{b}{a+b}$. For example, suppose a coin be allowed to fall upon a table. The fraction representing the probability that it will come to rest heads up is $\frac{1}{2}$; and that it will rest tails up is also $\frac{1}{2}$. The sum of these two fractions equals 1, or $p + q = 1$. From this it appears that unity minus the probability that the event will happen expresses the probability that it will fail to happen; that is, $q = 1 - p$. Also, of course, $p = 1 - q$.

When we cannot determine *a priori* how many ways an event can happen, we may experiment by taking a number of samples, or a large number of trials or observations, to determine the number of times the event happens, and fails to happen, in the number of trials. Then we may apply a well-known theorem to determine the probability that the event will happen in any one trial; namely, if a given event happens r times in n trials, n being a very large number, the probability that the event will happen in any one trial is, $p = \frac{r}{n}$. The probability that this event will fail to happen in any one trial is, $q = \frac{n-r}{n}$. Thus, assume for the moment that from observations we have determined that out of 85,441 persons aged 30, 720, on the average, die within one year. The ratio of deaths to the total number is $\frac{720}{85441}$, or from the above theorem, the probability that a person now aged 30 will die within one year is $\frac{720}{85441}$. The ratio of survivals to the total number is $\frac{84721}{85441}$. The sum of these two fractions is unity, and unity minus one of them equals the other one. Allowing p_x to represent the probability that a person now aged x will survive one year, and q_x the probability that a person now aged x will die within one year,

$$p_x + q_x = 1, \quad p_x = 1 - q_x, \quad \text{and} \quad q_x = 1 - p_x.$$

Events are said to be independent when the happening of one of them is in no way influenced by the happening, or by the failure, of any of the others. The probability that two or more independent events will happen simultaneously or in sequence is the product of the probabilities of the single events. For instance, suppose two coins, a dime and a nickel, are pitched simultaneously. What fraction expresses the probability that both will come to rest heads up? Any

one of four joint events may be the result of this experiment. Both coins may rest heads up; both, tails up; dime, head up, and nickel, tail; or the reverse may take place. Using *H* for heads up, and *T* for tails up, the joint events may conveniently be set down as follows:

Dime	Nickel	
H	H	(1)
H	T	(2)
T	T	(3)
T	H	(4)

Now since each one of these four events is as likely to happen as any other one, it appears that $1/4$ represents the probability that both coins will rest heads up. Also, $1/4$ represents the probability that any one of the other events will result. The fraction, $1/4$, however, can be arrived at in a manner much simpler than that of setting down all possible results and counting the number of conditions that satisfy the requirements. For instance, the probability that the dime will rest heads up is represented by $1/2$. The same is true of the nickel. Now if $1/2$ be multiplied by $1/2$, the result will be $1/4$, the probability that both will come to rest heads up, as before.

Also, the probability that some one or other of a set of mutually exclusive events will happen, is the sum of the probabilities of the single events. From the illustration of the two coins, $\frac{1}{4}$ represents the probability of one of the joint events happening; $\frac{1}{4} + \frac{1}{4}$, or $\frac{1}{2}$, is the probability that either one of two of the joint events will happen. Likewise, the probability that one of a specified three of the four events will occur is $\frac{3}{4}$; and $\frac{4}{4}$, or 1, is the probability that one of the four events will occur.

If $1/4$ represents the probability that both coins in the above illustration will rest tails up, what fraction expresses the probability that there will be a head (one or more) up? The answer, of course, is $1 - 1/4 = 3/4$, since $1/4$ represents the probability that there will be no head up, $1 - 1/4$ must represent the probability that there will be at least one, since the sum of all the probabilities equals unity, as before. To check the accuracy of this conclusion, observe the number of instances in which one or more heads appear in the illustration.

Returning to life contingencies, suppose $\frac{718}{89032}$ represents the probability that a man aged 25 will die within one year, and that $\frac{720}{85441}$ is the probability that a man aged 30 will die in the same period. One of four events may happen to this pair within the year. They may both

die, both survive, the younger die and the older survive, or the reverse may take place. Arranging them in the same manner as the coins:

A (Aged 25). *B* (Aged 30).

$$\frac{718}{89032} \times \frac{720}{85441} = \text{prob. both will die.}$$

$$\frac{718}{89032} \times \frac{84721}{85441} = \text{prob. younger will die, older survive.}$$

$$\frac{88314}{89032} \times \frac{720}{85441} = \text{prob. older will die, younger survive.}$$

$$\frac{88314}{89032} \times \frac{84721}{85441} = \text{prob. both will survive.}$$

Further:

$$1 - \left(\frac{718}{89032} \times \frac{720}{85441}\right) = \text{prob. at least one will survive.}$$

$$1 - \left(\frac{88314}{89032} \times \frac{84721}{85441}\right) = \text{prob. at least one will die.}$$

$$\left(\frac{718}{89032} \times \frac{84721}{85441}\right) + \left(\frac{88314}{89032} \times \frac{720}{85441}\right) = \text{prob. one or the other but not both will die.}$$

Other probabilities might be shown, but enough have been given for illustrative purposes.

To avoid more formal discussions, and at the same time to illustrate a very practical principle, let us return to the illustration concerning the pitching of a single coin. The greatest uncertainty exists as to whether a coin which is freely pitched will come to rest heads up or tails up. In view of the fact that the chances in this case are even, however, it may be reasoned that if a coin were pitched repeatedly, it would tend to come to rest heads up in approximately 50 per cent of the trials. In actual experience, this is the case, the tendency often making itself evident in so few as 50 or 100 trials. If the number of trials be increased, the discrepancy between an equal number of heads and tails will, of course, become greater and greater, but the ratio of the number of heads to the total trials will approach 50 per cent more closely. Likewise, the statement that $\frac{720}{85441}$ represents the probability that an individual aged 30 will die within one year, would be of little practical value to the business man if it were applied to one individual only. Used as a ratio to determine the number of deaths expected in a large

group, however, it, together with other and similar ratios, becomes of the utmost importance. For instance, a life insurance company may safely insure 28,480 persons aged 30, acceptable for life insurance, for one year, on the presumption that not more than 240 of them will die within the year. In other words, it may reasonably expect that the ratio of actual deaths to the insured group will correspond closely enough for practical purposes to $\frac{720}{85441}$, the ratio in another large group, and which expresses the probability that a single member of the group will die within one year. It is well known that if enough trials are made or enough risks are taken, the actual experience expressed as a ratio will tend to coincide with the ratio indicating the probable experience.

The Valuation of Risks.—Assuming the number of risks to be such that actual experience will coincide closely enough with the probable for practical purposes, probabilities which have been accurately determined may be used to ascertain the values of various hazards. For instance, suppose that an individual is at such an age that the probability of death within one year is $\frac{1}{100}$. If a life insurance company agrees to pay \$1000 if this life fails within one year, the value of the risk is $\frac{1}{100} \times 1000$, or \$10. Neglecting interest, expenses, and profits, then, \$10 should be the premium charged by the company for assuming the individual's risk for one year. If the company writes, say, 100,000 of such contracts, it will have enough to assure the coincidence of actual to probable experience, and, therefore, may expect to pay 1000 claims of \$1000 each, or a total of \$1,000,000. If it collects \$10 from each one of the 100,000 persons as a premium, it will just have \$1,000,000 with which to pay the claims.

It is thus seen that the theory of probabilities may be used to find the value of life contingencies. If, however, events insured against are likely to be postponed for a considerable length of time in the future, interest must be taken into account when their *present* value is to be ascertained. If the insurance company in the above example were to collect the \$1,000,000 premiums in advance, and if it were not to pay any of the claims until the end of the year, it would gain \$30,000 in interest even if it only realized 3 per cent. If the company is to operate without profit as was provided in the original assumption, \$10 is too high a premium to ask of each person insured. The present value of \$10 discounted for one year at 3 per

cent, or 10 ÷ 1.03, or $9.71 (from Table II) is the proper premium, for this sum increased at 3 per cent for one year will amount to the required $10; that is, the company may collect $971,000 from the group of 100,000 persons at the beginning of the year, invest this sum at 3 per cent, and have $1,000,130, or a little more than enough to pay the claims at the end of the year.

Attention may now be given to the compilation and tabulation of data to which the laws of probability may be applied in determining the value of life contingencies.

CHAPTER IV

MORTALITY TABLES

We have seen that the ratio, $\frac{720}{85441}$, may be safely used to determine the probable number of deaths that will occur within one year in a group different in number from 85,441, but containing the characteristics of the group of 85,441. For observation of the death rate for insurance purposes, therefore, that group of lives should be selected which is most likely to contain all the characteristics of the group to which the results of the observation are to be applied. Thus, when mortality statistics are collected and tabulated to serve as the basis for determining the probable future death rate for life insurance purposes, they should be scrutinized searchingly for the sake of avoiding inaccuracies and of determining whether there is a sufficient resemblance between the lives from the records of which the original data were compiled, and those which the company expects to insure in the future. Records of past mortality amongst insured lives, therefore, are the most satisfactory source of data from which to compile such statistics.

Final tabulations of mortality statistics, for convenience in determining death rates, are called mortality tables. The most satisfactory sources from which to take data for the construction of such tables are, as noted above, the records of life insurance companies, though general population statistics have furnished a number of well-known tables. Recourse must be had to these when the experience of companies is so limited that insufficient data are available from their records. General population statistics were necessarily the basis of the first mortality tables.[1] There are several objections to such statistics as sources of data, though some tables based upon them are quite safe for insurance purposes, and the trend of mortality rates in the general population is a matter of great interest to actuaries. The objections are,—that, in many communities, registers of births and deaths lack thoroughness and accuracy; that, adjustments must

[1] See the author's "History of Life Insurance," pp. 17 et seq., for a discussion of early mortality tables.

be made when the number of births exceeds the number of deaths during the period under observation; that, immigration and emigration must be properly noted and appraised; and that, special influences, such as medical or other selection, which might affect the mortality of insured lives must be estimated. In other words, if observations are taken from general population records, the group of lives observed would not be similar to that which the company expects to write.

The essence of a mortality table consists in two columns of figures, one showing the number dying each year, and the other showing the number who survive, so that the rate of mortality is apparent. For convenience, a certain number of persons, such as 100,000, at an age low enough for all practical insurance purposes, is assumed as a starting point and called the radix. The number who die within one year, on the average, out of this group, and at that age, are set down opposite the number living at the beginning of the year. The deaths during the year are then deducted from the number starting that year, and the result is set down under the original number as the number living at the beginning of the second year. This process is repeated year after year until all are dead. The arrangement of the figures is shown in the Appendix where the American Experience table, which is now in general use in this country, is given.[2] The convenience of this arrangement becomes apparent when the table is used to any extent in determining the values of contingencies, as is brought out later.

While this method of arranging the table shows a hypothetical group of individuals entering a certain age and traces the history of this group year by year until all have died, it must not be assumed that a table is ordinarily constructed by observing 100,000 individuals aged 10 until all have died. A table can be constructed by observing previous rates of mortality. For example, suppose it is learned by an examination of the records of a number of life companies that, within the past few years, 6000 persons were insured at age 10 who did not lapse or surrender their policies within one year. For an investigation of this kind, years of unusually favorable or unfavorable mortality, might well be avoided. Now since insurance companies in this country require applicants to state their ages to nearest birthday, some of these persons were nearly six months younger than age 10 at date of insuring, some were nearly six months older, others at shorter periods of time under or over that age, and a few at the exact age of 10. The average age of

[2] See Appendix B.

entry into the group, nevertheless, will be sufficiently close to the exact age of 10 for practical uses. Of those in this group who die within one year following age 10, however, some will be only a little more than $9\frac{1}{2}$, while others will be nearly $11\frac{1}{2}$ years old at death, so that the average age at death will be $10\frac{1}{2}$ years, the same as would be the case if ages of death were taken as of last birthday. Suppose further that at the end of one year, following age 10, 42 of these persons are found to have died. It may be concluded that $\frac{42}{6000}$, or, reduced to decimal, .007, represents the probability that a person aged 10 will die within one year.

Suppose it is learned, on referring to the records, that 60 persons in a group of 7500 aged 11, died within a year; $\frac{60}{7500}$, or .008, would then represent the probability that a person aged 11 will die within one year. At age 12, suppose the probability of death in one year to be .009. The results of this operation, repeated until the yearly probabilities of dying have been determined for each age up to the age of the oldest life on the books of the companies, may be used to construct a table with any desired number as a radix. Difficulties may be encountered at the lower and higher ages because too few persons may be found there to permit generalization as to what might be expected in a larger group. Estimates may then be made and the results compared with general population statistics.

Having prepared the death rates, we may now proceed to the construction of the table as follows:

Assume 100,000, or any other hypothetical figure, as the radix at age 10. If 42 deaths occur within one year in a group of 6000, how many deaths might be expected if the number under observation were 100,000? The ratio of deaths to the total number may safely be considered as the same in both groups. Hence, $\frac{42}{6000} = \frac{x}{100{,}000}$, or $x = 700$ deaths. If .007 represents the probability of death at age 10, how many deaths may be expected out of 100,000 lives? From the application of simple arithmetic, $100{,}000 \times .007 = 700$, the number of deaths, as before. The number of deaths is then subtracted from the number assumed in the beginning, to find the number assumed to be living at the beginning of the second year. $100{,}000 - 700 = 99{,}300$ are assumed to be left alive at age 11 of 100,000 living at age 10. At age 11 the probability of death found above is .008. $99{,}300 \times .008 = 794$, the number dying at age 11, of the 99,300 assumed to have started that year. Similarly, $99{,}300 - 794 = 98{,}506$, the number assumed to be living at age 12. At that age, $98{,}506 \times 009 = 887$, the number of

deaths. It should be noted that the expression "dying at age 10" is used here to mean dying between ages 10 and 11. For convenient use, the results may then be tabulated as follows:

Age	Assumed Number Living	Number of Deaths During the Year
10	100,000	700
11	99,300	794
12	98,506	887
13	97,619	etc.
etc	etc	

While the above serves well enough for illustrative purposes, in an actual attempt at construction the resulting table would probably show much more irregular results. If sufficient data were available in the original investigation, this would probably not be the result; the more complete the data the smoother will be the resulting table. If the table, however, should be too rough for practical use, various statistical methods are available for smoothing it so that regular probabilities of death will be shown over the span of life. The process of smoothing the rough table is known as graduation.

Central Death Rates.—The symbol q_x is used to represent the probability that a life now aged x will die within one year following age x. For instance, if x is 30, $\frac{720}{85441} = q_x$, according to the American Experience table; l_x is the symbol used to represent the number living at age x, d_x is the number dying between ages x and $x + 1$, and p_x is the probability of surviving one year, which in the above illustration would be $\frac{84721}{85441}$. l_{x+1} represents the number who attain age $x + 1$, or age 31 in this illustration. For convenience, then, these symbols will be used instead of writing down the figures each time. Then $p_x = \frac{l_{x+1}}{l_x}$, and $q_x = \frac{d_x}{l_x} = \frac{l_x - l_{x+1}}{l_x}$. These expressions correctly represent the probabilities of death and survival within one year when l_x represents a group starting at exact age x. It was pointed out above that when ages are given to nearest birthday, l_x accurately represents a group starting the year x. When ages are given as at the last birthday, however, a group giving their ages as x, would in fact be one-half year older, on the average, than if the ages were given to nearest birthday,

because some are nearly $x + 1$ years old, others are just past age x, the actual average age being $x + \frac{1}{2}$ years. In many census reports, including the United States census, with the exception of that of 1890, ages were taken as at last birthdays. Such statistics, therefore, when used to measure life contingencies, show the death rates based on the number living at the middle, rather than at the beginning of the year.

Likewise, in determining the number of deaths at age x it has been customary to take ages as of last birthday, so that d_x from the census returns represents the number dying at exact age $x + \frac{1}{2}$, on the average, just as it does when the statistics are taken from the records of insured lives. The fraction formed by taking as a numerator the number dying at exact age $x + \frac{1}{2}$, on the average, and as a denominator, the number living at age $x + \frac{1}{2}$ is said to represent the mean or central death rate; that is, the average death rate throughout the year from age x to $x + 1$. The mean or central death rate is expressed by the symbol m_x. Therefore, if the value of m_x is determined from the census at all ages, the values of q_x in terms of m_x must be found before the yearly probabilities of death can be ascertained. Assuming that the death rate is uniform throughout the year, this may be easily done. Supposing the number living at exact age 35 is 81,822, and the number of deaths in one year is 732, one-half of these deaths, or 366, may be assumed to occur during the first six months of the year, thus leaving 81,822 − 366, or 81,456 living at age $35\frac{1}{2}$. Then

$$m_x = \frac{732}{81456},$$

whereas

$$q_x = \frac{732}{81822}.$$

Using the symbols,

$$q_x = \frac{d_x}{l_x},$$

whereas

$$m_x = \frac{d_x}{l_x - \frac{1}{2}d_x} = \frac{d_x}{l_{x+\frac{1}{2}}}.$$

To find the value of q_x, in terms of m_x,

$$q_x = \frac{d_x}{l_x} = \frac{d_x}{l_{x+\frac{1}{2}} + \dfrac{d_x}{2}} = \frac{2d_x}{2l_{x+\frac{1}{2}} + d_x} = \frac{2\dfrac{d_x}{l_{x+\frac{1}{2}}}}{2 + \dfrac{d_x}{l_{x+\frac{1}{2}}}} = \frac{2m_x}{2 + m_x}.$$

Whence,

$$p_x = \frac{2 - m_x}{2 + m_x}.$$

The yearly probabilities of death and the probabilities of survival may be found, then, from central death rates determined beforehand and used to construct the rough table as explained above.

Graduation.—Probabilities of death that are determined from observations of a small number of persons may be expected to exhibit irregularities. In order to obtain a mortality table that will not lead to anomalous results, it is necessary to submit the rough table to a process known as graduation. The object of graduation is to remove, as nearly as possible, irregularities that may be accidental, without disturbing any characteristics that may be peculiar to the statistics investigated.

Both from what seems reasonable and from most collections of statistics, it appears that the rate of mortality tends to fall for a considerable period following infancy. According to the American Experience table, it increases gradually above age 10, until, as old age is approached it begins to become greater very rapidly.

When the results of any investigation, then, show irregularities, such as a very low or decreasing rate of mortality, followed by a sudden and disproportionate increase, an inspection of the results is necessary in order to determine whether there be not at work some unknown or unappraised forces affecting in a peculiar manner the group from which the statistics were taken. In the absence of unusual conditions, enough observations having been made, it will generally be found that the application of some method of graduation whereby the excesses will be made to fill up the deficiencies, will produce a uniform table. Graduation may be accomplished by several methods, the discussion of which is beyond the scope of this book.

The Construction of a Select Table.—If the statistics from which a mortality table is constructed are taken from general population records, the resulting table is said to be a " population " table. It is, of course, a table showing the probabilities of death and survival among a group that has not submitted to a medical examination. When statistics are compiled from records of insured lives, and those who have been insured five years (or some other period) are excluded, an " ultimate " table will result. Medical and other selection in life insurance is desirable in that it renders less probable an overbalance of those in poor health, thus preventing a disproportionate number of death claims. In most investigations in this country the benefits of medical and other selection accruing to the company in the form of fewer claims per thousand insured have been found to disappear after approximately five years. Consequently, persons who have been before the medical examiner within

the past five years have been excluded from the statistical investigation which precedes the construction of an ultimate table from the records of insured lives. This procedure was followed by Mr. Sheppard Homans in constructing the American Experience table. If the statistics of lives recently examined are combined with those of persons who have been without medical examination for many years, the result is called an *aggregate* table. If a table with, for example, six columns is made up showing separately for each age in the first column the rate of mortality in the year following the medical examination; in the second column, the rate of mortality in the second year after the examination, and so on to the sixth column which represents the ultimate rate of mortality, the result is said to be a "select" table. A select table, therefore, will show a low probability of death the first year, compared with the probability at the same attained age as shown by an ultimate table; a little higher one the second year; an increasingly higher one, each of the years until the probability reaches that shown by the ultimate table the sixth year and upwards. Suppose that in a group of freshly selected lives, the probability of death occurring the first year is found to be one-half of the probability according to the ultimate table. Suppose that it is 65 per cent of the probability shown by the ultimate table the second year, 75 per cent the third, 85 per cent the fourth, 95 per cent the fifth, and 100 per cent the sixth year and thereafter. These are the percentages assumed in the provisions of the former New York Insurance law governing valuation by the select and ultimate method. Taking the American Experience table as an example of an ultimate table, a select table may be constructed from it on the basis of the above percentages. If we suppose the number living at age 40 to be 78,106, of a group of how many persons aged 39 should these be the survivors if the probability of death during the intervening year is 95 per cent of the probability derived from the ultimate table? The probability according to the ultimate table is .009586, 95 per cent of which is .0091067; whence the probability of survival by the select table is 1 − .0091067, or .990893. Now if there are 78,106 survivors out of a group which yielded this probability of survival, the group must have been 78,106 ÷ .990893, or 78,822. Likewise, taking the probability of death at 85 per cent of the ultimate table, at age 38 the probability of survival may be found and divided into the number living at 39, or 78,822 ÷ .992003 = 79,458, the number living at 38. Continuing for ages 37, 36 and 35, the results are 80,013, 80,488, and 80,850. In a group of 80,850 persons aged 35 who have just passed the medical examination, 80,488 may be expected to survive one year; 80,013, two years; 79,458, three years; 78,822, four years;

78,106, five years; 77,341, six years, etc., following the ultimate table after the first five years. When the same operation is performed for each age in the ultimate table, a select table results. Such a table is given in the Appendix. To find the number of survivors at the end of each year out of a medically selected group starting at any age, which is designed by $l_{[x]}$, according to this table, read to the right in the column headed $l_{[x]+1}$ for the number surviving one year, $l_{[x]+2}$ for those surviving two years, etc., until the column headed l_{x+5} is reached, then down this column which shows the survivors according to the ultimate table. It will be noted that when the symbol x is placed in brackets, it indicates the age of entry upon the company's records, that is, the age when the group is medically examined. (See Appendix D.)

Since a medical examination is required of persons desiring life insurance policies, it might be thought that premiums should be based on a select table. It has been customary, however, to charge premiums according to ultimate tables. If an ultimate table is used the benefits of light mortality the first few years assist in meeting the heavy initial expenses. Should there still be a saving after meeting these expenses from the various sources indicated in Chapter XII, it would, in a participating company, inure to the benefit of the policyholders generally instead of to the new policyholders alone.

Application of the Theory of Probabilities to Mortality Tables.— The application of the theory of probabilities to mortality tables for measuring life contingencies may be shown by a few examples. From the American Experience table the probability that a person now aged 35 will die within one year is $\frac{732}{81822}$; within two years it is $\frac{732 + 737}{81822}$; during the second year of life hence, $\frac{737}{81822}$; during the tenth year, $\frac{812}{81822}$; and during the last year of life it is $\frac{3}{81822}$; since, according to the table, only 3 out of 81,822 persons living at age 35 die between ages 95 and 96. The probability that a person aged 35 will survive one year is $\frac{81090}{81822}$; two years, $\frac{80353}{81822}$; ten years, $\frac{74173}{81822}$; to age 95, $\frac{3}{81822}$. It will be noted that the probability of surviving to age 95 is the same as the probability of death between ages 95 and 96; since, according to the table, a person who survives to 95 is certain to die within one year. The probability that two lives aged 30 and 40, respectively, will fail within one year, by the law of compound probability, is $\frac{720}{85441} \times \frac{765}{78106}$;

within two years, $\frac{720+721}{85441} \times \frac{765+774}{78106}$; during the second year hence, $\frac{721}{85441} \times \frac{774}{78106}$; during the tenth year, $\frac{756}{85441} \times \frac{927}{78106}$; during the fifty-sixth year, $\frac{1292}{85441} \times \frac{3}{78106}$. The older life must, according to the table, have certainly failed before the end of the fifty-sixth year, hence both could not die in any subsequent year. The probability that both of two lives aged 30 and 40, respectively, will survive one year is $\frac{84721}{85441} \times \frac{77341}{78106}$; ten years, $\frac{78106}{85441} \times \frac{69804}{78106}$; fifty-five years, $\frac{5485}{85441} \times \frac{3}{78106}$. (See Appendix B.)

Suppose it were desired to find the probability that neither of two lives aged 30 and 40 will fail during the fifth year hence. The probability that the younger will not fail is, $1 - \frac{729}{85441}$; and the probability that the older will not fail is, $1 - \frac{812}{78106}$. The probability that neither will fail, therefore is, $\left(1 - \frac{729}{85441}\right) \times \left(1 - \frac{812}{78106}\right)$.

The Expectation of Life.—The expectation of life, or the average after-lifetime, is of considerable interest to laymen. It has long been used in an effort to place a value on human life in legal disputes such, for instance, as damage suits. To the actuary, however, it is hardly of other than academic interest. It may sometimes be employed to compare the characteristics of different mortality tables, but it is never used for the scientific computation of either annuity or insurance rates, or of any other monetary values.

To find the expectation of life, or the average after-lifetime, divide the aggregate number of years lived by a group setting out at a certain age, by the number in the group. For instance, referring to the table and choosing one of the higher ages such as age 92, to avoid a tedious problem in addition, it is observed that 79 persons live one year; 21 live another year, and 3 live still another year. In other words, the number of full years lived by the group of 216 persons at age 92, according to the table, is $79 + 21 + 3$, or 103 years in the aggregate. $103 \div 216 = .48$. This is known as the *curtate* expectation. It will at once be observed, however, that, on the assumption of a uniform death rate throughout the year, the 137 persons who die betwen ages 92 and 93, live one-half year, on the average,

during that year. In other words, the 137 who die contribute 68.5 years in the aggregate to the 79 full years lived by those who survive to age 93. Likewise, the 58 dying at 93 contribute 29 years, those at 94, 9; and those at 95, 1½. $68.5 + 29 + 9 + 1.5 = 108$ years. $108 \div 216 = .5$, which should be added to the curtate expectation in order to arrive at the *complete* expectation of life, which in this illustration is $.48 + .5$, or .98. Since those who die in any year, live six months on the average within the year of death, to find the complete expectation, or average after-lifetime, at age 92, add the number of survivors at the end of each full year, divide the sum by the number living at age 92, and add 0.5 to the result. The formula for the complete expectation is:

$$\mathring{e}_x = \frac{1}{2} + \frac{l_{x+1} + l_{x+2} + l_{x+3} + \text{etc.}}{l_x} \text{ (to the end of the table).}$$

The curtate expectation:

$$e_x = \frac{l_{x+1} + l_{x+2} + l_{x+3} + \dots}{l_x}.$$

From this it appears that the curtate expectation is the sum of the yearly probabilities of survival.

Using the table,

$$\frac{79}{216} = .365, \text{ probability of surviving one year;}$$

$$\frac{21}{216} = .097, \text{ probability of surviving two years;}$$

$$\frac{3}{216} = .014, \text{ probability of surviving three years;}$$

the sum of which = .48, the curtate expectation.

Or, $\frac{79}{216} + \frac{21}{216} + \frac{3}{216} = \frac{103}{216} = .48$, as before.

The *vie probable*, or probable lifetime, is a somewhat peculiar expression of certain French actuaries, by which is meant the period beginning at age x and extending to an age-date at which it will be equally probable that a person now aged x will be either alive or dead. In other words, the end of the period is the moment at which the number setting out at age x, according to the mortality table, is reduced by exactly one-half. To find this moment, add the number of deaths each year from age x until the sum equals $\frac{1}{2}l_x$. It is also called the equation of life. The *most probable after-lifetime* is the period from the given age x to the year

in which the number of deaths in the group l_x is greatest. The *joint-life curtate expectation* of two persons aged x and y is the average number of full years that will pass while both are living. Thus out of $l_x + l_y$ persons, the total number of pairs, one aged x and the other y, will be $l_x \times l_y$, written l_{xy}. To illustrate, suppose 21 persons, A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, and U, are 94 years of age; and that Messrs. X, Y, and Z, are 95. X may be paired with A, or B, or C, etc. Likewise Y may be paired with A, or B, or C, etc.; and Z may be similarly combined with each of the 21 aged 94, so that the sum of all possible pairs is equal to 3 × 21. One year hence, according to the table, the total number of pairs will be $l_{x+1} \times l_{y+1}$. In the above illustration, $l_{y+1} = 0$, so there can be no pairs after the older of x and y has reached the extreme length of life. At any earlier age, however, l_{y+1} has a value greater than zero. For convenience, $l_{x+1} \times l_{y+1}$, $l_{x+2} \times l_{y+2}$, etc., may be written ${}^1l_{xy}$, ${}^2l_{xy}$, etc. The total years of joint-life, then, will be the sum of ${}^1l_{xy} + {}^2l_{xy} \times {}^3l_{xy}$, etc., to the end of the table for the older life. The average number of full years both x and y will survive, therefore, or the joint-life curtate expectation, is

$$e_{xy} = \frac{{}^1l_{xy} + {}^2l_{xy} + {}^3l_{xy} + \dots}{l_{xy}}.$$

From this expression, as in the formula for finding the curtate expectation of a single life, it appears that the sum of the compound probabilities of joint survival each year may be added to find the joint-life curtate expectation. Also, the expectation of x after the death of y is the sum of the compound probabilities that x will survive and y not survive each year, or $e_{y|x} = e_x - e_{xy}$.

CHAPTER V

Net Single Premiums: Pure Endowments and Annuities: Commutation Columns D and N

The purchaser of an insurance or annuity contract may pay for it in one lump sum sufficiently large to cover its entire cost. The amount of such a payment is called the single premium for the contract. Annuity contracts are frequently purchased with single premiums; insurance policies, usually, with periodic premiums. Of the latter, annual premiums are the most common, although such premiums are frequently paid in semi-annual or quarterly installments; and in industrial insurance, weekly premiums are customary. The determination of single premiums is an essential preliminary step in approaching the methods of calculating periodic premium rates.

Net premiums are those which are exactly sufficient to pay the claims as they arise, should the company's mortality experience happen to coincide precisely with the rate indicated by the mortality table upon which its premiums are based, and should the net rate of interest earned by the company on its investments exactly correspond to the rate assumed in its premium calculations. Net premiums do not include charges to meet expenses or profits. In practice, therefore, to cover expenses, a sum called the loading is usually added to the net premium. This and the net premium together form the gross premium. Gross premiums are often called "office premiums," since they are the ones that appear in agents' rate-books, and are quoted to prospective purchasers of insurance or annuity contracts. For easy comprehension of net annual premiums, an understanding of the calculation of net single premiums is necessary; for determining gross or office premiums, an understanding of both net single and net annual premiums.

Assumptions Underlying Rate-Making.—Before attempting to demonstrate the methods of calculating premiums, certain assumptions underlying rate-making must be set forth. As pointed out in Chapter III, the company must assume a rate of interest which it is certain of realizing on its investments over a long period of years.

The assumed rate of interest, therefore, must be conservatively low. The second assumption is that premiums will be paid in advance; that is, at the beginning of the policy-year, in the case of annual premiums, and at the date when the contract becomes effective, in case it is paid for by a single premium. This assumption is in accordance with the general practice of companies. The third assumption is that claims will be paid at the end of the policy-year in which they arise. This assumption was originally made when it required about three months to render proof of death, and when companies were permitted by contract to defer payment of the claim for three additional months after receiving satisfactory proof that a claim had arisen. For many years, however, it has been customary for companies to pay claims promptly upon receiving proof of death, which, owing to improvements in communication facilities, may now be rendered to the company within a few days after death. The assumption is therefore contrary to fact, since claims are paid approximately six months, on the average, before the time at which, in the determination of premium rates, it is assumed they will be paid. By following this false assumption, the actuary calculates premiums which would be proper if his company were to retain the use of money paid out on claims for a period of approximately six months longer than it actually does. If the face of a policy is $1000, and the interest earned is 4 per cent, the actuary assumes that the company will earn approximately $20 more than it actually earns; because it pays claims nearly six months, on the average, before the ends of the policy-years in which they occur. A known error is not a serious one, however, for measures may be taken to overcome it. Since mortality tables usually indicate a mortality rate higher than that which will actually be experienced, and since the assumed rate of interest is so conservatively low, losses, due to premiums being too low because of this false assumption, are more than counterbalanced. This being true, and because of the labor and expense involved in computing new tables of rates, reserves, etc., based on the more correct assumption that claims will be paid in the middle of the policy-year, actuaries have refrained from making such tables. Furthermore, as is shown later, once premiums for insurances payable at the end of the policy-year of death have been computed, it is not difficult to make an additional calculation showing premiums for insurances payable at the moment of death. The fourth assumption is that the death rate is uniform throughout the policy-year. This assumption corresponds accurately enough to experience at the usual ages when ordinary insurance is

carried, but at very young and very advanced ages it is contrary to experience. In industrial insurance, policies on lives younger than 10 are frequently granted on the weekly-premium plan. Now if 600 deaths are expected at the rate of 50 each month, a higher death-rate in the early part of the policy-year, before many weekly premiums are collected, might prove embarrassing to the company. Special arrangements are made, therefore, to take care of this contingency. Beyond age 10, an increasing death rate presents no such problem, since if the company collects one-twelfth of the premiums the first month and is called upon to pay fewer than one-twelfth of the claims arising during the policy-year, it will be in a position stronger than if the death-rate were uniform throughout the policy-year. Regarding annuities payable more often than annually, the reverse would be true, the losses due to the false assumption being heavier at the extreme ages.

Net Single Premiums for Pure Endowments.—The assumptions being known, and the face, kind of policy, age of the insured person, mortality table and interest rate to be used having been ascertained, the problem of determining the net single premium for a policy may be attacked. One of the simplest operations in premium calculations is the determination of the net single premium for a pure endowment. As previously stated, a pure endowment is payable only in the event of survival. Suppose a ten-year pure endowment of \$1000 be issued to each of 81,822 persons aged 35. According to the American Experience table, the company must be prepared to pay out \$74,173,000 ten years hence, since 74,173 persons survive that period. The company, however, need not collect \$74,173,000 now, since that amount will not be needed for ten years. The company need collect only a sum which when put at interest for ten years will amount to \$74,173,000. Assuming that the company will be able to realize 3 per cent net on its investments, $74{,}173{,}000 \div (1.03)^{10}$, would be the sum which should be collected, according to Chapter III, above. This sum is to be collected from 81,822 persons now aged 35, hence the amount to be collected from each one would be $\frac{74{,}173{,}000 \div (1.03)^{10}}{81822}$. The value of this expression remains the same if it is written $\frac{74173}{81822} \times 1000 \div (1.03)^{10}$. Observe that in this expression the amount at risk on an individual life, or \$1000, is multiplied by the probability of the risk maturing as a claim, or $\frac{74173}{81822}$, and the result is discounted for the period of time elapsing before maturity. The rule is, multiply the amount at risk

by the probability that the event insured against will happen, and discount the result for the period of time extending from the beginning of the contract to the date when this event may happen.

To complete the above operation, recourse may be had to interest Table II. To divide a sum by $(1.03)^{10}$, will yield the same result as to multiply the same sum by $\left(\frac{1}{1.03}\right)^{10}$, which equals v^{10}, or 0.74409391, when i is .03. Hence, $\frac{74173}{81822} \times 1000 \times .74409391 = \$674.53$, the net single premium.

While this is the net single premium for a ten-year pure endowment policy of \$1000 issued at age 35, and while it is customary to quote rates at so much per \$1000 of insurance, in calculating net rates it is the usual practice to consider unity, or the figure 1, as the face value. Thus the above illustration would become,

$$\frac{74173}{81822} \times 1 \times .74409391 = 0.67453.$$

Calculations are then more easily made, and the premium for an insurance or annuity of 1 may easily be converted into the premium for 1000 by shifting the decimal point three places to the right.

Since the multiplication of any expression by 1 does not change its value, the above expression of the premium for 1 of insurance may be written $\frac{74173 \times .74409391}{81822} = .67453$. 81822, however, is l_x, the number living at age x, according to the table, when x is 35. And 74,173 is l_{x+10}, or the number living at age 35 + 10, or 45, and .74409391 is the value of v^{10} when interest is at 3 per cent. The term of years, or in this case, 10, may be represented by n. The general expression of the net single premium for a pure endowment of 1, then, becomes,

$$_nE_x = \frac{v^n l_{x+n}}{l_x}.$$

Since $\frac{l_{x+n}}{l_x}$, is the probability that the insured person will survive the period of n years, or $_np_x$, the final form is:

$$_nE_x = \frac{v^n l_{x+n}}{l_x} = v^n \frac{l_{x+n}}{l_x} = v^n {}_np_x.$$

Annuities.—As explained in Chapter III, the present value of an annuity certain is $v + v^2 + v^3 + v^4 +$ etc., to the end of the annuity period. When the annuity payments depend upon the survival of the

annuitant, however, and cease upon his death, their present value must be found not only by discounting each future payment back to the present time at the assumed rate of interest, but also by discounting each future payment for the probability involved. Thus if an annuity payment of 1 falls due one year hence, provided a person now aged 35 lives to receive it, its present value is,

$$vp_x = \frac{vl_{x+1}}{l_x} = \frac{.97087379 \times 81090}{81822} = 0.96218811,$$

if interest is at 3 per cent. In other words, the problem is: what sum must the company collect now in order to pay \$1 to a man, at present aged 35, one year hence if he is then living? $\frac{81090}{81822}$ represents the probability that the company will be obliged to pay the \$1. The value of the risk, then, is $\frac{81090}{81822} \times 1$. The company, however, is going to collect the premiums now, and in any event it will not pay the claim for one year. If the company can earn 3 per cent on the money collected during the year, it need only collect the value of the risk discounted for one year at 3 per cent, or $\frac{81090}{81822} \times 1 \times .97087379$. This annuity payment, therefore, is similar in all respects except in the term, to the pure endowment illustrated in the preceding paragraph. If the present value of a second annuity payment of 1 due two years hence upon the survival of a person now aged 35 is desired, it may be found by multiplying $\frac{80353}{81822}$, the probability that the company will be called upon to pay, by 1, the amount at risk, and multiplying the result by .94259591, the discounted value of 1 due two years hence, if interest is taken at 3 per cent compounded annually. The expression is:

$$v^2{}_2p_x = v^2\frac{l_{x+2}}{l_x} = \frac{.94259591 \times 80353}{81822} = .92567291.$$

If still a third annuity payment is to be made three years hence provided a person now aged 35 survives the three years, the amount which the company must collect now in order to meet the payment when and if it falls due, is:

$$v^3{}_3p_x = v^3\frac{l_{x+3}}{l_x} = \frac{.91514166 \times 79611}{81822} = .89041263.$$

If the company agrees to pay a three-year life annuity of \$1 to a person now aged 35, it must collect a sum sufficient to meet all three of the payments, when and if each falls due. Obviously

the sum of the present values of the three annuity payments found above, or \$2.77827365, constitutes the net single premium which the company should charge, according to the American Experience table and 3 per cent. This is usually spoken of as the "net cost," or "present value" of the annuity.

The net cost of a ten-year annuity of 1 issued at age 35 according to the American Experience table and 3 per cent, might be found as follows:

$$\frac{97087379 \times 81090}{81822} = 0.96218811, \text{ present value, first annuity-payment}$$

$$\frac{.94259591 \times 80353}{81822} = 0\ 92567291, \text{ present value, second annuity-payment}$$

$$\frac{91514166 \times 79611}{81822} = 0\ 89041263, \text{ present value, third annuity-payment}$$

$$\frac{.88848705 \times 78862}{81822} = 0\ 85635716, \text{ present value, fourth annuity-payment}$$

$$\frac{.86260878 \times 78106}{81822} = 0\ 82343283, \text{ present value, fifth annuity-payment}$$

$$\frac{.83748426 \times 77341}{81822} = 0\ 79161924, \text{ present value, sixth annuity-payment}$$

$$\frac{.81309151 \times 76567}{81822} = 0\ 76087088, \text{ present value, seventh annuity-payment}$$

$$\frac{.78940923 \times 75782}{81822} = 0\ 73113605, \text{ present value, eighth annuity-payment}$$

$$\frac{.76641673 \times 74985}{81822} = 0\ 70237538, \text{ present value, ninth annuity-payment}$$

$$\frac{.74409391 \times 74173}{81822} = 0\ 67453347, \text{ present value, tenth annuity-payment}$$

Total..... . 8 11858712, present value of all of the annuity payments, which is the net single premium, or net cost of the annuity.

The present value of a whole-life annuity issued at age 35 on the basis of the American Experience table and 3 per cent, could be found by continuing the above operation to the end of the mortality table. If issued at any age other than 35, the value of l_x would of course be different and the values of the probabilities involved would therefore be changed. The net cost, however, of any life annuity for any term of years and on any mortality and interest basis, could be determined in this manner.

It will be readily appreciated that the calculation of the present values of annuities by the method illustrated above is a very tedious process, especially when calculating machines are not available. So, while the illustration may serve to make clear the principles involved, in actual practice, recourse is had to much shorter methods. The first step towards reducing the labor involved is to reduce the number of divisions by the radix, l_x (which in this case is 81,822). Since the common denominator of all of the ten fractions in the illustration of the annuity calculation above is 81,822, their numerators may first be added and the results divided by 81,822 to arrive at the net cost. Thus:

$$
\begin{aligned}
.97087379 \times 81090 &= 78728.15563110 \\
.94259591 \times 80353 &= 75740.40915623 \\
.91514166 \times 79611 &= 72855.34269426 \\
.88848705 \times 78862 &= 70067.86573710 \\
.86260878 \times 78106 &= 67374.92137068 \\
.83748426 \times 77341 &= 64771.87015266 \\
.81309151 \times 76567 &= 62255.97764617 \\
.78940923 \times 75782 &= 59823.01026786 \\
.76641673 \times 74985 &= 57469.75849905 \\
.74409391 \times 74173 &= 55191.67758643 \\
\hline
\text{Total.........} & \quad 664278.98874154
\end{aligned}
$$

This number, which is the sum of the ten numerators, may then be divided by the common denominator, or 81,822, to arrive at the present value of the annuity. Thus, $664278.98874154 \div 81822 = 8.118587$, the present value as found before.

It will be noticed that a life annuity appears as a series of pure endowments, one due one year hence, if the annuitant survives this period, another due two years hence, if the annuitant survives two years, etc., etc. In the ten-year annuity there are ten separate amounts of 1 each exposed to ten different hazards involving the probability of surviving one year, two years, three years, etc.; and the value of each risk is discounted for a different period of years. Using symbols, the series expressing the value of an annuity for any term of years and issued at any age (indicated by $|_n a_x$) may be set down as follows:

$$
\begin{aligned}
|_n a_x &= E_x + {}_2E_x + {}_3E_x + {}_4E_x + \ldots {}_nE_x; \\
&= vp_x + v^2{}_2p_x + v^3{}_3p_x + v^4{}_4p_x + \ldots v^n{}_np_x; \\
&= \frac{vl_{x+1} + v^2l_{x+2} + v^3l_{x+3} + v^4l_{x+4} + \ldots v^nl_{x+n}}{l_x}
\end{aligned}
$$

On first inspecting this expression, students are sometimes led to believe that it is applicable only to temporary life annuities. A moment's reflection, however, is sufficient to make it clear that by taking the number of years, n, as a number sufficiently great to cover the remaining span of life from the annuitant's present age to the end of the mortality table, the expression may be made to serve for the valuation of whole-life annuities also. A whole-life annuity is usually designated by the symbol a_x, and its value indicated:

$$a_x = \frac{vl_{x+1} + v^2l_{x+2} + v^3l_{x+3} + v^4l_{x+4} + \text{etc., to end of table}}{l_x}.$$

In the discussion of interest tables, Chapter III, above, it was pointed out that Table IV, which shows the present value of an annuity certain of 1 per annum, was derived from Table II, which shows the present value of 1 due at the end of one, two, three, etc., years in the future, by adding the values of Table II. (See Appendix.) Similarly a table of life annuity values may be derived from a table showing the values of a series of pure endowments of 1 due at the end of each year from age x. Starting, for convenience, at the extreme age of the mortality table, the value of a whole-life annuity at age 95, is, of course, 0. At age 94, it is (according to the American Experience table and 3 per cent), vp_{94}, or $\frac{.97087379 \times 3}{21} = .13869625$. In other words, if the person is now 94, the company must ask \$0.1387 in order to pay him \$1 if he survives to age 95. If this person were 93, how much must the company ask in order to pay him \$1 two years later, or at age 95? The answer is $\frac{.94259591 \times 3}{79} = .03579478$. If the company were also to pay this person, now aged 93, \$1 upon reaching 94, and \$1 upon reaching 95, it must collect \$0.03579478 to pay the \$1 at 95 if the annuitant survives two years, and it must also collect, $\frac{.97087379 \times 21}{79}$, or \$0.25808037, to pay the \$1 due at 94 if the person survives one year. For both annuity payments, then, the company must charge

$$.25808037 + .03579478, \quad \text{or} \quad \$0.29387515,$$

which is the value of an annuity at 93. The same result, however, may be attained in another manner. Observe that at age 93 the company must charge enough to provide \$1 at 94 if the person survives, and also enough to provide \$0.1387 at 94 to take care of a possible claim of \$1 at 95; that is, the net single premium, or net cost, of a whole-

life annuity at 93, is the net single premium for a pure endowment of $1 + 0.1387$, due one year hence. Thus,

$$\frac{.97087379 \times 21 \times 1.13869625}{79} = \$0.29387515,$$

the value of the annuity at 93, as before.

The present values of the simple endowments composing annuities issued at each age of the mortality table having been computed, it appears that a table showing the present values of whole-life annuities may be derived from them. An inspection of the illustration given above will reveal the fact that when an annuitant has survived one year, and the first \$1 is due, the value of his annuity is \$1 plus the present value of an annuity at an age one year older than his age at the outset, or in the case of a whole-life annuity, $1 + a_{x+1}$ dollars. Then the value at the outset one year earlier is the present value of a pure endowment of $1 + a_{x+1}$ dollars due in one year. Or,

$$a_x = E_x(1 + a_{x+1}) = vp_x(1 + a_{x+1}).$$

Using the symbols, the process of making a complete table of whole-life annuity values may be shown as follows:

$$\begin{aligned} a_{95} &= vp_{95} &&= 0. \\ a_{94} &= vp_{94} &&= 0.13869625 \\ a_{93} &= vp_{93}(1 + a_{94}) &&= 0.29387515 \\ a_{92} &= vp_{92}(1 + a_{93}), \text{ etc.,} \end{aligned}$$

back to the beginning of the mortality table. (A table of whole-life annuity values appears in the Appendix.)

Though a table of whole-life annuity values is quite practicable, one showing the values of temporary annuities at all ages and for all possible terms of years would constitute a cumbersome set of figures. Considerable, too, would be the labor of determining, in the above manner, the value of every temporary annuity which might at some time be desired. The work thus involved is sufficiently great to have caused actuarial mathematicians to exercise their ingenuity in devising shorter methods of arriving at desired results. It is still necessary to understand the use of these labor-saving devices and tables in order to get an adequate understanding of the actual determination of net premiums for annuity and insurance contracts.

A comprehensive explanation of the origin and use of these expedients is facilitated by the knowledge of a few of the more elementary

principles of algebra. Taking the general expression for the value of a whole-life annuity,

$$a_x = \frac{vl_{x+1} + v^2 l_{x+2} + v^3 l_{x+3} + \text{etc.}}{l_x},$$

it has been shown that the values of a have a certain relation to each other; so that if one value is given, the next lower in age may be derived from it. Likewise, a little reflection will make clear the fact that the numerator of any value of a will consist of the values of l for all ages higher than x (the present age of the annuitant). The first value of l in the numerator will be modified by v, the second by v^2, etc. Thus,

$$a_{x+1} = \frac{vl_{x+2} + v^2 l_{x+3} + v^3 l_{x+4} + \text{etc.}}{l_{x+1}} \quad . \quad . \quad . \quad . \quad (1)$$

and,

$$a_{x+2} = \frac{vl_{x+3} + v^2 l_{x+4} + v^3 l_{x+5} + \text{etc.}}{l_{x+2}} \quad . \quad . \quad . \quad . \quad (2)$$

It is seen in these expressions that the index of v and the suffix of l do not agree. Except for the omission of the first, the values of l remain the same, but each value of l is modified by changing powers of v as the series progresses from a_x to a_{x+1}, a_{x+2}, a_{x+3}, etc., to the end of the mortality table. Thus in No. (1), above, l_{x+3} is modified by v^2, whereas in No. (2), l_{x+3} is modified by v. If by some means the index of v could be made to correspond to the suffix of l, the value of a_{x+1} could be determined by simply omitting the first expression in the numerator of the fraction showing the value of a_x, and by changing the denominator to l_{x+1}. This cannot be done, yet it is possible to modify both numerator and denominator so that the indices of v and the suffixes of l will be the same. Again referring to the original expression of the value of a_x, suppose both the numerator and denominator of the fraction composing the second term of the equation be multiplied by v^x, which will not change its value. Thus,

$$\begin{aligned} a_x &= \frac{vl_{x+1} + v^2 l_{x+2} + v^3 l_{x+3} + \text{etc.}}{l_x} \\ &= \frac{vl_{x+1} + v^2 l_{x+2} + v^3 l_{x+3} + \text{etc.}}{l_x} \times \frac{v^x}{v^x} \\ &= \frac{v^{x+1} l_{x+1} + v^{x+2} l_{x+2} + v^{x+3} l_{x+3} + \text{etc.}}{v^x l_x}. \end{aligned}$$

Now suppose both the numerator and the denominator of the second

term of the fraction showing the value of a_{x+1}, are multiplied by v^{x+1}, which again will not alter its value:

$$a_{x+1} = \frac{vl_{x+2} + v^2l_{x+3} + v^3l_{x+4} + \text{etc.}}{l_{x+1}}$$

$$= \frac{vl_{x+2} + v^2l_{x+3} + v^3l_{x+4} + \text{etc.}}{l_{x+1}} \times \frac{v^{x+1}}{v^{x+1}}$$

$$= \frac{v^{x+2}l_{x+2} + v^{x+3}l_{x+3} + v^{x+4}l_{x+4} + \text{etc.}}{v^{x+1}l_{x+1}}.$$

Observe that the indices of v agree with the suffixes of l in each term in the numerator, that the index of v also agrees with the suffix of l in the denominator (each corresponding to the age of the annuitant), and that the numerator of the fraction expressing the value of a_{x+1} is exactly like the numerator of the fraction expressing the value of a_x, except that the first term is omitted. Hence if the value of a_{x+2} were desired, it is:

$$a_{x+2} = \frac{v^{x+3}l_{x+3} + v^{x+4}l_{x+4} + v^{x+5}l_{x+5} + \text{etc.}}{v^{x+2}l_{x+2}},$$

and the formula is found to be general, thus:

$$a_{x+n} = \frac{v^{x+n+1}l_{x+n+1} + v^{x+n+2}l_{x+n+2} + v^{x+n+3}l_{x+n+3} + \text{etc.}}{v^{x+n}l_{x+n}}.$$

Commutation Column D.—In each of the terms of the numerator and the denominator of the above fraction, the index of v agrees with the suffix of l whose value it modifies. Hence a table showing the value of v^xl_x at each age could be conveniently used to find the value of a whole-life annuity, or of a temporary annuity at any age. Since the value of v^xl_x is so often used as a denominator, suppose the value of v^xl_x be called D_x. Starting at the end of the table, the value of D_x when x is age 95, would be, $v^xl_x = v^{95}l_{95}$. From interest table II, $v^{95} = 0.06032032$, when interest is at 3 per cent and from the American Experience table, $l_{95} = 3$. $v^{95}l_{95} = 0.06032032 \times 3 = 0.18096096$, the figure found in the D_x column (see Appendix), opposite age 95. Likewise, $v^{94}l_{94} = 0.06212993 \times 21 = 1.30472853$, the number in the D_x column opposite age 94; and $v^{93}l_{93} = 0.06399383 \times 79 = 5.05551257$, the value of D_x when x is 93. Continuing, the D_x column of the commutation columns may be readily constructed.

Substituting D_x for $v^x l_x$ the value of a whole-life annuity may be expressed:

$$a_x = \frac{v^{x+1}l_{x+1} + v^{x+2}l_{x+2} + v^{x+3}l_{x+3} + \text{etc.}}{v^x l_x}$$

$$= \frac{D_{x+1} + D_{x+2} + D_{x+3} + \text{etc.}}{D_x}.$$

To illustrate, suppose x = age 93. Then the value of an annuity of 1 issued at age 93 would be:

$$a_{93} = \frac{D_{94} + D_{95}}{D_{93}} = \frac{1.30472853 + 0.18096096}{5.05551257}$$

$$= 0.29387515, \text{ as previously found.}$$

In the discussion of pure endowments, above, the net single premium or present value of a pure endowment was found to be: ${}_nE_x = \frac{v^n l_{x+n}}{l_x}$. Now multiplying both numerator and denominator by v^x,

$${}_nE_x = \frac{v^{x+n}l_{x+n}}{v^x l_x};$$

but

$$v^x l_x = D_x,$$

and

$$v^{x+n}l_{x+n} = D_{x+n}.$$

Hence the value of a pure endowment becomes,

$${}_nE_x = \frac{D_{x+n}}{D_x}.$$

Returning to the problem of determining the present value of a ten-year pure endowment issued at age 35, and referring to the commutation column headed D_x:

$${}_nE_x = \frac{D_{x+n}}{D_x} = \frac{D_{35+10}}{D_{35}} = \frac{19614.20}{29078.18} = 0.674533,$$

as before.

Thus by performing the simple division of one number found in the commutation column at the end of the pure endowment period, by another found at the beginning of that period, the value of the endowment is at once derived.

Commutation Column N.—Though a table showing the values of D_x at all ages would be very useful for saving time in making calculations, it is plain that a summation might be advantageously made and the results tabulated in a form convenient to use. Thus in the above illustration, it was necessary to add the values of D_{94} and D_{95}. This is no great task at the older ages of the D_x column, but at the younger ages, the work would be quite tedious, and altogether unnecessary. Once the D_x column has been formed, it is a matter of addition to sum the values of D above each age, and thus form a table showing at sight the numerators of the formula, just as the D_x column shows the denominators. Since the sums of these values in the D_x column constitute the numerator of the fraction, the column showing the results is designated N_x. Starting at the end of the D_x column, the values of N_x may be determined in the following manner:

Following the English notation adopted by the Congress of Actuaries,

$$N_x = D_{x+1} + D_{x+2} + \text{etc.}$$

Hence:

$$N_{95} = D_{96} = 0$$

$$N_{94} = D_{95} = 0.18096$$

$$N_{93} = D_{94} + D_{95} = 1.48569$$

$$N_{92} = D_{93} + D_{94} + D_{95} = 6.54120$$

$$N_{91} = D_{92} + D_{93} + D_{94} + D_{95} = 20.77855, \text{ etc.}$$

It will be noted at once that the value of N_{91} is the same as at age 92, except that the value of D_{92} is added. Hence the construction of the N_x column may be still further simplified as follows:

$$N_{95} = D_{96} = 0.$$

$$N_{94} = D_{95} = 0.18096$$

$$N_{93} = D_{94} + 0.18096 = 1.48569$$

$$N_{92} = D_{93} + 1.48569 = 6.54120$$

$$N_{91} = D_{92} + 6.54120 = 20.77855, \text{ etc.}$$

Returning to the original formula, an expression may be derived, whereby the value of a whole-life annuity may be set forth in such terms

as will permit the use of commutation columns in making actual calculations, as follows:

$$a_x = \frac{vl_{x+1} + v^2l_{x+2} + v^3l_{x+3} + \text{etc.}}{l_x}$$

$$= \frac{v^{x+1}l_{x+1} + v^{x+2}l_{x+2} + v^{x+3}l_{x+3} + \text{etc.}}{v^x l_x}$$

$$= \frac{D_{x+1} + D_{x+2} + D_{x+3} + \text{etc.}}{D_x}$$

$$= \frac{N_x}{D_x},$$

and,

$$a_{x+n} = \frac{N_{x+n}}{D_{x+n}}.$$

Example.—Using the commutation columns based on the American Experience table of mortality and 3 per cent, calculate the present value, or net cost of a whole-life annuity of $1 issued at age 93.

Solution:

$$a_{93} = \frac{N_{93}}{D_{93}} = \frac{1.48569}{5.05551} = \$0.293875.$$

Example II.—What would be the value of the above annuity if it were issued at age 35?

Solution:

$$a_{35} = \frac{N_{35}}{D_{35}} = \frac{550082.48}{29078.18} = \$18.9174.$$

Annuities Due.—Annuities whose first payment is due at once are not issued, because it would be absurd to pay a sum to an insurance company only to have it immediately returned. Nevertheless it is necessary to understand annuities-due in order to calculate insurance premiums that are paid periodically. It is evident that once the value of a whole-life annuity of 1 whose first payment is due one year hence has been determined, the value of a whole-life annuity of 1 with the first payment due at once can be readily ascertained. It should be greater by just one payment. Thus a whole-life annuity-due of 1 may be expressed, $1 + a_x$. To find the value of a whole-life annuity-due of 1, therefore, calculate the value of a whole-life annuity and add 1 to the result. To derive the value directly from the commutation columns:

$$1 + a_x = 1 + \frac{N_x}{D_x} = 1 + \frac{D_{x+1} + D_{x+2} + D_{x+3} + \text{etc.}}{D_x}$$

$$= \frac{D_x}{D_x} + \frac{D_{x+1} + D_{x+2} + D_{x+3} + \text{etc.}}{D_x}$$

$$= \frac{D_x + D_{x+1} + D_{x+2} + D_{x+3} + \text{etc.}}{D_x}$$

$$= \frac{N_{x-1}}{D_x}.$$

EXAMPLE.—Calculate the present value of a whole-life annuity-due of $1 issued at age 35, using commutation columns based on the American Experience table and 3 per cent. (See Appendix C.)

Solution:

$$1 + a_{35} = \frac{N_{x-1}}{D_x} = \frac{579160.66}{29078.18} = \$19.9174.$$

American Columns.—In some of the older American publications the value of a whole-life annuity is expressed by A_x instead of a_x, and A_x in those works usually signifies an "annuity-immediate," an expression which their authors often used to signify an annuity-due. The D column was constructed as described above, but the N column was constructed so that N_x included the value of D_x, instead of starting with D_{x+1}. Thus N_{93} would be: $D_{93} + D_{94} + D_{95} = 6.54120$, instead of 1.48569, the value of N_{93} as most generally used at the present time. Hence N_{94} of the old American notation is equivalent to N_{93}, of the English notation now almost universally adopted; and N_x, old American notation, is equivalent to N_{x-1}, modern notation. The following illustration shows the difference:

	Old American Notation	English and New American Notation
N_{91}	52 14422	20 77855
N_{92}	20 77855	6 54120
N_{93}	6 54120	1 48569
N_{94}	1 48569	0 18096
N_{95}	0.18096	0 00000

Hence, $\frac{N_x}{D_x}$, using the older American tables, gives the present value of a whole-life annuity-due. The reason for the difference in notation is that English actuaries had first to deal with annuities, whereas in America the development of insurance for the most part preceded that of annuities. Care must be exercised to make sure which tables are being used, in order to avoid confusion. The commutation tables given in this book are based on the modern English system.

In this work, only such commutation tables are given as are necessary for purposes of illustration. Columns giving the values at different rates of interest, based on different mortality tables, and for more than one life, are available.

Deferred Annuities.—Though immediate life annuities are the most common type of annuity, other kinds are sometimes issued, as was indicated in Chapter II. Among these the deferred annuity offers several advantages, among which the most important are: (1) it permits a person to provide a life income after the business-retirement age, and (2) it may be paid for by periodic premiums, immediate life annuities, naturally, being sold only on the single premium plan. In other words, deferred annuities furnish the means of scientifically saving enough to purchase a life income, in the form of a whole-life annuity-due, when, as and if needed.

Before the periodic premium can be determined, however, it is desirable to calculate the net single premium, or net cost, or "present value" of the deferred annuity. This can be done in two ways. Thus suppose it were desired to find the net cost or present value of a deferred annuity of 1, issued at age 35, the first payment to be made by the company at age 65, provided the annuitant lives to receive it. This is called a 29-year deferred annuity, since at 64 it becomes a whole-life annuity and at 65 it becomes a whole-life annuity-due. This is an important point, for in using commutation columns, n, the period of deferment, is in this case 29, not 30. One method of conveying the idea underlying this problem is to consider each probability separately. Thus, what fraction expresses the probability that the company will be called upon to make the first annuity payment of 1? It is the probability that the annuitant who is now aged 35 will survive 30 years, or to age 65. From the American Experience table of mortality, this probability is $\frac{49341}{81822}$; but the company is going to collect the premium now, and it will not have to pay in any event for 30 years. Hence,

$$v^{30} \times \frac{49341}{81822} = \frac{.41198676 \times 49341}{81822} = 0.248439,$$

interest being at 3 per cent. The second annuity payment will be due 31 years hence, or when the annuitant has reached 66, if he survives. Its present value, therefore, is $\frac{.39998715 \times 47361}{81822} = 0.231524$. So the operation might be continued, discounting each payment back to age 35 both for the probability and compound discount involved, until the

end of the mortality table is reached. The sum of the present values of all the annuity payments would be the net cost of the deferred annuity.

A solution involving less work, however, is possible. Thus as noted above, the deferred annuity becomes an ordinary whole-life annuity at age 64. In other words, if the annuitant should survive to age 64, the company must then have on hand the value of a whole-life annuity at that age, in order to begin paying 1 at 65 if the person survives one year, 1 at 66 if he survives another year, etc. To find the present value of such a deferred annuity issued at 35, therefore, first find the present value of an ordinary whole-life annuity at 64. It is:

$$a_{64} = \frac{N_{64}}{D_{64}} = \frac{68645.33}{7725.774} = 8.8852.$$

Since, however, the annuitant is only 35 now, the company may never need the 8.88, because the annuitant may not survive. The probability of surviving from 35 to 64 is $\frac{51230}{81822}$. The company is collecting the premium now, yet it will not need the present value of a whole-life annuity for 29 years, in any event. Hence the 8.8852 becomes a pure endowment, whose present value, when $v = \frac{1}{1.03}$, is:

$$v^{29} \times \frac{51230}{81822} \times 8.8852 = \frac{.42434636 \times 51230 \times 8\ 8852}{81822} = 2.3607.$$

This is still too laborious a process. Using symbols, the present value of a deferred annuity, which is represented by $_{n|}a_x$, and which really means a whole-life annuity deferred n years (in this case 29 years), may be expressed: $_{n|}a_x = {}_nE_x a_{x+n}$. $_nE_x$ represents the present value of the pure endowment of 1 (in the above illustration, $v^{29} \times \frac{51230}{81822}$), and a_{x+n} the value of the deferred whole-life annuity (a_{35+29}, or 8.8852 in the illustration). Now, as previously noted, $_nE_x = \frac{D_{x+n}}{D_x}$, and $a_{x+n} = \frac{N_{x+n}}{D_{x+n}}$.

Hence:

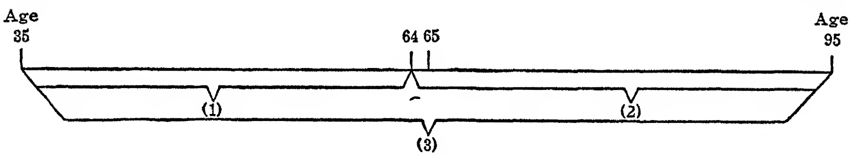
$$\begin{aligned} {}_{n|}a_x &= {}_nE_x(a_{x+n}) \\ &= \frac{D_{x+n}}{D_x} \times \frac{N_{x+n}}{D_{x+n}} \\ &= \frac{N_{x+n}}{D_x}. \end{aligned}$$

Referring to the commutation columns, the present value of the 29-year deferred annuity issued at age 35 is easily solved:

$$_{n|}a_x = \frac{N_{x+n}}{D_x} = \frac{N_{35+29}}{D_{35}} = \frac{N_{64}}{D_{35}} = \frac{68645.324}{29078.18} = 2.3607,$$

as before. The simple division of the number found in column N at the end of the period of deferment, by the number found in column D at the beginning of that period, involves much less labor than the somewhat tedious processes shown above for purposes of illustration.

Application of the Columns to Temporary Annuities.—The value of a whole-life annuity issued at any age must be equal to the value of a deferred life annuity issued at the same age, *plus* the value of a temporary life annuity to run during the period of deferment. Therefore, the value of a temporary, life annuity must be the value of a whole-life annuity, *less* the value of a deferred life annuity effective at the end of the period during which the temporary annuity is to run. The following diagram may serve to illustrate these statements:



(1) Value, 29-year annuity, issued at age 35, payable at ages 36 to 64 inclusive.

(2) Value, whole-life annuity, issued at age 35, but deferred 29 years, first payment by company at age 65.

(3) Value, whole-life annuity, issued at age 35, first payment to be made by the company at age 36.

NOTE.—If the annuity is 1 per annum, these values on the basis of the American Experience table and 3 per cent are as follows:

(1)	16.5567
(2)	2.3607
(3)	18.9174

Now the value of a whole-life annuity has been found to be, $a_x = \frac{N_x}{D_x}$, and the value of a deferred annuity is, $_{n|}a_x = \frac{N_{x+n}}{D_x}$. It follows, therefore, that the value of a temporary annuity (written $|_n a_x$), is,

$$|_n a_x = \frac{N_x}{D_x} - \frac{N_{x+n}}{D_x} = \frac{N_x - N_{x+n}}{D_x}.$$

Returning once more to the problem previously solved, on page 60, suppose it be now proposed to find the value of a 10-year annuity of 1 issued at age 35, by using the commutation columns. The result is quickly attained as follows:

$$|_n a_x = \frac{N_x - N_{x+n}}{D_x} = \frac{N_{35} - N_{45}}{D_{35}} = \frac{550082.48 - 314008.78}{29078.18}$$

$$= 8.118587,$$

the result obtained above with much more effort than that involved in the simple subtraction and division of the numbers found in the commutation columns.

Conversion Formulas.—Life, Temporary and Deferred Annuities.— It appears that, since the values of temporary and deferred annuities are but parts of the value of an immediate life annuity, when any two of these three values have been determined, the other may be derived from them. Thus:

$$a_x = |_n a_x + {}_n|a_x$$

$${}_n|a_x = a_x - |_n a_x$$

$$|_n a_x = a_x - {}_n|a_x$$

Annuities Certain and Continuous.—The value of an annuity whose payments are guaranteed for a certain number of years, and are then to continue throughout the after-lifetime of the annuitant beyond the guaranteed period, may be considered as being made up of two parts. First, the present value of an annuity certain for the guaranteed period, and second, the present value of a whole-life annuity deferred until the end of the guaranteed period. Interest table IV yields at once the present value of the annuity certain, and if the annuity is for 1 per annum payable at the end of the contract-year, the fraction, $\frac{N_{x+n}}{D_x}$, may be applied to the commutation columns to find the present value of the deferred annuity. For example, what is the present value of an annuity of 1, issued at age 35, certain for 20 years and continuous, on the basis of the American Experience table and 3 per cent? From interest table IV, the value of 1 per annum payable at the end of each year for 20 years is 14.87747486. This sum is sufficient to provide payments of 1 per annum to the annuitant at ages 36 to 55, inclusive, or to the annuitant's estate in case of his death before the end of this period, provided the company can earn 3 per cent net on funds left in its possession. A whole-life annuity issued at 35 and deferred 20 years, would provide the first payment of 1 at age 56, or at the end of one year

following the period of deferment, provided the annuitant survived one year beyond the period of deferment. Referring to the columns for the value of $\frac{N_{x+n}}{D_x}$, or in this instance $\frac{N^{55}}{D^{35}}$, it is found to be,

$$\frac{157436.15}{29078.18} = 5.41423672.$$

If the company collects both of these sums, or,

$$14.8775 + 5.4142 = 20.2917,$$

it could afford to grant both benefits. $20.2917, then, is the net cost of an annuity of $1 certain for 20 years and continuous, according to the mortality table and interest rate indicated.

Suppose a person aged 35 has $10,000 and wishes to spend it for an annuity certain for 20 years and continuous. On the same mortality and interest basis as above, what should be the amount of each annuity payment? If $20.2917 is sufficient to provide for an annuity of $1 certain for 20 years and continuous, $10,000 should provide an annuity as much greater than $1 as 10,000 is greater than 20.1917. Or

$$\frac{10000}{20.1917} = \frac{x}{1};$$

$$x = \$495.25.$$

It should be noted that this would be the amount of each annuity payment only if the annuity certain and continuous were granted at *net* single premium rates. No allowance for expenses is made in any of the preceding calculations.

Suppose the problem be modified somewhat to make it correspond to one which frequently arises in practice. Thus suppose a person dies and leaves $10,000 of insurance payable to a beneficiary aged 35, in the form of an annuity certain for 20 years and continuous. As noted in Chapter II, this mode of settlement is often referred to as settlement by means of a "continuous installment." How may the amount of each payment be determined?

In such cases as this, it is usually provided that the first payment to the beneficiary shall be made by the company *at once*, upon the death of the insured person, instead of one year later. Assuming such an arrangement, the first step in the solution of the problem is to find out how much would be required to make payments of $1 each. The present value of $1 paid at once at age 35 would obviously be $1. Then there will follow nineteen more guaranteed payments

at intervals of one year each, whose present value, from interest table IV, is \$14.32379911. The present value of twenty guaranteed payments of \$1 would therefore be \$15.323799. In this case, the guaranteed payments will be made from ages 35 to 54, inclusive, if the beneficiary survives, or to the beneficiary's estate in case of death before the end of the guaranteed period. If the beneficiary survives to age 54, however, the company must possess the present value of a whole-life annuity, so as to begin making payments at 55 and thereafter, subject to the beneficiary's survival. The whole-life annuity, therefore, is only deferred 19 years, or from 35 to 54, in this instance. Hence:

$$\frac{N_{x+n}}{D_x} = \frac{N_{54}}{D_{35}} = \frac{170140.03}{29078.18} = 5.85112.$$

Then, $5.8511 + 15.3238 = \$21.1749$, the amount necessary to pay \$1 at once, \$1 each year for the next 19 years, and \$1 each year the beneficiary survives thereafter. Consequently:

$$10000 : 21.1749 :: x : 1;$$

$$x = 10000 \div 21.1749 = \$472.26,$$

the amount per annum of the annuity-due, certain for twenty payments, and continuous.

Annuities Payable More Often than Annually.—Consider two whole-life annuities of 1 each granted to an annuitant now aged x. Suppose that one of them is an ordinary whole-life annuity with the first payment due one year hence, the second two years hence, etc., subject to the annuitant's survival. Its present value is clearly a_x. Supposing the other differs from an ordinary whole-life annuity in that its first payment is due six months hence, the second $1\frac{1}{2}$ years hence, the third $2\frac{1}{2}$ years hence, etc., subject to survival in each instance, as before; i.e., that the second annuity is payable at the middle of the contract-year; the value of this second annuity is clearly greater than a_x, because its first payment is to be made six months before the end of one year. If its first payment were to be made at once, or twelve months before the end of one year, it would be an annuity-due, the value of which has been determined as $a_x + 1$. Since, however, the first payment of 1 is to be made six months before the end of the year, or $\frac{1}{2}$ year earlier than the first payment of an ordinary whole-life annuity falls due, the value of this second annuity is considered to be $a_x + \frac{1}{2}$. This is not quite accurate, since the first payment of 1 should be discounted for six months at the assumed rate of interest and multiplied by the probability of the

annuitant's survival for six months from age x; the second payment of 1 should be discounted $1\frac{1}{2}$ years and multiplied by the probability of surviving the $1\frac{1}{2}$ years, etc. The probabilities of surviving half-years are not used, however, and so the minor elements of error are ignored in practice, the value of the annuity payable on the half-year being taken as $a_x + \frac{1}{2}$. The value of both of the annuities is, therefore, $(a_x + \frac{1}{2}) + a_x$. The symbol for a whole-life annuity of 1 payable semi-annually is $a_x^{(2)}$. The two annuities assumed at the beginning of this paragraph really make up one annuity of 2, one-half of which is payable semi-annually. Hence:

$$2a_x^{(2)} = (a_x + \tfrac{1}{2}) + a_x$$

$$a_x^{(2)} = \frac{a_x}{2} + \frac{1}{4} + \frac{a_x}{2}$$

$$= \frac{2a_x}{2} + \frac{1}{4}$$

$$= a_x + \tfrac{1}{4}.$$

Now suppose there are four whole-life annuities of 1, one of which is payable at the end of each quarter of the contract-year, subject to survival. This would be called a whole-life annuity of 4, payable quarterly. Following the same line of reasoning pursued above, the value of the annuity of 4 payable quarterly would be:

$4a_x^{(4)} = (a_x + \frac{3}{4})$, the value of the annuity of 1 payable end 1st quarter;

$+ (a_x + \frac{2}{4})$, the value of the annuity of 1 payable end 2d quarter;

$+ (a_x + \frac{1}{4})$, the value of the annuity of 1 payable end 3d quarter;

$+ (a_x)$, the value of the annuity of 1 payable end contract year;

$= (a_x + \frac{3}{4}) + (a_x + \frac{2}{4}) + (a_x + \frac{1}{4}) + (a_x)$.

Clearing of parentheses and dividing by 4:

$$a_x^{(4)} = \frac{a_x}{4} + \frac{3}{16} + \frac{a_x}{4} + \frac{2}{16} + \frac{a_x}{4} + \frac{1}{16} + \frac{a_x}{4};$$

$$= \frac{4a_x}{4} + \frac{6}{16}$$

$$= a_x + \tfrac{3}{8}.$$

Assuming an annuity of m payable, $1/m$th at the end of the first

mth part of the year, another $1/m$th payable at the end of the second mth part of the year, etc.; its value is:

$$a_x^{(m)} = a_x + \frac{m-1}{2m}.$$

Thus if $m = 2$,

$$a_x^{(m)} = a_x + \frac{m-1}{2m} = a_x + \frac{1}{4};$$

and if $m = 4$,

$$a_x^{(m)} = a_x + \frac{m-1}{2m} = a_x + \frac{4-1}{2 \times 4} = a_x + \frac{3}{8}, \text{ as above.}$$

If the annuity were payable monthly, m would be 12, and the value of an annuity of 1 payable monthly would be:

$$a_x^{(m)} = a_x + \frac{m-1}{2m};$$

$$a_x^{(12)} = a_x + \frac{11}{24}.$$

In practice, the value of an annuity payable monthly is often taken as $a_x + \frac{1}{2}$.

If the annuity were payable momently, that is, if it were what is known as a continuous annuity, its value would be expressed:

$$\bar{a}_x = a_x + \tfrac{1}{2},$$

because $\frac{m-1}{2m}$ approaches $\frac{1}{2}$, as m is made to approach infinity. To illustrate, suppose $m = 1000$. Then,

$$\frac{m-1}{2m} = \frac{999}{2000};$$

and if $m = 1{,}000{,}000$,

$$\frac{m-1}{2m} = \frac{999999}{2000000}.$$

Joint-life Annuities.—The present value of a joint-life annuity payable at the end of each year that both of two persons aged x and y survive, may be determined in exactly the same manner as the value of an annuity based on a single life, except that the probabilities involved are compound probabilities and the calculations continue only until the older of the two has reached the end of the mortality table. In other words, the value of a joint-life annuity of 1 may be found by multiplying 1 by the compound probability that both

x and y will survive one year, and discounting the result for one year at the assumed rate of interest to arrive at the present value of the first annuity-payment; then multiplying the compound probability that both will survive two years by 1 and discounting for two years, etc., to the end of the table for the older of the two persons. The sum of the results is the net single premium. Resorting to the symbols,

$$a_{xy} = v\left(\frac{l_{x+1}}{l_x} \times \frac{l_{y+1}}{l_y}\right) + v^2\left(\frac{l_{x+2}}{l_x} \times \frac{l_{y+2}}{l_y}\right) + v^3\left(\frac{l_{x+3}}{l_x} \times \frac{l_{y+3}}{l_y}\right) + \text{etc.},$$

to the end of the table for the older life.

Since l_{xy} is the common denominator of the fractions on the right side of this equation,

$$a_{xy} = \frac{vl_{x+1\,:\,y+1} + v^2l_{x+2\,:\,y+2} + v^3l_{x+3\,:\,y+3} + \text{etc.}}{l_{xy}}$$

Let x be the older of the two lives, and multiply both the numerator and the denominator of the above fraction by v^x. The result is:

$$a_{xy} = \frac{v^{x+1}l_{x+1\,:\,y+1} + v^{x+2}l_{x+2\,:\,y+2} + v^{x+3}l_{x+3\,:\,y+3} + \text{etc.}}{v^x l_{xy}}$$

$$= \frac{D_{x+1\,:\,y+1} + D_{x+2\,:\,y+2} + D_{x+3\,:\,y+3} + \text{etc.}}{D_{xy}}$$

$$= \frac{N_{xy}}{D_{xy}},$$

which may be applied to special commutation columns based on two lives, to determine the values of joint-life annuities directly.

Reversionary Annuities.—Since a reversionary annuity, or survivorship annuity, as it is often called, provides for the payment of an annuity to a beneficiary who outlives the insured person, beginning at once upon the death of the latter and continuing for the whole after-lifetime of the beneficiary, if the beneficiary dies first, the contract comes to an end, the company having earned the premium. How shall the net single premium for such a contract be determined? If the insured person were to die at once, the company must have on hand the present value of an annuity-due based on the life of the beneficiary, or $1 + a_x$, x being the beneficiary's age, and the reversionary payments, 1 each. As long as they are both living, however, the company will not be called upon to make any payments, so it need not collect the full value of $1 + a_x$ as a single premium. It will need $1 + a_x$ reduced by the net single premium for a joint-life annuity-due based on both lives. Students

sometimes get confused at this point, reasoning that the company will not have to pay so long as both insured and beneficiary live, nor if the beneficiary should die. The difficulty is usually cleared up at once by explaining that the probabilities of the beneficiary's survival were considered in determining the value of a_x in the first place.

Sometimes the beginner more readily understands this calculation by looking upon it in this manner; suppose the beneficiary now has a whole-life annuity-due which will pay her \$1 at once, and \$1 at the end of each year she continues to survive. Its value would be $1 + a_x$. It will pay her the \$1 at the end of each year she and her husband both survive, and will continue to pay her the same if she should survive her husband. It may be considered as being composed of two annuities, the first payable to her during their joint-lifetime, and the second also payable to her during her after-lifetime, if she survives him. Now if the company is going to pay only one of these annuities, it need charge for only one of them. If it is going to pay only the second of them, which is what it agrees to do under a reversionary annuity contract, it need collect the value only of a whole-life annuity-due based on the beneficiary's present age, *less* the value a joint-life annuity-due based on both lives. This operation yields the same result, however, as though an ordinary joint-life annuity on both lives were subtracted from an ordinary life annuity on the life of x, the beneficiary. Thus:

$$
\begin{aligned}
(1 + a_x) - (1 + a_{xy}) &= 1 + a_x - 1 - a_{xy}; \\
&= a_x - a_{xy} \\
&= \frac{N_x}{D_x} - \frac{N_{xy}}{D_{xy}}.
\end{aligned}
$$

In applying these expressions, it is necessary to remember that N_{xy} and D_{xy} require special joint-life commutation columns.

Last Survivor Annuities.—Suppose an annuity of 1 payable to x during his lifetime, with succession to y, if y survives x. Such an annuity, as explained in Chapter II, above, is sometimes purchased by an elderly person to provide a life income to himself, and to his beneficiary in case she survives him. It is, then, an annuity for the life of the survivor of the two. Such may, of course, be granted on three or more lives. The value of a last-survivor annuity based on two lives, expressed by the symbol, $a_{\overline{xy}}$, is evidently greater than a_x, but less than $a_x + a_y$. Now $a_x + a_y$ is sufficient to purchase an annuity of 2 during the joint-lifetime of x and y, to continue as an annuity of 1 to the survivor of the two persons, following the death of one of them. The

last-survivor annuity provides just these benefits, except that the annuity is only 1 during the joint-lifetime. The last survivor annuity is less valuable than $a_x + a_y$, therefore, and it is less valuable by the amount required to purchase a joint-life annuity of 1 on the two lives, x and y. Hence:

$$a_{\overline{xy}} = a_x + a_y - a_{xy};$$

and x being the older of the two,

$$a_{\overline{xy}} = \frac{N_x}{D_x} + \frac{N_y}{D_y} - \frac{N_{xy}}{D_{xy}}.$$

Again, the last term in this expression requires joint-life commutation columns.

Annuity Values not Based on Expectation of Life.—A misconception which was once quite commonly held, and which still persists among some who have not devoted careful thought or study to the rudiments of this subject, is that the present value of a whole life annuity is the same as the present value of an annuity certain for the expectation of life. The curtate expectation is implied in this misconception, although a person who knows the difference between the curtate and the complete expectation would probably not fall into such an error.

If interest were not taken into consideration, this conception would be quite correct. Thus, suppose each of a group of 81,822 persons aged 35 were granted a whole-life annuity of \$1. At the end of one year, or at age 36, the company would have to be prepared to pay \$1 to each one of the survivors, whose number according to the American Experience table of mortality, would be 81,090. Two years hence, the number of dollars to be paid out would be 80,353, since that is the number of persons who survive two years, according to the table. The total number of dollars the company must be prepared to pay out to this group, therefore, would be the same as the total of the number of persons living at the end of one year, plus those surviving two years, three years, etc., to the end of the mortality table. This total, divided by 81,822, would yield the number of dollars paid to each person, on the average; and this number of dollars would be equal to the number of years in the curtate expectation. (See Chapter IV.) At age 35 the *curtate* expectation is 31.28, and \$31.28 would be the present value of an annuity of \$1 issued at age 35, if no allowance were made for interest, because 31.28 would be the number of dollars the company must pay, on the average, to

each person. In other words, some persons will die soon after age 35, whereas others will live to be 80 or 90, the average number of years lived beyond age 35 being 31.28. Thus the probabilities involved in a whole-life annuity are accurately taken care of in the curtate expectation.

Interest being considered, however, the present value of an annuity certain of 1 for the curtate expectation would be $20.0004, approximately, from interest table IV, when interest is at 3 per cent. It will be seen that in this operation, 31 years is the longest period for which any one payment of $1 is discounted; that is, in an annuity certain for 31 years, all of the payments will be made within that prescribed period. In a whole-life annuity, some of the annuity payments will be made beyond the period marked off as the expectation, and these should be discounted for a longer term than 31 years. The last one should be discounted for 60 years to get its true present worth. Now, the longer the period over which a sum is discounted, the less will be its present value. So, while the average *number* of payments on a whole-life annuity issued at 35 will be 31.28, according to the table, the same as the number of payments on an annuity certain for the expectancy, the *present value* of the payments to be made on the life annuity will be less than the present value of the annuity certain, because some of the payments on the life annuity must be discounted for a longer period of time. Thus the present value of a whole-life annuity of 1 issued at age 35 has been determined as $18.9174.

The following sometimes assists in clearing up this matter. Suppose an annuity certain of $1, to continue 31.28 years, is granted to each of 81,822 persons. At the end of one year, the company must pay out $81,822, $1 to each person, or to each person's estate. Likewise, the company must pay $81,822 at the end of each other year in this period. At the end of the period, however, the company makes its last payment, having made 31.28 payments in all. The sum of the payments, each discounted for the time to run before it is due, will be their present value.

If the company grants a whole-life annuity of 1 to 81,822 persons aged 35, however, it will not be called upon to pay out $81,822 at the end of one year, but only $81,090, since only 81,090 persons survive that year, according to the table. At the end of the second year it will be obliged to pay out still fewer dollars, only 80,353 persons surviving to receive their payments. So the company will pay out fewer dollars, in this case, during the first 31.28 years than in the

case of annuities certain, but some of its life-annuity payments will continue far beyond the curtate expectation of 31.28 years, so that the average number of payments made per individual will be 31.28, as before. It is the present value of these delayed payments, which is lower than if they were to be made earlier, that renders the present value of the whole-life annuity less than the present value of an annuity certain for the curtate expectation of life. It is less by a still greater amount than the value of an annuity certain for the complete expectation, which is one-half year longer than the curtate expectation. Also, the higher the rate of interest, and the younger the age, the greater will be the discrepancy.

CHAPTER VI

Net Single Premiums: Insurances: Columns C and M

Term and Whole-life Insurance.—As previously mentioned, insurance policies are usually quoted at so much per $1000 of protection. Premiums, however, are usually calculated in the first place for 1 of protection because of the convenience with which the figure 1 may be handled in mathematical operations, and the ease with which premiums for 1 of insurance may be converted into premiums for $1000, or for any other amount, of protection. Suppose, then, it is desired to find the net single premium for a five-year term insurance of 1, issued at age 35 according to the American Experience table of mortality with interest at 3 per cent. If the company were to issue such a policy to each of 81,822 persons living at age 35, the company must prepare to pay 732 claims within one year, according to the table. The company will, however, collect the net single premiums now at the beginning of the contract, and, by assumption, will pay claims only at the end of the policy-year in which they occur. The company will therefore be called upon to pay 732 one year hence, according to this assumption and according to the mortality table. Hence at the present moment the company must collect the present value of these claims, which according to interest table II is, $.97087379 \times 732 = 710.67961428$. Now the company will also be called upon to pay the claims that arise during the second year. These, according to the table, will be 737 in number; and again, by assumption, they will be paid at the end of the policy-year in which they mature, or in this case, two years hence. To provide for the payment of these anticipated claims, the company must collect their present value, which, with interest at 3 per cent, is, $.94259591 \times 737 = 694.69318567$.

Arranging in tabular form:

$.97087379 \times 732 =$	710 67961428,	present value, first year's claims
$.94259591 \times 737 =$	694 69318567,	present value, second year's claims
$.91514166 \times 742 =$	679 03511172,	present value, third year's claims
$.88848705 \times 749 =$	665 47680045,	present value, fourth year's claims
$.86260878 \times 756 =$	652.13223768,	present value, fifth year's claims
Total........	3402.01694980,	present value, all five years' claims

This total, then, is the sum the company must collect from the group of 81,822 persons living now at age 35, in order to pay 1 to each one's estate in case of death during the next five years. The amount collected per individual would therefore be $3402.01694980 \div 81822 = 0.04157826$, the net single premium. If the amount of insurance on the face of the policy were \$1000, the net single premium could be immediately found by moving the decimal point three places to the right, \$41.58 being the net single premium to the nearest cent.

Using the symbols $|_n A_x$ to represent the net single premium for a temporary insurance of 1, d_x to represent the number dying during the first year following age x, d_{x+2}, the number dying during the second year, etc., the above process of finding the net single premium for a five-year term policy of 1 of insurance may be set down:

$$|_5 A_x = \frac{vd_x + v^2 d_{x+1} + v^3 d_{x+2} + v^4 d_{x+3} + v^5 d_{x+4}}{l_x}.$$

Whole-life Insurance.—To find the net single premium for a whole-life insurance of 1, issued at age 35, proceed to discount future claims in the same manner as in the case of the term policy illustrated above, but continue the discounting to the end of the mortality table, before dividing the present value of all anticipated future payments by 81,822, the number living at the age of issue. This operation would yield the net single premium for 1 of insurance. Multiply it by 1000, or set the point three places to the right, to obtain the net single premium for a \$1000 whole-life policy. Hence, the rule for finding the net single premium is: find the present value of all future claims when and as they will occur, according to the mortality table, and divide the result by the number living, according to the table, at the age of issue.

The net single premium for 1 of insurance is represented by the symbol A_x, in the case of whole-life insurance, and by $|_n A_x$, in the case of term insurance. From the above discussion, then, if d_x is used to represent the number dying during the first year following age x, d_{x+1}, the number dying during the second year following age x, etc., the net single premium for a temporary insurance of 1 to run for any term of years becomes:

$$|_n A_x = \frac{vd_x + v^2 d_{x+1} + v^3 d_{x+2} + \ldots v^n d_{x+n-1}}{l_x};$$

and the net single premium for a whole-life policy becomes:

$$A_x = \frac{vd_x + v^2 d_{x+1} + v^3 d_{x+2} + \text{etc.}}{l_x}.$$

The same results may be obtained by using the probabilities involved, which thus far in this discussion have been allowed to remain in the background. Thus the net single premium for the five-year term insurance might be found as follows:

$$\frac{732}{81822} \times 1 \times \ 97087379 = \frac{710.67961428}{81822}$$

$$\frac{737}{81822} \times 1 \times \ 94259591 = \frac{694.69318567}{81822}$$

$$\frac{742}{81822} \times 1 \times \ 91514166 = \frac{679\ 03511172}{81822}$$

$$\frac{749}{81822} \times 1 \times .88848705 = \frac{665\ 47680045}{81822}$$

$$\frac{756}{81822} \times 1 \times \ 86260878 = \frac{652\ 13223768}{81822}$$

Total. . . . . $\dfrac{3402\ 01694980}{81822} = 0.04157826$, the net single premium, as before.

In other words, the problem may be looked upon in this manner: If a company agrees to pay 1 to the estate of a person now aged 35, if that person should die within the next five years, the face of the policy, or 1, is exposed to five separate and distinct hazards. The company may be obliged to pay because death happens within one year. $\dfrac{732}{81822}$ represents the probability that this event will occur. The probability of occurrence multiplied by the amount at risk yields the value of the risk, if no interest is involved. By assumption, however, the company will collect the premium now and will not pay the claim for one year in any event. Hence the value of the risk discounted for one year, yields the present value of the company's agreement to pay 1 if death should occur the first year. Likewise, $\dfrac{737}{81822} \times 1 \times .94259591$, or the probability that the company will have to pay the claim because of death during the second year, multiplied by the amount at risk, and discounted for the time to run prior to payment, gives the amount the company should ask for exposing 1 to the risk of the insured person's death during the second year following age 35. So the process may be repeated until the end of the term is reached. If the term were made long enough to extend to the end of the mortality table, the net single premium for a whole-life policy would appear. Hence, as with annuities, multiply the amount at risk by each probability of its occurrence,

and discount the result for each period of time the money may remain under the company's control.

Commutation Columns C and M.—As in the case of annuities, the elementary methods of calculation are used only for purposes of illustration. In actual computations they would be entirely too tedious, even when the modern computing machines are available. Hence methods have been devised for the construction and use of auxiliary tables, by which premiums may be determined with much less effort. Thus, multiplying the fraction on the right-hand side of expression,

$$A_x = \frac{vd_x + v^2d_{x+1} + v^3d_{x+2} + v^4d_{x+3} + \text{etc.}}{l_x}, \text{ by } \frac{v^x}{v^x},$$

the following result is obtained:

$$A_x = \frac{v^{x+1}d_x + v^{x+2}d_{x+1} + v^{x+3}d_{x+2} + v^{x+4}d_{x+3} + \text{etc.}}{v^x l_x}.$$

The values of $v^x l_x$ at each age have been determined and tabulated in the commutation column headed D_x, as previously explained. The value of the denominator of the fraction, then, may be learned at once by referring to the commutation tables. Additional columns must be constructed, however, before the value of the numerator can be as readily ascertained. The first step in the construction of these additional columns is to ascertain the value of $v^{x+1}d_x$ at all ages, and set down the results opposite age x. Starting at the end of the table for convenience of illustration, at age 95,

$$v^{x+1}d_x = v^{96}d_{95} = .05856342 \times 3 = .17569026.$$

Then $v^{x+1}d_x$, when x is 94, would be:

$$v^{x+1}d_x = v^{95}d_{94} = .06032032 \times 18 = 1.08576576.$$

Then continuing back to age 10, the complete column of values of $v^{x+1}d_x$ would appear. This column is headed C_x and is spoken of as commutation column C. It is so seldom necessary to use this column that it is not included in the commutation tables in this book. A column which is derived from column C, however, is very often used, hence a few of the values of column C are herewith set down:

Age	C_x	Age	C_x
90	26 13805	93	3 60354
91	16 21475	94	1 08577
92	8.76715	95	.17569

Substituting C_x for $v^{x+1}d_x$ wherever it appears in the numerator, and substituting D_x for $v^x l_x$ in the denominator, of the fraction, the net single premium for a whole-life insurance of 1 now becomes:

$$A_x = \frac{C_x + C_{x+1} + C_{x+2} + C_{x+3} + \text{etc.}}{D_x}.$$

The convenience of having the values of $C_x + C_{x+1}$ + etc., summed to the end of the table from each age immediately becomes apparent, the sum showing the numerator of the fraction at once. Therefore, column M has been formed, by summing the values of C_x. Thus, again starting at the end of the table, the last figure in column M is .17569, the same as in the C_x column. Add the last two figures in column C to get 1.26146, the figure in the M_x column opposite age 94. Add to this, the figure in the C_x column opposite 93, to get 4.86499, the value of M_x when x is 93. (The addition produces 4.86500, due to tabulating only to five decimal places.) Then to this add the value of C_x at 92, and 13.63215, the value of M_x at 92 will appear. Continuing back to age 10, column M of the commutation tables, is constructed. Then, since $M_x = C_x + C_{x+1}$ + etc., the fraction expressing the net single premium for a whole-life insurance of 1 becomes:

$$A_x = \frac{C_x + C_{x+1} + C_{x+2} + \text{etc.}}{D_x} = \frac{M_x}{D_x}.$$

Now if x is 35, the net single premium is:

$$A_{35} = \frac{M_{35}}{D_{35}} = \frac{12209.42}{29078.18} = 0.41988.$$

For a \$1000 policy issued at age 35, the net single premium would be, \$419.88.

Furthermore,

$$A_{x+n} = \frac{M_{x+n}}{D_{x+n}}.$$

Hence if $x = 35$ and $n = 5$,

$$A_{40} = \frac{M_{40}}{D_{40}} = \frac{11000.40}{23943.9} = 0.45942.$$

For a term insurance of 1, it has been shown that:

$$
\begin{aligned}
|_n A_x &= \frac{vd_x + v^2 d_{x+1} + \ldots v^n d_{x+n-1}}{l_x} \\
&= \frac{v^{x+1}d_x + v^{x+2}d_{x+1} + \ldots v^{x+n}d_{x+n-1}}{v_x l_x} \\
&= \frac{C_x + C_{x+1} + C_{x+2} + \ldots C_{x+n-1}}{D_x}
\end{aligned}
$$

Since, however, $C_x + C_{x+1} + C_{x+2} +$ etc., to the end of the table equals M_x, the value of M_x may be looked upon as the sum of two series, one consisting of $C_x + C_{x+1} + C_{x+2} +$ etc., up to and including C_{x+n-1}; the other beginning with $C_{x+n} + C_{x+n+1} +$ etc., on to the end of the table. The value of either of these two series subtracted from the value of M_x, must yield the value of the other. The value of the second series has been found to be M_{x+n}. Hence,

$$C_x + C_{x+1} + C_{x+2} + C_{x+3} + \ldots C_{x+n-1} = M_x - M_{x+n};$$

and

$$|_nA_x = \frac{C_x + C_{x+1} + C_{x+2} + \ldots C_{x+n-1}}{D_x}$$

$$= \frac{M_x - M_{x+n}}{D_x}.$$

Now to find the net single premium for the five-year term insurance issued at age 35 from the commutation columns, Appendix C:

$$|_nA_x = \frac{M_x - M_{x+n}}{D_x}$$

$$|_5A_{35} = \frac{M_{35} - M_{40}}{D_{35}}$$

$$= \frac{12209.42 - 11000.40}{29078.18} = 0.041578,$$

or \$41.58 per \$1000 of insurance, as before.

It is possible to purchase a whole-life policy now at age x but have it become effective only after a period of n years. Such a contract is known as a deferred insurance, and the present net single premium for it would be the present value of a pure endowment due at the end of n years whose face is the net single premium for the whole-life policy if issued at age $x + n$. Hence:

$$_{n|}A_x = {}_nE_x\frac{M_{x+n}}{D_{x+n}} = \frac{D_{x+n}}{D_x} \times \frac{M_{x+n}}{D_{x+n}} = \frac{M_{x+n}}{D_x}.$$

Single Premium from Annuity Values.—It will be recalled that the present value of a life annuity-due, of 1, is $1 + a_x$, as found in Chapter V. An annuity-due of 1 provides for the payment of 1 *at the beginning* of every contract-year the annuitant enters. A life annuity, on the other hand, provides for the payment of 1 to the annuitant *at the end of every contract-year he completes.* Its value is a_x, or 1 less than the value of an annuity-due, as shown in Chapter V. Now suppose a third type of life annuity which provides for the payment of 1

to the annuitant, or to his estate, *at the end of every year he enters.* The value of this contract is less than that of an annuity-due, because the first payment will not be made until the end of one year; the second payment will not be made until the end of the second year, and then only if the annuitant is living at the beginning of that year, etc. It is greater than the value of an annuity, however, because the first payment must be made at the end of the first year whether the annuitant survives the year or not, and the second payment must be made at the end of the second year whether the annuitant survives that year or not, provided he is living at its beginning, etc.; since the contract calls for a payment at the end of each year he enters. Its value is evidently the same as the value of an annuity-due, except for the fact that the first payment is delayed one year. In other words it is an annuity-due granted now at age x but beginning one year hence. Therefore its present value is the value of an annuity-due based on the present age x, discounted for one year; that is, its value is $v(1 + a_x)$.

Now, at some time in the future the annuitant will enter a contract-year, and fail to complete it alive. The above annuity whose value is $v(1 + a_x)$, will pay his estate 1 at the end of that year. An ordinary annuity whose value is a_x, will not pay his estate anything at the end of that year. The difference between $v(1 + a_x)$ and a_x, therefore, is the present value of an agreement to pay 1 at the end of the year in which death occurs, which is the net single premium for a whole-life insurance of 1. Hence:

$$A_x = v(1 + a_x) - a_x.$$

Example.—Calculate the net single premium for a whole-life insurance of 1 issued at age 35 according to the American Experience table and 3 per cent.

Solution.—In Chapter V, values, when $x = 35$, were found as follows:

$$a_x = a_{35} \qquad = 18.9174$$

$$1 + a_x = 1 + a_{35} = 19.9174.$$

From interest Table II, $v = .97087379$, when $i = .03$. Therefore:

$$A_x = v(1 + a_x) - a_x$$

$$A_{35} = (.97087379 \times 19.9174) - 18.9174$$

$$= 19.33728 - 18.9174$$

$= 0.41988$, the net single premium for 1 of insurance, or \$419.88 for \$1000 of insurance, as before.

It appears, then, that the value of A_x could be determined from column N, of the commutation tables, without the aid of column M. Thus it was found in Chapter V that

$$1 + a_x = \frac{N_{x-1}}{D_x}, \quad \text{and} \quad a_x = \frac{N_x}{D_x}.$$

Hence:

$$A_x = v(1 + a_x) - a_x = v\frac{N_{x-1}}{D_x} - \frac{N_x}{D_x} = \frac{vN_{x-1} - N_x}{D_x}$$

From this expression it appears that column M could have been constructed from the values in column N; for it has now been found that,

$$A_x = \frac{vN_{x-1} - N_x}{D_x} = \frac{M_x}{D_x}.$$

Whence,

$$M_x = vN_{x-1} - N_x.$$

If $x = 35$, and $i = .03$, $M_{35} = vN_{34} - N_{35}$

$= (.97087379 \times 579160.7) - 550082.5$

$= 12209.4$, the value as found in column M opposite age 35.

The relation of single premiums to annuities is of sufficient importance to warrant further explanation. Since $v = \frac{1}{1+i}$, the above expression of the net single premium for a whole-life insurance of 1, may be modified as follows:

$$\begin{aligned} A_x &= v(1 + a_x) - a_x \\ &= \frac{1 + a_x}{1 + i} - a_x \\ &= \frac{1 + a_x}{1 + i} - \frac{(1 + i)a_x}{1 + i} \\ &= \frac{1 + a_x - a_x - ia_x}{1 + i} \\ &= \frac{1 - ia_x}{1 + i} = \frac{1}{1 + i} - \frac{ia_x}{1 + i}. \end{aligned}$$

If interest is at 3 per cent, $\frac{i}{1+i} = \frac{.03}{1.03} = .02912621$, which is one

year's discount, for, $1 - .97087379 = .02912621$. Substituting d (one year's discount) for $\frac{i}{1+i}$, and continuing:

$$A_x = \frac{1}{1+i} - \frac{ia_x}{1+i}$$
$$= v - da_x$$
$$= 1 - d - da_x$$
$$= 1 - d(1 + a_x)$$

The problem, then, of computing the net single premium for an insurance of 1 may be conceived of in this manner: If the insured were to die at once, the value of the insurance would be 1. Since, on the average, however, the insured will live a considerable period of time, the single premium is less than the face of the policy. How much less? In computing net single premiums, the company allows a discount of d each year the money remains under its control. It will have the net single premium under its control during the lifetime of the insured. By assumption, it will retain control of this money at least one year. The discount, d, then, becomes an annuity-due for the lifetime of the insured, the present value of which is allowed the insured in the form of a reduction from the face of the policy. The difference between the face and the present value of annuity-due of d is the net single premium.

Example.—Calculate the net single premium as of age 35 for a whole-life insurance of 1, according to the American Experience table and 3 per cent.

Solution:

$$A_x = 1 - d(1 + a_x)$$
$$= 1 - .02912621 \times 19.9174$$
$$= 1 - .580118$$
$$= .41988, \text{ or \$419.88 per \$1000 of insurance, as before.}$$

Endowment Insurance.—As explained in Chapter II, an endowment insurance policy provides for the payment of its face value in case the insured person dies during the endowment period, and also in case he survives to the end of that period. The basis for the determination of the net single premium for such a contract has already been given. The net single premium sufficient to provide for the first contingency, that of death during the term, is evidently the

net single premium for a term insurance policy to run for the endowment insurance period; the net single premium for the second contingency, that of survival to the end of the term, is the net single premium for a pure endowment maturing at the end of the endowment insurance period. The sum of these two premiums, therefore, would be the net single premium for the endowment insurance.

An endowment insurance policy is thus a combination of a term insurance policy and a pure endowment. Using symbols to which the commutation columns may be at once applied, it has been shown that the net single premium for a term insurance policy may be expressed:

$$|_nA_x = \frac{M_x - M_{x+n}}{D_x},$$

and the present value of a pure endowment may be stated:

$$_nE_x = \frac{D_{x+n}}{D_x}.$$

The value of both of these would be their sum; hence the present value of an endowment insurance of 1, or,

$$|_nAE_x = \frac{M_x - M_{x+n}}{D_x} + \frac{D_{x+n}}{D_x} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x}.$$

Example.—Calculate the net single premium for a 20-year endowment insurance policy of $1000 issued at age 35 according to the American Experience table of mortality with interest at 3 per cent.

Solution:

$$|_nAE_x = \frac{M_x - M_{x+n} + D_{x+n}}{D_x},$$

Referring to columns M and D in the Appendix:

$$|_{20}AE_{35} = \frac{12209.42 - 7748.344 + 12703.88}{29078.18}$$

$= .590303$, the net single premium for an endowment insurance of 1, or $590.30 for a 20-year endowment insurance policy of $1000.

To find the net single premium for a double endowment of 1; that is, one that promises to pay 1 if death should occur during the period, and 2 in case of survival, multiply the net single premium for the pure endowment by 2. The value of the double endowment thus becomes,

$$\frac{M_x - M_{x+n}}{D_x} + \frac{2D_{x+n}}{D_x}.$$

The net single premium for a semi-endowment of 1, which promises 1 in the event of death during the period, but only $\frac{1}{2}$ in case of survival to the end of it, may be found by taking $\frac{1}{2}$ of the value of the pure endowment element. Thus:

$$|_{n}A_x + \frac{1}{2}{}_nE_x = \frac{M_x - M_{x+n}}{D_x} + \frac{D_{x+n}}{2D_x}.$$

Installment Insurance.—Suppose an insurance policy provides for the payment of ten annual installments of $100 each, beginning upon the happening of an event insured against. The amount which the company must have on hand upon the happening of the event, in order to meet these ten payments, is their present value. Thus, upon the happening of an event insured against, the company must have $100 to pay out at once, the present value of $100 to be paid out one year later, the present value of $100 to be paid two years later, etc. The present value of a ten-year annuity-due and certain may be ascertained from interest table IV. At 3 per cent, it is 8.7861, for ten annual payments of 1, or $878.61 for ten annual payments of $100, the first payment to be made at once. $878.61, therefore, is the amount of insurance for calculation purposes. Hence when the net single premium for 1 of insurance has been determined, no matter what may be the type of policy, the net single premium for 1 of insurance multiplied by 878.61 gives the single premium for the contract to be issued. At a different rate of interest, or for installments of different amounts or over different periods of time, the present value of the installments would, of course, be greater or less than 878.61, as the case may be; the amount could be ascertained by consulting interest table IV.

Suppose a policy allows the beneficiary to choose between a cash payment of $1000 at once upon the happening of the event insured against, or ten annual installments beginning upon the happening of such an event. Since the beneficiary may choose the $1000 cash, $1000 must be considered as the amount of insurance. Premium computation, therefore, is exactly the same as if no option were offered by the company. Should the beneficiary, however, elect to receive ten annual installments in lieu of $1000, each installment should be greater than 100. As explained in a previous chapter, the amount of each installment should be as much greater than 1, as 1000 is greater than the present value of 1. At 3 per cent, therefore, each of the ten installments should be $\frac{1000}{8.7861} = \$113.81$.

The Continuous-installment Option.—If the policy offers the option of a cash settlement or settlement by means of a continuous installment, or annuity-due, certain and continuous, the face of the policy, which may be demanded by the beneficiary at the happening of an event insured against, is again the amount of insurance, and premium calculations are no different from what they would be if no option were offered. If the continuous installment is elected, the face of the policy is treated as the present value of an annuity-due, certain for the specified number of years (usually twenty) and continuous, beginning at once upon the death of the insured person, and calculated according to the age of the beneficiary when the first payment falls due. The method of calculation was explained in Chapter V.

Continuous-installment Insurance.—When the policy leaves no option, but definitely states that, upon the death of the insured person, annual installments of stipulated amounts will be paid to a named beneficiary, certain for a number of years and continuous during the after-lifetime of that beneficiary beyond the guaranteed period, the net single premium must be based upon two lives, that of the beneficiary and that of the insured person. For example, suppose a contract is granted to a person now aged 35, which promises to pay 1 per annum for 20 years certain and continuous beginning upon the insured person's death, to a beneficiary now aged 30. What should be the net single premium? The benefit may be considered as two separate benefits; one, an insurance policy for an amount equal to the present value of the 20 guaranteed payments; and the other, a reversionary annuity to the beneficiary for her after-lifetime beyond the guaranteed period which begins with the insured's death. The method of calculating the net single premium for an insurance payable in annual installments has been demonstrated in this chapter, and the determination of the present value of a reversionary annuity is explained in Chapter V. It is important to note, however, that the reversionary annuity becomes effective only at the end of twenty years after the death of the insured. To find the present value of this benefit, the present value of a life annuity on the life of the beneficiary at age 30, deferred 19 years, should be diminished by the present value of a joint-life annuity on the life of the beneficiary at age 49 and the life of the insured at age 35, to arrive at the true present value of the reversionary annuity benefit. This present value, added to the net single premium for the commuted value of the installments certain, yields the net single premium for the continuous-

installment policy, when the installments are to be paid annually in advance.

The net single premium obtained in this manner is a little more than sufficient to provide for a monthly income policy, one-twelfth of whose annual installment is paid monthly. Thus, suppose the net single premium has been determined for a continuous-installment policy which promises to pay $600 upon the death of the insured, a like sum each year thereafter until 20 payments have been made, the payments then being continued throughout the remainder of the beneficiary's lifetime. This premium is slightly greater than would be necessary to provide a payment of $50 upon the death of the insured, a like sum each month thereafter until 240 payments have been made, and continuing throughout the after-lifetime of the beneficiary. From the principles explained previously it would be possible to calculate the present value of a reversionary annuity payable monthly. The present value, also, of 240 monthly installments is quite easily obtained. The difference between the net single premium obtained in this manner, however, and that calculated on the basis of annual payments, is so slight that the additional calculations are scarcely worth while.

The Monthly-income Option.—When the policy provides a definite face value of, say, $1000, which may be demanded by the beneficiary upon the death of the insured, or which may be settled, either at the option of the beneficiary or of the insured, in the form of installments certain and continuous, the amount of each installment depending upon the age of the beneficiary when the claim falls due, the difference between annual and monthly payments should be considered. Premium calculations, as explained above, remain the same as if no option were offered. To determine the amount of each installment when they are to be paid annually, the face value is divided by the present value of a whole-life annuity-due, certain and continuous, based on the beneficiary's attained age, as was also explained above. When the installments are to be paid monthly, to arrive at the correct amount of each installment, the face value should be divided by an annuity-due, certain and continuous, based on the beneficiary's attained age but payable monthly. The amount may be approximated, however, by dividing the amount of an annual installment, as if the payments were to be made annually, by the present or commuted value of 1 per month for 12 months, the first payment of 1 being due at once.

Income or Guaranteed-interest Bonds.—While so-called guaranteed-interest bonds are seldom made use of in this country, there are several types of them that are of sufficient interest to merit brief explanations. It is evident that instead of making a cash settlement of the amount insured when a claim arises, the company could, should the beneficiary or the insured desire such an arrangement, afford to keep the money in its possession, pay interest on it to the beneficiary at the rate assumed in premium computations for any definite period of time or for the remainder of the beneficiary's life-time, and then pay the full face value, without any addition to the regular net single premium. It could even afford to make this type of settlement participating, in that it would be able to pay the beneficiary the net rate of interest realized on its investments. If the company wishes to *guarantee* a rate of interest *higher* than the assumed rate which it may reasonably expect to earn, however, the difference between the guaranteed rate and the assumed rate is an additional benefit, for which the company should charge an extra premium in addition to the regular net single premium. Thus suppose the policy provides that upon the happening of some event insured against, the company will pay \$60 at once, \$60 each year thereafter until 20 payments have been made, and then \$1000 at the end of 20 years. The question to be answered first in the computation of the net single premium for this contract, is: what sum must be considered as the amount of insurance? In other words, how much must the company have on hand when one of the events insured against happens, in order to meet these payments? If the company assumes it will be able to earn 3 per cent compounded annually on funds under its control, the present value of the \$1000 due 20 years hence from the time the claim arises, is \$553.67, from interest table II. The present value of the \$60 due at once, is evidently \$60 at the time the claim arises. The present value of the remaining 19 payments of \$60 each is $14.32379911 \times 60 = \$859.43$, from interest table IV. The present value of all of the 20 payments of \$60 each is, therefore, $60 + 859.43$, or \$919.43. At the time the claim arises, then, the present value of all the payments which the company must make, is, $553.67 + 919.43$, or \$1473.10. Using this sum as the face of the policy, or "bond," compute the net single premium in the ordinary manner. This type can be granted on the term, whole-life, endowment insurance, or other plan, but premium calculations for an insurance of 1 on the several plans have already been explained.

The face value of \$1473.10, which would be used in this instance

for calculating the net single premium, may be ascertained in another manner. The full value of $1000 may be considered as if it were due at the time the claim arises. If the company has that amount under its control, it could pay $30 from interest earned on the $1000 at the end of one year and $30 each year thereafter until 19 payments have been made, and then the $1000 at the end of 20 years, without an additional premium. The company, however, agrees to pay $60 at once, and $60 per annum thereafter until 20 payments have been made, and then the $1000 at the end of 20 years. Hence it must charge an extra premium for this first payment of $60 at once and for the *extra* annual payments of $30 per year for the next 19 years, the first of which is due one year following the date when the claim arises. The present value of the extra payments of $30 per annum payable at the end of each year, for 19 years, the first to be made one year after the claim arises, is, $14.32379911 \times 30 = \$429.71$. The company will not need quite so much, however, since it will hold the $1000 during the twentieth year following the date of the claim, and could therefore pay $1030, 20 years hence, whereas it is only obligated to pay $1000. The $429.71 may therefore be reduced by the present value of $30 due 20 years hence, or by $.55367575 \times 30$, or $16.61. Hence $429.71 - 16.61 = \$413.10$, the amount necessary for the company to meet all of the extra payments of $30 above the interest earnings on the $1000, except the first payment of $60 due at once when the claim arises. The $60 at once added to the present value of the other payments above interest earnings at the assumed rate of 3 per cent, or $413.10, equals $473.10, the present value of the total benefits in excess of the $1000 face value. This sum plus the face, or $473.10 + 1000$, yields $1473.10, the total amount of insurance, as before.

Another type of bond might provide for the payment of $1000 at once upon the happening of an event insured against, $60 at the end of each year thereafter for 20 years, and another $1000 at the end of the 20-year period. Here the amount of insurance is $1000, the present value of the $1000 due at once,

+ 892.65, the present value of $60 per annum at the end of each year for 20 years,
+ 553.67, the present value of $1000 due 20 years hence,

= 2446.32, the total amount.

Still another type of policy might provide for the payment of $60

at once to a beneficiary upon the death of the insured, \$60 per annum thereafter for the after-lifetime of the beneficiary, and then \$1000 at the latter's death. In this case, the net single premium would be the sum of the net single premiums for the following benefits: 1000 + 60, due at once upon the death of the insured + [(the present value of a whole-life annuity of 30 to the beneficiary) — (a joint-life annuity of 30 on the lives of both insured and beneficiary)] — the net single premium for an insurance of 30 on the life of the beneficiary, since one year's interest will have accumulated on the \$1000 held by the company during the last year of the beneficiary's life, and the company will only pay \$1000 at the end of that year,—not \$1030.

Joint-life Insurance.—As explained in Chapter II, a joint-life insurance policy becomes payable by the company upon the happening of the first death among the joint-lives insured. Thus suppose an insurance of 1 is granted to x and y jointly, x now being 35, and y, 30 years of age. There are two methods of arriving at the probability that the claim will arise during the first year. First, it may be taken as the sum of the three compound probabilities that x will die within, and y, survive, the first year, that y will die within, and x survive, that year, and that both will die within one year. A much easier method of arriving at the same result, however, is to subtract the compound probability that both will survive that year, from 1; that is, if both survive that year, the company will not be called upon to pay the claim at the end of it; whereas, under any other circumstances, the claim will arise. The probability insured against during the first year, then, is, $1 - \left(\frac{81090}{81822} \times \frac{84721}{85441}\right)$, according to the American Experience table of mortality. The probability that the claim will arise the second year is the probability that the first death of the two will occur that year, which, it may be demonstrated, is the probability that both will survive to the beginning of that year, *less* the probability that both will survive to its close. Thus, $\left(\frac{81090}{81822} \times \frac{84721}{85441}\right) - \left(\frac{80353}{81822} \times \frac{84000}{85441}\right)$, is the probability that the claim will arise during the second year. Then, continuing until the older life reaches the end of the mortality table, the probability of the claim arising in each separate year will have been determined. Thus, the operation would be:

$$1 - \left(\frac{81090}{81822} \times \frac{84721}{85441}\right) \times 1 \times .97087379 =$$

present value, 1st year's risk;

$$\left[\left(\frac{81090}{81822}\times\frac{84721}{85441}\right)-\left(\frac{80353}{81822}\times\frac{84000}{85441}\right)\right]\times 1\times .94259591 =$$

present value, 2d year's risk;

$$\left[\left(\frac{80353}{81822}\times\frac{84000}{85441}\right)-\left(\frac{79611}{81822}\times\frac{83277}{85441}\right)\right]\times 1\times .91514166 =$$

present value, 3d year's risk,

etc., to the end of the mortality table for the oldest life.

The sum of the present values of all of the risks would be the net single premium. For a term policy, the operation would merely be shortened. Thus for a 10-year term policy, the process need only be continued until the present value of the 10 separate years' risks have been determined.

As in the case of policies on single lives, however, this method of procedure is entirely too tedious for practical use. Inspecting the first few figures in the above demonstration and substituting symbols for their values, it will be observed that the common denominator of the fractions expressing the compound probabilities is in each instance, $l_x l_y$, or l_{xy}. Since $\frac{l_{xy}}{l_{xy}} = 1$, the present value of the first year's risk may be expressed,

$$\left(\frac{l_{xy}}{l_{xy}} - \frac{l_{x+1\,:\,y+1}}{l_{xy}}\right)\times .97087379.$$

This, however, is equal to $\frac{v(l_{xy} - l_{x+1\,:\,y+1})}{l_{xy}}$, when $v = \frac{1}{1.03}$. The present value of the second year's risk, then, may be expressed,

$$\frac{v^2(l_{x+1\,:\,y+1} - l_{x+2\,:\,y+2})}{l_{xy}},$$

and the value of all of the years' risks, or the net single premium, may be expressed:

$$A_{xy} = \frac{v(l_{xy} - l_{x+1\,:\,y+1}) + v^2(l_{x+1\,:\,y+1} - l_{x+2\,:\,y+2}) + \text{etc.}}{l_{xy}}$$

Multiplying the right-hand side of this equation by $\frac{v^x}{v^x}$, which does not change its value, it appears that:

$$\begin{aligned} A_{xy} &= \frac{v^{x+1}(l_{xy} - l_{x+1\,:\,y+1}) + v^{x+2}(l_{x+1\,:\,y+1} - l_{x+2\,:\,y+2}) + \dots}{v^x l_{xy}} \\ &= \frac{C_{xy} + C_{x+1\,:\,y+1} + C_{x+2\,:\,y+2} + \dots}{D_{xy}} \\ &= \frac{M_{xy}}{D_{xy}}. \end{aligned}$$

Commutation tables based on joint-lives may then be made use of to ascertain the value of $\frac{M_{xy}}{D_{xy}}$, and the net single premium may be readily determined. Likewise, with proper modifications, the formulas for insurances, annuities, and endowments, based on single lives, can be applied to determine all of the usual joint-life values.

Insurance Payable at the Moment of Death.—So far, in the discussion of insurance premiums, it has been assumed that claims will be paid at the end of the policy-year in which they occur. This is one of the assumptions on which practical calculations are usually based, and its use is continued for reasons explained in Chapter V. In practice, American companies pay their losses very promptly after receiving due proof that a legitimate claim has arisen. When the death rate is approximately uniform throughout the year, therefore, it appears that claims are paid about six months earlier, on the average, than is assumed in calculating the net single premium. To calculate the net single premium for a policy payable at the moment of death, which, on the average, will occur very nearly at the middle of the policy-year, it is necessary to take into account the interest which would have been earned had the policy not been paid until the end of the policy-year. A_x represents the net single premium for a whole-life insurance of 1, payable at the end of the policy-year in which death occurs. If the insurance of 1 were to be paid six months earlier, or at the approximate moment of death, on the average, six months' interest on 1 would be lost by the company. At 3 per cent, the interest lost would be 0.015, or $\frac{.03}{2}$. If this is to be lost, it may be looked upon as an extra benefit conferred upon the insured, and for this benefit, the correct net single premium may be determined. Since A_x multiplied by the amount of insurance yields the net single premium for that insurance, $.015 \times A_x$ or $.015A_x$, is the net single premium for the extra benefit conferred when insurances are made payable at the moment of death. The regular net single premium for 1 of insurance, is A_x. The sum of these two, or $A_x + .015A_x$, then, would be the net single premium for a whole-life insurance of 1, payable at the moment of death instead of at the end of the policy-year in which death occurs; but,

$$A_x + .015A_x = A_x(1 + .015) = A_x\left(1 + \frac{03}{2}\right).$$

Hence the expression, approximately correct, showing the value of a whole-life insurance of 1 payable at the moment of death, is:

$$\overline{A}_x = A_x\left(1 + \frac{i}{2}\right).$$

Approximation of Term from Age and Premium.—As explained in Chapter IX, it often happens that a policyholder will have occasion to discontinue the payment of his premiums on an insurance policy. Under such circumstances one of the options offered by the companies is that he may accept a term insurance policy in lieu of a cash settlement of his surrender value, the term to extend as far into the future from his attained age as the cash value, used as a net single premium, will carry it. In practice, then, it is quite often convenient to know how to determine the term through which a given net single premium will carry a policy from some specified age.

The symbol for the net single premium for a term insurance policy of n years' duration, is $|_n A_x$, and its value in terms of the commutation symbols, is $\dfrac{M_x - M_{x+n}}{D_x}$, as shown above. Now supposing the term, n, is unknown, and letting it be represented by n', the net single premium for a term-insurance policy issued at age x, but with the length of the term unknown, may be expressed:

$$|_{n'} A_x = \frac{M_x - M_{x+n'}}{D_x}.$$

The net single premium being given, the values of all of the above terms are either known, or directly ascertainable from the commutation columns, except $M_{x+n'}$. Now if the value of $M_{x+n'}$ could be found, the term of years elapsing between M_x and $M_{x+n'}$ is evidently the length of time the term policy is to run. Clearing the above equation of fractions,

$$D_x(|_{n'} A_x) = M_x - M_{x+n'}.$$

Transposing,

$$M_{x+n'} = M_x - D_x(|_{n'} A_x).$$

The value of $|_{n'} A_x$ being given, and those of M_x and D_x having been ascertained from the commutation columns, the multiplication of the value of D_x by that of $|_{n'} A_x$, and the subtraction of the result from the value of M_x, may be performed, and the value of $M_{x+n'}$ will appear. If it is desired to approximate the term only to the nearest year, reference may be had to column M to find the value in that column which most nearly corresponds to the ascertained value of $M_{x+n'}$. The age at which this value is found in column M is the age at which the insurance ceases. The age at which the term begins (age x) being given, and the age at which it ceases (age $x + n'$) having been ascertained, the length of the term at once appears.

EXAMPLE.—Given \$41.58 as the net single premium for a \$1000 term policy issued at age 35 according to the American Experience table and 3 per cent, to approximate the term to the nearest full year.

If \$41.58 is the net single premium for a term insurance of \$1000, .04158 must be the net single premium for a temporary insurance of 1; expressed by $|_{n'}A_x$, since the term is unknown. Referring to the commutation columns for the values of M_x and D_x at age 35,

$$\begin{aligned} M_{x+n'} &= M_x - D_x(|_{n'}A_x) \\ &= 12209.42 - (29078.18 \times .04158) \\ &= 12209.42 - 1209.07 \\ &= 11000.35. \end{aligned}$$

Referring again to column M, the value in that column nearest 11000.35, is 11000.40, which is opposite age 40. The policy which is issued at 35 ceases at age 40, therefore, and so the term is 5 years.

In the above example, the term was approximately 5 years even, because the net single premium for a 5-year term policy at age 35 was chosen. It would have been exactly even if a sufficient number of decimal places had been taken in the values of M_x, D_x and $|_{n'}A_x$. In practice it seldom happens that the term will be an exact number of full years. It will usually be for a number of years and then for a certain number of days longer. To find approximately the number of days additional, take the difference between the values in column M at the exact ages below and above the value of $M_{x+n'}$, and the difference between the value of M at the age below $M_{x+n'}$, and $M_{x+n'}$. The ratio of this last difference to the first shows what proportion of the 365 days additional the policy should remain in force.

EXAMPLE.—Suppose the net single premium for a \$1000 term insurance policy issued at age 35 on the basis of the American Experience table and 3 per cent, is \$43.14. For 1 of insurance, then, the premium would be .04314, which is the value of $|_{n'}A_x$. From the formula:

$$\begin{aligned} M_{x+n'} &= M_x - D_x(|_{n'}A_x) \\ &= 12209.42 - (29078.18 \times .04314) \\ &= 12209.42 - 1254.43 \\ &= 10954.99. \end{aligned}$$

Now, 10954.99, falls between the values in column M opposite ages 40 and 41. It is at once evident that the policy should run for

5 years plus a certain number of days additional. The difference between the value of M at age 40, and its value at age 41, is

$$11000.40 - 10772.72, \quad \text{or} \quad 227.68.$$

Hence if the value of $M_{x+n'}$ had been 227.68 less than 11000.40, the value of M at 40, the term should have been 6 years, because $M_{x+n'}$ would then have corresponded to the value of M at age 41, but the actual value of $M_{x+n'}$ is 10954.99, or 45.41 less than the value of M at 40. The policy should extend $\frac{45.41}{227.68}$ of a year beyond age 40, therefore, or, $\frac{45.41}{227.68} \times 365 = 73$ days, approximately. The term, therefore, is five years and seventy-three days.

A slight inaccuracy is involved in these calculations since they are based on the assumption of a uniform death-rate throughout the policy-year. Strict adherence to the facts involves an increasing death-rate throughout the policy-year, at ages above 10. The error is on the safe side, from the company's point of view, however, since on this assumption it is calculated that the premium will be expended, and the policy will therefore cease, a little sooner than the rate of mortality makes necessary.

Approximation of Endowment Periods.—Suppose the age of the insured and the net single premium of an endowment insurance of 1 be given, to approximate the term to maturity. As found above, the formula for determining the net single premium for an endowment insurance of 1 from commutation columns, is,

$$|_n AE_x = \frac{M_x - M_{x+n} + D_{x+n}}{D_x}.$$

When the length of the endowment period is unknown, the formula may be written

$$|_{n'} AE_x = \frac{M_x - M_{x+n'} + D_{x+n'}}{D_x}.$$

Clearing of fractions, $D_x(|_{n'}AE_x) = M_x - M_{x+n'} + D_{x+n'}$. Transposing the unknown quantities to the left-hand side of the equation,

$$M_{x+n'} - D_{x+n'} = M_x - D_x(|_{n'}AE_x).$$

Now find the values of M_x and D_x from the commutation columns, and perform the multiplication and subtraction indicated by the right-hand side of the equation. The value of $M_{x+n'} - D_{x+n'}$ will then have been determined, although the value of neither $M_{x+n'}$ nor $D_{x+n'}$ is

known. Next subtract the values of D from the values of M as found in the columns at ages at which it seems likely, from inspection, that the endowment might mature. Before many such subtractions have been made, a value next above and one next under the ascertained value of $M_{x+n'} - D_{x+n'}$, will have been found. If the endowment period is to be approximated to the nearest whole year only, the value of M minus the value of D which is nearest the value of $M_{x+n'} - D_{x+n'}$, will be found opposite the age at which the endowment matures. The age at which it begins, age x, deducted from the age at which it ends, age $x + n'$, will of course yield, n', the number of years in the endowment period, as was the case in the term insurance explained above. If the approximation of a number of days additional is required, proceed in the same manner as in estimating the length of a term policy, remembering that the difference between the differences in the values of M and D at the age next higher and next lower than the exact age at which the endowment terminates, would represent a full year's extension of the endowment term.

EXAMPLE.—Suppose an endowment insurance of 1 is issued at age 35, the net single premium for which is 0.58712, on the basis of the American Experience table and 3 per cent. Find the period.

Solution:

$$\begin{aligned} M_{x+n'} - D_{x+n'} &= M_x - D_x({}_{\overline{n'|}}AE_x) \\ &= 12209.42 - (29078.18 \times .58712) \\ &= -\,4862.96. \end{aligned}$$

Observing differences between the values of M and D in the commutation tables, the above value of $M_{x+n'} - D_{x+n'}$ is found to lie between ages 55 and 56. Thus:

$$M_{55} - D_{55} = -\,4955.54$$

$$M_{56} - D_{56} = -\,4595.52.$$

The endowment period is therefore somewhat longer than 20 years. Now,

$$(M_{55} - D_{55}) - (M_{56} - D_{56}) = -\,360.02,$$

$$(M_{55} - D_{55}) - (M_{x+n'} - D_{x+n'}) = -\quad 92.58.$$

Hence,

$$\frac{92.58}{360.02} \times 365 = 94 \text{ days, approximately.}$$

The endowment period would therefore be 20 years and 94 days. The

endowment period for a \$1000 endowment insurance policy would of course be the same, if the net single premium were 1000 times greater than the premium used in this illustration, or \$587.12.

Approximation of Annuity Periods.—Suppose the present value of a temporary life annuity of 1 issued at age 35 is 8.118, on the basis of the American Experience table and 3 per cent. What is the term?

It has been found above that, $|_n a_x = \dfrac{N_x - N_{x+n}}{D_x}$, which may be written, $|_{n'} a_x = \dfrac{N_x - N_{x+n'}}{D_x}$, when the term is unknown.

Clearing of fractions,

$$D_x(|_{n'} a_x) = N_x - N_{x+n'}.$$

Transposing,

$$N_{x+n'} = N_x - D_x(|_{n'} a_x).$$

Substituting the given value of $|_{n'} a_x$, and the values of N and D_x as ascertained from the commutation columns,

$$\begin{aligned} N_{x+n'} &= 550082.5 - (29078.18 \times 8.118) \\ &= 550082.5 - 236073.7 \\ &= 314008.8 \end{aligned}$$

Inspecting column N, this value is found opposite age 45, which, therefore, is the age at which the annuity ceases. Hence the term is 10 years from age 35.

CHAPTER VII

Net Level Premiums: Commutation Columns R and S: Conversion Tables

The Net Natural Premium.—The net natural premium rate is one which is just sufficient to pay for insurance from one premium date to the next. Thus if premiums are to be paid annually, the net natural premium is just sufficient to pay for the current year's insurance. A policy issued on this basis is called a yearly-renewable term policy, and the net natural premium rate for it is called a yearly renewable term rate. Such a policy is looked upon as if it were granted for one year only; that is, as a one-year term policy, to be renewed, however, for one year at the end of each successive year, provided the insured is living and pays the premium for each year according to his attained age, without an additional medical examination. Since the probability of death within one year increases as age advances, at most insurable ages, the net natural premium must also increase as the policy advances in age. Following the rule previously developed; that is, multiplying the amount at risk by the probability that the event insured against will occur, and discounting the result for the period of time the money remains under the company's control, the determination of the net natural premium at each age becomes a mere exercise in arithmetic. Thus the net natural premium for a one year insurance of 1 at age 35, on the basis of the American Experience table of mortality and interest at 3 per cent, is: $.008946 \div 1.03 = .008685$. In this operation, .008946 is the probability of death within one year following age 35, as found from the mortality table. For a \$1000 policy the rate would be \$8.69. At age 36, the rate would be: $.009089 \div 1.03 = .008824$, or \$8.82 per \$1000 of insurance. At age 94, the rate would be $.857143 \div 1.03 = .832178$ or \$832.18 per \$1000 of insurance. At age 95 it would be $1 \div 1.03 = .97087379$, or \$970.87 per \$1000 of insurance.

It thus appears that a curve showing the trend of the net natural premium rate from age ten upwards would run a little under, but follow quite closely, the mortality curve. It is plain that the net natural premium or yearly renewable term plan is impracticable at

the extreme ages. Even at age 60 or 65, the net natural premium is high enough to become burdensome to most of those persons who are likely to desire this plan because of the relatively low premiums in the younger years. If a large group of persons at the younger ages should insure on the net natural premium plan, therefore, it is evident that as these persons grow older and as the contribution of each surviving member of the group increases, circumstances will compel some to discontinue their insurance. Others will drop out of their own volition. Those in poor health, however, will remain as long as possible, thus causing the actual mortality to exceed that expected. Now if it were possible to compel or induce young lives to join the group in sufficient numbers to maintain the same average age from year to year, a premium equal to the average annual premium for the group could be exacted from each one, and this premium need not increase. The young would then pay more than their death rates require, and the old would pay less. In other words the younger members would pay for the excess mortality amongst the older members. It is difficult, however, to secure young members on these terms, in the absence of compulsion or some powerful inducement, other than insurance, such as is held out to workers in an industrial or business organization, where, in the form of group insurance, the yearly renewable term plan has had a considerable modern development. With this exception, the net natural premium plan may be looked upon only as a means of offering cheap, temporary protection at the younger ages. It has been found necessary, therefore, to devise some means whereby each member may be charged only for the risk assumed on his own life. The net single premium accomplishes this purpose, but single premiums are impracticable in most instances, because very few persons could afford an adequate amount of protection if they were compelled to pay in full for it at once, and many who could afford it prefer to invest their funds in other enterprises. To meet this situation, it is necessary to calculate annual premium rates which are equivalent to their respective net single premiums.

The Net Annual Level Premium.—*Ordinary Life Insurance.*—It is evident that the company must charge either the net single premium or its equivalent, and that the net natural premium plan, in other than group insurance, does not prove successful in practice. So it is necessary to calculate a premium which will remain the same each year. Such a premium is called the net level premium. This means that the insured will be charged more than mortality necessitates in

the younger years, and less in the older years. The determination of the net annual level premium, then, involves finding an annual premium which is equivalent to the net single premium.

Since the annual premium for an ordinary life policy continues until the death of the insured, it may be looked upon as an annuity-due which is purchased by the company from the insured person, and for which the company pays by granting an insurance policy. The annual level premium is an annuity-due because the first premium is payable at once at the beginning of the contract, and annually thereafter. Considering the net annual premium, then, as an annuity-due, if its present value is equal to the net single premium for the insurance, it appears that the insured and the company have made a fair bargain. The insured agrees to pay so much at once and the same sum each year thereafter so long as he lives, the present value of the premiums payable over the span of life being equal to the net single premium; for this the company agrees to pay the amount of insurance, upon the death of the insured person. The present value of a life annuity-due of 1, has been expressed, $1 + a_x$. The present value of an annuity-due of P_x (the symbol for the net annual level premium) would be expressed, therefore, by $P_x(1 + a_x)$. Since the present value of future premiums due, must equal the net single premium, $P_x(1 + a_x) = A_x$. Dividing by $1 + a_x$, $P_x = \frac{A_x}{1 + a_x}$. Thus the net annual level premium for an ordinary life insurance of 1 is equal to the net single premium for a life insurance of 1 divided by the present value of a whole-life annuity-due of 1. If the insurance were for \$1000, the value of A_x would be multiplied by 1000 to arrive at the net single premium, which would in turn give P_x, a value 1000 times as great as it would have if the insurance were for 1. Hence when the value of P_x for 1 of insurance has been determined, move the decimal point three places to the right to get the annual premium for 1000 of insurance. It has been found in Chapters V and VI, above, that $A_{35} = .41988$, and $1 + a_{35} = 19.9174$. Hence: $P_{35} = \frac{A_{35}}{1 + a_{35}} = \frac{.41988}{19.9174} = .021081$, or \$21.08 per annum for \$1000 of insurance.

In Chapter VI, it was shown that $A_x = \frac{M_x}{D_x}$. In Chapter V, it was shown that $1 + a_x = \frac{N_{x-1}}{D_x}$. Hence, in terms of commutation symbols,

$$P_x = \frac{A_x}{1 + a_x} = \frac{M_x}{D_x} \div \frac{N_{x-1}}{D_x} = \frac{M_x}{D_x} \times \frac{D_x}{N_{x-1}} = \frac{M_x}{N_{x-1}}.$$

EXAMPLE.—Calculate the net annual level premium for an ordinary life insurance of 1 issued at age 35 on the basis of the American Experience table of mortality with interest at 3 per cent. (See Appendix.)

Solution:

$$P_x = \frac{M_x}{N_{x-1}},$$

$$P_{35} = \frac{M_{35}}{N_{34}} = \frac{12209.42}{579160.7} = .021081,$$

or \$21.08 per annum for \$1000 of insurance, as before.

The relations of single premiums to annuities, as worked out in Chapter VI, may be used in determining net level premiums directly from the present values of annuities, without first calculating the equivalent net single premium. For instance, it was shown that $A_x = 1 - d(1 + a_x)$, and it has just been shown that $P_x = \frac{A_x}{1 + a_x}$. Substituting for A_x in this equation,

$$P_x = \frac{1 - d(1 + a_x)}{1 + a_x}$$

$$= \frac{1}{1 + a_x} - \frac{d(1 + a_x)}{1 + a_x} = \frac{1}{1 + a_x} - d.$$

EXAMPLE.—The present value of a whole-life annuity-due of 1 at age 35, is 19.9174, according to the American table and 3 per cent, and d is .02912621, if $i = .03$. Hence the net level premium for an ordinary life insurance of 1 issued at age 35 on the same mortality and interest basis, would be:

$$P_x = \frac{1}{1 + a_x} - d$$

$$= \frac{1}{19.9174} - .02912621$$

$$= .0502073 - .029126$$

$= .021081$, or \$21.08 per \$1000 of insurance, as before.

Again,

$$P_x = \frac{1}{1 + a_x} - d,$$

$$P_x + d = \frac{1}{1 + a_x},$$

$$(1 + a_x)(P_x + d) = 1,$$

$$1 + a_x = \frac{1}{P_x + d},$$

$$a_x = \frac{1}{P_x + d} - 1.$$

Given the level premium, the single premium may be derived from it. Thus,

$$P_x = \frac{A_x}{1 + a_x},$$

$$A_x = P_x(1 + a_x).$$

If $P_x = .021081$, and $1 + a_x = 19.9174$, as is true at age 35:

$$\begin{aligned} A_{35} &= P_{35}(1 + a_{35}) \\ &= .021081 \times 19.9174 \\ &= .41988. \end{aligned}$$

Likewise, if the single premium only were known, the level premium might be derived directly from it without calculating the value of the annuity-due as follows:

$$A_x = 1 - d(1 + a_x),$$

$$a_x = \frac{1 - A_x}{d} - 1,$$

$$\begin{aligned} P_x &= \frac{A_x}{1 + a_x} \\ &= \frac{A_x}{1 + \left(\frac{1 - A_x}{d} - 1\right)} \\ &= \frac{dA_x}{1 - A_x}. \end{aligned}$$

EXAMPLE.—Given $A_x = .41988$ to find P_x.

Solution:

$$\begin{aligned} P_x = \frac{dA_x}{1 - A_x} &= \frac{.02912621 \times .41988}{1 - .41988} \\ &= .02108. \end{aligned}$$

It is clear, then, that the three functions, the annuity, the single premium, and the annual premium, are so closely related to one another that when any one of them is known, together with the rate of interest, the other two may be derived from it.

Limited-payment Life Insurance.—In the first expression of the value of the net annual level premium for an ordinary life insurance of 1, given above, the annual premium was set forth in terms of the single premium and the present value of the corresponding annuity-due. The

expression is, $P_x = \frac{A_x}{1 + a_x}$. Observe that in this expression, the level premium is stated as being equal to the net single premium divided by the present value of an annuity-due of 1 for the premium-paying period, which in this case extends throughout the whole of life. As explained in Chapter II, however, many persons prefer to pay for their insurance during a limited period of years. Twenty years is the most popular period, and a whole-life policy to be paid for by 20 annual premiums, is called a 20-payment life policy. Since in the case of a 20-payment life policy the insured person is under obligations to pay premiums only for twenty years, his premium payments may be looked upon as an annuity-due for 20 years only. The present value of these payments, then, must be equal to the net single premium for a whole-life policy, since the company must collect either the net single premium or its equivalent. In Chapter V, above, the symbol used to express the present value of a temporary life annuity of 1, is, $|_n a_x$. A temporary life annuity-due of 1 may be expressed, $1 + (|_{n-1} a_x)$, since the first payment is made at once and there remain $n - 1$ payments, subject to survival. In the case of a 20-payment life policy, one premium payment is made at once, to be followed by 19 similar annual payments, subject to survival. Suppose the annual premium for a limited-payment life policy is represented by tP_x, in which t represents the term of the premium-paying period. The present value of the premiums, therefore, would be $tP_x[1 + (|_{t-1} a_x)]$. Since this must equal the net single premium,

$$_tP_x[1 + (|_{t-1} a_x)] = A_x,$$

$$_tP_x = \frac{A_x}{1 + (|_{t-1} a_x)}.$$

As before, the net single premium is divided by the present value of an annuity-due of 1 for the premium-paying period, to arrive at the net level premium. In commutation symbols, the present value of a temporary annuity due is expressed, $\frac{N_{x-1} - N_{x+n-1}}{D_x}$. In this instance, the period of the annuity is represented by t. Hence

$$_tP_x = \frac{M_x}{D_x} \div \frac{N_{x-1} - N_{x+t-1}}{D_x} = \frac{M_x}{N_{x-1} - N_{x+t-1}}.$$

Example.—Calculate the net annual premium for a 20-payment life policy of $1000 issued at age 35, according to the American Experience table and 3 per cent. (See Appendix C.)

Solution:

$$
{}_tP_x = \frac{M_x}{N_{x-1} - N_{x+t-1}}
$$

$$
{}_{20}P_{35} = \frac{M_{35}}{N_{34} - N_{54}}
$$

$$
= \frac{12209.42}{579160.7 - 170140.0}
$$

$$
= .0298504, \text{ or \$29.85 per \$1000 of insurance.}
$$

Temporary and Deferred Insurance.—The same rule, which requires the division of the net single premium by the present value of an annuity-due of 1 for the premium-paying period to arrive at the net annual premium, obtains in term insurance also. Using the symbol, $|_nP_x$, to represent the net annual level premium for a term insurance of 1.

$$
|_nP_x = \frac{|_nA_x}{1 + (|_{n-1}a_x)}.
$$

In commutation symbols, the net single premium is expressed:

$$
|_nA_x = \frac{M_x - M_{x+n}}{D_x},
$$

as shown in Chapter VI. Hence:

$$
|_nP_x = \frac{|_nA_x}{1 + (|_{n-1}a_x)}
$$

$$
= \frac{M_x - M_{x+n}}{D_x} \div \frac{N_{x-1} - N_{x+n-1}}{D_x}
$$

$$
= \frac{M_x - M_{x+n}}{N_{x-1} - N_{x+n-1}}.
$$

Example.—Calculate the net annual premium for a five-year term insurance policy of \$1000 issued at age 35, according to the American Experience table and 3 per cent.

Solution:

$$
|_nP_x = \frac{M_x - M_{x+n}}{N_{x-1} - N_{x+n-1}},
$$

$$
|_5P_{35} = \frac{M_{35} - M_{40}}{N_{34} - N_{39}}
$$

$$
= \frac{12209.42 - 11000.40}{579160.7 - 444394.4} = .008971,
$$

or \$8.97 per \$1000 of insurance.

Deferred insurance policies are seldom issued in this country. The net annual premiums for such policies, however, may be easily determined, if such is desired, once the single premiums have been calculated as explained in Chapter VI. The net single premium for a whole-life insurance of 1 deferred n years is expressed, ${}_n|A_x$. If premiums are to be paid annually in advance for the period of deferment, let the annual premium be represented by, ${}_nP_n|A_x$. Following the rule of dividing the net single premium by the present value of an annuity-due of 1 for the premium-paying period, it appears that,

$${}_nP_n|A_x = \frac{{}_n|A_x}{1 + ({}_{|n-1}a_x)}.$$

In terms of the commutation symbols, ${}_n|A_x = \dfrac{M_{x+n}}{D_x}$, from Chapter VI.

Hence:

$$\begin{aligned} {}_nP_n|A_x &= \frac{M_{x+n}}{D_x} \div \frac{N_{x-1} - N_{x+n-1}}{D_x} \\ &= \frac{M_{x+n}}{N_{x-1} - N_{x+n-1}}. \end{aligned}$$

Endowments.—Using the expression P_nE_x to represent the net annual premium for a pure endowment of 1, issued at age x and payable at the end of n years,

$$P_nE_x = \frac{{}_nE_x}{1 + ({}_{|n-1}a_x)} = \frac{D_{x+n}}{D_x} \div \frac{N_{x-1} - N_{x+n-1}}{D_x} = \frac{D_{x+n}}{N_{x-1} - N_{x+n-1}}.$$

The net annual premium for an endowment insurance policy of 1 for a period of n years, may be expressed, P_nAE_x, or $P_{x\overline{n}|}$. For convenience, the present value of a temporary life annuity-due of 1 is often expressed, $1 + a_{x\overline{n-1}|}$, instead of $1 + ({}_{|n-1}a_x)$, and ${}_nAE_x$ may be written $A_{x\overline{n}|}$. The net annual premium for an endowment insurance of 1 for n years may be expressed:

$$\begin{aligned} P_{x\overline{n}|} &= \frac{A_{x\overline{n}|}}{1 + a_{x\overline{n-1}|}} \\ &= \frac{M_x - M_{x+n} + D_{x+n}}{D_x} \div \frac{N_{x-1} - N_{x+n-1}}{D_x} \\ &= \frac{M_x - M_{x+n} + D_{x+n}}{N_{x-1} - N_{x+n-1}}. \end{aligned}$$

EXAMPLE.—Calculate the net annual premium for a twenty-year endowment insurance policy of $1000, issued at age 35, according to the American Experience table with interest at 3 per cent.

Solution:

$$P_{x\overline{n|}} = \frac{M_x - M_{x+n} + D_{x+n}}{N_{x-1} - N_{x+n-1}},$$

$$P_{35\,\overline{20|}} = \frac{M_{35} - M_{55} + D_{55}}{N_{34} - N_{54}}$$

$$= \frac{12209.4229 - 7748.3442 + 12703.88}{579160.66 - 170140.03}$$

$$= \frac{17164.9587}{409020.63}$$

$= .041965$, or \$41.97 per \$1000 of insurance.

Endowments may be paid for during a shorter period of time than the endowment period itself. Thus to find the net annual premium for a 20-payment, 30-year endowment, divide the net single premium for the 30-year endowment by the present value of an annuity-due of 1 for the premium-paying period.

Deferred Annuities.—Immediate whole-life annuities are usually purchased on the single-premium plan; deferred annuities, nearly always by periodic premiums. Thus a person aged 35 may wish to pay a certain sum annually for 30 years in order to provide for a definite income in the form of an annuity to begin at age 65, if he should survive to that age. To find the net annual premium for a deferred whole-life annuity, divide the single premium, by the present value of a temporary annuity-due for the premium paying period. Allowing $P_{n|}a_x$ to represent the net annual premium, its value in terms of the single premium and the corresponding annuity-due, may be expressed:

$$P_{n|}a_x = \frac{{}_{n|}a_x}{1 + a_{x\overline{n-1|}}} = \frac{N_{x+n}}{N_{x-1} - N_{x+n-1}}.$$

This is the usual form in which this expression appears in Actuarial literature, and when the premiums are to be paid at the beginning of each year during the period of deferment; that is, when the number of premiums is the same as the number of years in the period of deferment, it is correct. When one additional premium is collected, however, the right-hand side of the expression becomes $\frac{N_{x+n}}{N_{x-1} - N_{x+n}}$, in which n is the period of deferment. Thus in this volume, as previously explained, an annuity issued at age 35, first payment to be made by the company at age 65, is considered as a whole-life annuity deferred 29 years from age of issue. The premium-paying period, however, usually extends from ages 35 to 64, inclusive, there being 30 premiums in all. The

expression, $\frac{N_{x+n-1}}{N_{x-1} - N_{x+n-1}}$, may be used in such a problem, if n is given a value of 30, to correspond to the number of annual premium payments, instead of 29, the period of deferment.

EXAMPLE.—Calculate the net annual premium for a whole-life annuity of 1, issued at age 35, first payment to be made by the company at age 65, subject to survival; the premiums to be paid annually in advance from ages 35 to 64, inclusive.

Solution:

$$P_{n|}a_x = \frac{N_{x+n-1}}{N_{x-1} - N_{x+n-1}}$$

$$= \frac{N_{64}}{N_{34} - N_{64}}$$

$$= \frac{68645.33}{579160.7 - 68645.33}$$

$= .13446$, or \$134.46 per annum to provide a deferred annuity of \$1000 per year.

Continuous-installment Insurance.—It will be readily understood that installment policies, policies containing the various settlement options, double and semi-endowment policies and income bonds, present no difficulties so far as level-premium calculation is concerned. Follow the rule and express the annual premium by a fraction whose numerator is the single premium and whose denominator is the present value of an annuity-due for the premium-paying period, set the fraction in terms of the commutation symbols, ascertain the values of the symbols from the commutation tables, and perform the arithmetical operation to arrive at the result. In continuous-installment policies, however, it is customary for the company to reduce the annual premium upon the death of the beneficiary, if the beneficiary's decease should precede that of the insured. As noted in Chapter VI, the benefits of a continuous installment policy on the ordinary-life plan are two in number. The net single premium for the guaranteed installments, divided by the present value of a whole-life annuity-due of 1, based on the life of the insured person, will yield the net annual premium for the guaranteed benefit. This is the annual premium which the company must continue to collect as long as the insured person survives, even though the beneficiary's life has failed. In addition, as long as both insured and beneficiary survive, the company must collect an annual premium whose present value, at the time the

policy is issued, is equal to the net single premium for the annuity which is to continue to the beneficiary if she survives the guaranteed period which begins upon the insured's death. The single premium for this annuity to the beneficiary, then, divided by the present value of a joint-life annuity of 1 on the lives of both insured and beneficiary, yields the net annual premium for this second benefit. The sum of the two annual premiums is the annual premium while both survive. This premium may be reduced upon the prior death of the beneficiary to the premium necessary to provide the guaranteed installments.

We may observe that when continuous installments are issued on the endowment insurance plan, the benefits separate into four parts. The single premium for the guaranteed payments, and the single premium for the annuity to the insured if he should survive the guaranteed period which begins upon the maturity of the endowment, should each be divided by an annuity-due of 1 based on the life of the insured and continuing for the premium-paying period. The single premiums for the other benefits may be divided by the present value of an annuity-due of 1 based on the joint-lives of the insured and the beneficiary and continuing for the premium-paying period, to arrive at the equivalent annual premiums. The sum of the annual premiums for all four benefits is the net annual premium necessary while both survive during the premium-paying period. If the beneficiary should die during the premium-paying period, the premiums may be reduced to an annual payment sufficient to provide the first two benefits only. Since, as explained on pages 94-95, the single premium for a continuous-installment policy whose benefits are paid annually, is slightly more than sufficient for one whose benefits are paid monthly, the annual premium is also a trifle more than sufficient to provide for a monthly income policy.

Joint-life Insurance.—The net annual premium for a joint-life policy may be found by dividing the net single premium by the present value of a joint-life annuity-due for the premium-paying period. The calculation of net single premiums for joint-life insurances, endowments, and annuities, has been previously explained.

Approximation of Term from Age and Annual Premium.—Suppose a person aged x agrees to pay a stipulated premium for a given number of years; for how many years will the premiums provide a term insurance of 1? The net annual premium for a term insurance was expressed above, as

$$|_nP_x = \frac{M_x - M_{x+n}}{N_{x-1} - N_{x+n-1}}.$$

When the term is unknown, n' may be used instead of n. The expression then becomes,

$$|_{n'}P_x = \frac{M_x - M_{x+n'}}{N_{x-1} - N_{x+n'-1}}.$$

In this expression, the value of x, $|_{n'}P_x$, M_x, and N_{x-1}, are either known or directly ascertainable from the commutation columns. It remains, therefore, to find the values of $M_{x+n'}$, and $N_{x+n'-1}$. Clearing and transposing,

$$M_{x+n'} - |_{n'}P_x(N_{x+n'-1}) = M_x - |_{n'}P_xN_{x-1}.$$

Now ascertain the value of the right-hand side of this equation, and choose a trial value for $x + n'$ near the age to which the insurance seems likely to run. Continuing until two values are found, one next under and one next over the ascertained value of $M_x - |_{n'}P_xN_{x-1}$, the value, as found from the columns, which most nearly approximates this ascertained value, is opposite the age at which the insurance should terminate, if an approximation in years only is desired. If an approximation in days is desired, the number of days additional tó the number of full years may be found by the process explained in Chapter VI for finding the days additional, in case the single premium is given to approximate the term.

Varying Benefits: Commutation Columns S and R.—Increasing and decreasing annuities are seldom used in insurance mathematics, but increasing insurances occasionally appear, since they are used in return premium calculations. In the case of annuities, it is customary to let $(va)_x$ represent the value of an annuity beginning at one sum designated as k, and annually increasing or decreasing by a sum indicated by h. Column S is constructed by setting opposite each age the sum of the values of $N_x + N_{x+1} + N_{x+2} +$ etc., to the end of Column N. The value of a varying annuity is then expressed,

$$(va)_x = \frac{kN_x \pm hS_{x+1}}{D_x}.$$

When k and h each equal 1, the value of an annuity commencing at 1 and increasing 1 per annum throughout life may be expressed:

$$(Ia)_x = \frac{N_x + S_{x+1}}{D_x}$$

$$= \frac{S_x}{D_x}.$$

Likewise, using the symbols $(vA)_x$ and $(IA)_x$ for the corresponding insurances,

$$(vA)_x = \frac{kM_x \pm hR_{x+1}}{D_x},$$

$$(IA)_x = \frac{R_x}{D_x}.$$

Values in Column R are derived by taking the sum of the values of $M_x + M_{x+1} +$ etc., to the end of Column M.

The latter expression of the value of an increasing insurance of 1 throughout life, represented by $(IA)_x$, occurs often enough in insurance mathematics to warrant further explanation. In such a benefit, 1 is the face amount of insurance the first year, 2 the second, 3 the third, etc. The probability of death during the first year multiplied by 1 and discounted for one year, plus the sum resulting when the probability of death during the second year is multiplied by 2 and discounted two years, plus, etc., to the end of the mortality table, will yield the net single premium.

From principles previously explained, the numerator of the fraction expressing this result, may take the form:

$$v^{x+1}d_x + 2v^{x+2}d_{x+1} + 3v^{x+3}d_{x+2} + \ldots,$$

which in turn may be expressed:

$$C_x + 2C_{x+1} + 3C_{x+2} + \ldots.$$

Now these values of C_x may be separated so as to represent the values of M_x, M_{x+1}, etc., as follows:

$$\begin{aligned} M_x &= C_x + C_{x+1} + C_{x+2} + \ldots \\ M_{x+1} &= \phantom{C_x + {}} C_{x+1} + C_{x+2} + \ldots \\ M_{x+2} &= \phantom{C_x + C_{x+1} + {}} C_{x+2} + \ldots \\ \text{etc.} & \qquad\qquad\qquad \text{etc.} \end{aligned}$$

It then appears that $C_x + 2C_{x+1} + 3C_{x+2} + \ldots$, is equal to $M_x + M_{x+1} + M_{x+2} + \ldots.$

As previously stated, however, the sum of $M_x + M_{x+1} + M_{x+2} +$ etc., is set down in Column R opposite age x. The numerator of the fraction thus becomes R_x. The denominator remains D_x, the same as explained in Chapter VI, since the probabilities are the same, and so $v^x l_x$ is the same as in ordinary single premium calculations.

The net single premium for an increasing insurance of 1 for the whole of life, divided by the present value of a whole-life annuity-due of 1 on

the life of the insured, will, of course, produce the net annual premium for the increasing insurance. In commutation symbols, the present value of the annuity-due, is $\frac{N_{x-1}}{D_x}$. Allowing $P(IA)_x$ to represent the net annual premium for an increasing insurance of 1 for the whole of life,

$$P(IA)_x = \frac{R_x}{D_x} \div \frac{N_{x-1}}{D_x} = \frac{R_x}{N_{x-1}}.$$

EXAMPLE.—Calculate the net annual premium for an increasing insurance of 1, issued at age 35, on the basis of the American Experience table with interest at 3 per cent. (See Appendix C.)

Solution:

$$\begin{aligned} P(IA)_x &= \frac{R_x}{N_{x-1}} \\ &= \frac{R_{35}}{N_{34}} \\ &= \frac{321361.63958}{579160.66} \\ &= .55487, \text{ or \$554.875 per annum for an increasing insurance of \$1000.} \end{aligned}$$

Return-premium Insurance.—Suppose an insurance of 1, which provides for the return of all annual premiums, *without interest*, upon the death of the insured person. Let the net annual premium be designated by the symbol π. The net single premium, or value, of such a contract may be separated into two parts; an insurance of 1, and an insurance of π which increases yearly by π. The net single premium for the first benefit is A_x, and for the second, $\pi(IA)_x$; that is, the net single premium for an increasing insurance of 1, is $(IA)_x$; hence the net single premium for an increasing insurance of π, would be $\pi(IA)_x$. The net single premium for the entire benefit, then, would be,

$$A_x + \pi(IA)_x.$$

This divided by the present value of an annuity-due, or $1 + a_x$, would yield the net annual premium for the entire benefit. Thus:

$$\pi = \frac{A_x + \pi(IA)_x}{1 + a_x}.$$

Clearing,

$$\pi(1 + a_x) = A_x + \pi(IA)_x.$$

Transposing and factoring,

$$\pi[1 + a_x - (IA)_x] = A_x,$$

$$\pi = \frac{A_x}{1 + a_x - (IA)_x}.$$

In commutation symbols this becomes:

$$\pi = \frac{\dfrac{M_x}{D_x}}{\dfrac{N_{x-1}}{D_x} - \dfrac{R_x}{D_x}}$$

$$= \frac{M_x}{D_x} \div \frac{N_{x-1} - R_x}{D_x}$$

$$= \frac{M_x}{N_{x-1} - R_x}.$$

Example.—Calculate the net annual premium for an insurance of 1, issued at age 35, on the basis of the American Experience table and 3 per cent; and which provides that upon the insured's death all net annual premiums that have been paid to the company will be returned without interest.

Solution:

$$\pi = \frac{M_x}{N_{x-1} - R_x}$$

$$= \frac{M_{35}}{N_{34} - R_{35}}$$

$$= \frac{12209.42}{579160.7 - 321361.64}$$

$= .04736$, or \$47.36 per \$1000 of insurance.

Pure Endowments and Deferred Annuities.—Calculations of premiums for the return of net single and net annual premiums for all the various forms of life contracts may be made, and such computations supply interesting exercises in insurance mathematics. It is also possible to calculate premiums which would permit the company to return not only the premiums themselves, but also a rate of interest on them. The entire interest earned on premiums held by the company could not be returned, however, since from interest earnings the company must derive an income sufficient to meet the cost of the regular feature of the contract. Thus in a whole-life insurance

of $1000 the company may return all the premiums collected, without interest, if the premiums are large enough to make the interest earned on them while under the company's control sufficient to pay for the insurance of $1000. If interest is to be returned, it must either be at a lower rate than that earned by the company; or, if it is at the same rate, only simple interest on the premiums can be returned, whereas the company earns interest compounded. In the latter case, the excess of compound over simple interest, must be equivalent to the premium for the regular benefit without the return feature. An explanation of the process of determining such a premium is found in the "Institute of Actuaries' Text Book," second edition, pp. 290, et seq.

When the regular net annual premium has been determined, the annual premium for the benefit with return of premiums without interest, may be determined directly from it. Thus suppose an insurance of 1 issued at age 35 with return of net annual premiums without interest. The net annual premium at that age has been found to be .02108, and the net annual premium for an increasing insurance of 1 for the whole of life, is .554875, as found above. These values may be used to find the return premium as follows:

Let π = the entire premium;

P_x = net annual premium for 1 of insurance;

$P(IA)_x$ = net annual premium for an increasing insurance of 1.

Now $\pi P(IA)_x$ must be the net annual premium for an increasing insurance of π, or in other words $\pi P(IA)_x$ is the extra premium which must be added to P_x to cover the return feature. Then:

$$\pi = P_x + \pi P(IA)_x$$

$$\pi[1 - P(IA)_x] = P_x,$$

$$\pi = \frac{P_x}{1 - P(IA)_x}$$

$$= \frac{.02108}{1 - .554875}$$

$$= .04736, \text{ or } \$47.36 \text{ per } \$1000 \text{ of insurance, as before.}$$

Conversion Tables

Single-premium Tables.—It has been shown above that

$$A = 1 - d(1 + a); \quad P = \frac{1}{1 + a} - d; \quad \text{and} \quad P = \frac{dA}{1 - A}.$$

The suffix x may be omitted for present purposes, since neither the age nor the rate of mortality need be considered in these relations of annual and single premiums to each other and to the corresponding annuity. In other words, if the annuity at any age and on any mortality basis has been determined, the corresponding annual and single premiums on the same age and mortality basis may be easily found, provided the rate of interest is known. These relations hold, regardless of the nature of the insurance, or of whether it is on one life or on many lives, if the annuity is of the same order. While these formulas for the conversion of annuity values into equivalent single and annual premiums may save a considerable amount of work, it is apparent that much additional labor might be saved if the corresponding values were worked out and tabulated. Thus if the work of preparing such conversion tables were accurately performed to a sufficient number of decimal places, a new mortality table could be put into use with much less work than if no conversion tables were available. For instance, commutation columns N and D might be formed on the new basis, and from these the value of a at all ages and for all types of annuities used in the calculation of ordinary premiums might be set down. Then by referring to conversion tables, the corresponding net premiums could be readily found and tabulated. Or when premiums are calculated in the ordinary manner, conversion tables might be used to check the accuracy of the results.

In 1856 William Orchard produced conversion tables giving premiums corresponding to annuity values differing by hundredths, from which single and annual premiums may be ascertained to one-thousandth of a unit. He also compiled tables of differences so that premiums equivalent to annuity values smaller than hundredths of a unit may be approximated. In the case of single premiums the tables were constructed in the following manner. If the value of A is known, the amount by which it should be increased or diminished if the value of the corresponding annuity a is increased or decreased by Δa, may be determined:

$$A = 1 - d(1 + a). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$A + \Delta A = 1 - d(1 + a + \Delta a). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Subtracting (1) from (2),

$$\Delta A = - d(\Delta a). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Since the annuity value decreases as the equivalent single premium increases, it appears that the difference, Δa, is subtractive, as shown by the last expression. Making Δa equal to .01, for example, it is found

that the value of ΔA becomes a fixed quantity when the value of i is given. Orchard took Δa equal to .01, hence ΔA became $-.01 \times d$ (from the last equation). With interest at 3 per cent, then, $d =$ .02912621, from interest table II of this volume. Then, $.01 \times d =$ .00029126. The construction of the table, then, is as follows:

Value of Annuity	Value of Insurance
0 00	.97087379
	− 00029126
0.01	.97058253
	− 00029126
0 02	.97029127
etc.	etc.

The formula may then be used to check the accuracy of the table at intervals.

Annual-premium Tables.—Annual-premium conversion tables, which show the net annual premium equivalent to the corresponding annuity, may be constructed in this manner:

$$P = \frac{1}{1+a} - d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$P + \Delta P = \frac{1}{1+a+\Delta a} - d. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Subtracting (1) from (2):

$$\Delta P = \frac{1}{1+a+\Delta a} - \frac{1}{1+a}. \quad . \quad . \quad . \quad (3)$$

Again it will be observed that the differences are subtractive, since $\frac{1}{1+a}$ is greater than $\frac{1}{1+a+\Delta a}$, in the last expression. In other words, the greater the annuity-value the smaller the corresponding annual premium. Making Δa equal to .01 as before, the quantities by which the initial value of P when $a = 0$, must be reduced when $a = .01$, .02, .03, etc., may be found by taking the differences of the series of reciprocals belonging to the successive values of $1 + a$. A table of reciprocals readily yields these differences. This series of differences is independent of the rate of interest, so when they are found for use with one rate, they may be used with any rate. The value of P, of course, differs with different interest rates.

For those who do not have a table of reciprocals at hand, a few examples worked from the foregoing may serve to show the construction.

Let $\Delta a = .01.$

When $a = 0,$

$$\Delta P = \frac{1}{1 + a + \Delta a} - \frac{1}{1 + a}$$

$$= \frac{1}{1.01} - 1$$

$$= .990099 - 1$$

$$= -\ .009901.$$

When

$$a = .01,$$

$$\Delta P = \frac{1}{1 + a + \Delta a} - \frac{1}{1 + a}$$

$$= \frac{1}{1 + .01 + .01} - \frac{1}{1 + .01}$$

$$= .980392 - .990099$$

$$= -\ .009707.$$

When

$$a = .02, \quad \Delta P = -\ .009518.$$

These values may now be tabulated:

a	$\frac{1}{1+a}$	$\Delta\frac{1}{1+a} = \Delta P$ (Subtractive)
0 00	1 000000	.009901
01	990099	.009707
02	.980392	.009518
etc.	etc.	etc.

Now if i is given, columns showing the values of P, may be added. Thus if $i = .03$; when $a = 0$, P must equal v, or .970874; and when $a = .01$, $\Delta P = .009901$, subtractive, hence $.970874 - .009901 = .960973$. Likewise, if i were .04; when $a = 0$, $P = .961538$; and when $a = .01$, $P = .961538 - .009901 = .951637$. Starting with

$P = v$, when $a = 0$, then, and continuously subtracting the differences in the third column from the successive values of P until values of a sufficiently great for all practical purposes have been reached, an annual-premium conversion table might be formed. Thus:

a	$\frac{1}{1+a}$	$\Delta\frac{1}{1+a} = \Delta P$	P (When $i = .03$)	P (When $i = .04$)
0 00	1 000000	.009901	.970874	.961538
01	990099	.009707	.960973	.951637
02	980392	009518	.951266	.941930
etc.	etc.	etc.	etc.	etc.

Abbreviations of the Tables.—While Orchard's "Conversion Tables" furnish convenient means of finding such values as described herein, there are certain abbreviated tables in the "Institute of Actuaries' Text Book," Part II, which show the value of A and P for unit values of A. The differences of A for differences of .1, .01, and .001 in the value a, and differences of P for differences of .1 in the value of a, are then given; so that values of A and P to fractional values of a may be found with sufficient accuracy. Being less voluminous, these tables are more often available. Their use may be conveniently shown by examples.

Example.—Given $a = 18.9174$ to find A and P, interest being at 3 per cent.

Solution for A:

A for 18		= .44660
Subtract		
For .9,	.02621	
For .01,	.000291	
For .007,	.0002039	
For .0004,	.00001165	.026716
A for 18.9174		= .41988

Solution for P:

P for 18		= .02351
Subtract		
For .9,	.00238	
For .0174,	.000045	.00243
		= .02108, P for 18.9174.

Note.—The .000045 is found as follows: 18.9174 lies between 18.9 and 19. The difference of P for .9, opposite 18 is .00238, from the table.

Then, P for 18.9 = .02351 − .00238 = .02113
P for 19 = .02087

P for difference between 18.9 and 19, or .1 = .00026
P for difference of .0174 = .000045

EXAMPLE 2.—Suppose $a = 14.271$. Find P when $i = .03$.

Solution.—When a is 14, $P = .03754$. From this subtract .00088 for the .2. The remaining value of a is .071, which lies between .2 and .3. When $\Delta a = .3$, $\Delta P = .00131$. $.00131 - .00088 = .00043$, which would also have been subtracted if the .2 had been .3. Since it is .071 greater than .2, however, only $\frac{71}{100}$ of this amount should be subtracted; that is, $\frac{71}{100} \times .00043 = .0003053$, to be subtracted for the .071. Hence:

P for 14		= .03754
Subtract		
For .2,	.00088	
For .071,	.00030	.00118
P for 14.271		= .03636

CHAPTER VIII

Gross Premiums: Loading

Up to this point, the discussion of rate-making has been confined to net premiums. As previously explained, net premiums, in theory, make no provision for the expenses necessary to the conduct of the business. The prevailing system in this country results in a net premium high enough to assure sufficiency, and in addition to create a surplus for contingent losses. This is due to the calculation of net premiums on such a conservative mortality and interest basis that they are more than sufficient to provide the benefits promised by the company. Expenses of management, then, constitute the most essential element in the problem of loading. Once the amount of the loading necessary to meet expenses has been determined, it may be added to the net premium to arrive at the gross or office premium which is quoted to the prospective policyholder.

While provision for expenses is the most important element in the question of loading, there are many other factors that must be considered. In a mutual, participating company, extra loadings are generally included in the gross premium with a view to creating an extra surplus from which dividends may be paid. This lends additional strength to the company, and generally results in satisfying the policyholder. In non-participating business such extra loadings are not practicable; for, although the profits to the stockholders might be enhanced thereby, the cost to the policyholders might be too great for the company to meet competition. Again, in participating business, any inequity between groups of policyholders in the matter of loading may be corrected in large measure by adjustments in the scale of dividends. In non-participating business, such an adjustment is of course impracticable; hence the problems incident to loading vary in the different types of business. While it cannot be too forcibly impressed upon the student that the sufficiency of the premium is essential to the stability of the entire structure of life insurance, yet competition must be given careful consideration, and there may be occasions when for temporary purposes benefits ought to be offered

at premium rates barely sufficient to cover the risk and the expenses. In such cases, prospective purchasers may choose that company which shows the most favorable record and offers the best terms at his age and for the type of contract he desires. Also, as is shown later, the heaviest expenses in connection with a contract ordinarily come at the outset, and must be met shortly after the policy is issued. These are some of the more important elements that enter into the problem of loading, in addition to the basic one of providing enough money to meet expenses.

Classification of Expenses.—There are numerous items of expense in connection with the business of life insurance, and many classifications of expenses have been made. For present purposes, expenses may be divided into four principal classes. First, new business expenses, which may include advertising, canvassing literature, a part of salaries, medical examination fees, inspection of risks, agents' first-year commissions, and other expenses incurred in getting new business. While some of these items might be placed under the head of general expenses which depend largely on the volume of business, the greater portion of them may be considered as being dependent upon the premium rate. Thus the heaviest item of expense in connection with new business is the agents' first-year commission. In this country, agents' first-year commissions on ordinary-life policies probably average as high as 50 per cent of the first premium. Commissions on other types of policies vary, so, while some of the initial expenses are not dependent upon the size of the premium, there is a sufficiently close relation between premiums and initial expenses to warrant the assumption that the latter vary with the size of the gross premium per unit of insurance.

The second group of expenses are incurred in keeping the business in force, and are called collection or renewal expenses. These consist primarily of agents' renewal commissions, although they sometimes include collection fees, exchange, taxes on premiums, etc. Renewal commissions stimulate the agent to make an effort to keep a policy in force, once he has placed it, and since the commissions consist of a percentage of the premium collected, renewal expenses, like initial expenses, may be considered as varying with the gross premium-rate. Renewal expenses, including renewal commissions, are naturally much less than initial expenses. As regards these two classes of expenses, then, it should be observed that both vary with the size of the gross premium per unit of insurance, but that the first year's expenses are the heavier.

The third class of expenses may be called general expenses, or expenses of management. They include those incident to general supervision, salaries to officers, actuarial, clerical, and legal services, the investigation and settlement of death claims. These do not vary with the premium rate per unit of insurance, but rather tend to vary with the amount of insurance in force.

The fourth class may be called investment expenses. It includes the cost of making and safeguarding investments, losses on investments, taxes and repairs on property constituting investments. Investment expenses, however, need not enter into the problem of loading, since they may be met by interest earnings, and are therefore provided for by assuming a conservatively low net rate of interest in net premium calculations.

Three principal classes of expenses remain, therefore, to be taken care of by means of loading. Of these, initial expenses occur but once, and at the outset. They constitute a heavy percentage of the initial gross premium. Renewal expenses constitute a low percentage of the gross premiums after the first. General management expenses do not depend on the premium rate, but rather on the size of the company as regards the amount of insurance in force, and so these expenses may be based on the unit of insurance in force instead of on the premium rate. The nature of these three classes of expenses gives rise to two major problems in connection with loading. The one, sometimes spoken of as the problem of equitable distribution of expenses, involves adjustments in the loading so that each policyholder will be required to pay the expenses which he causes the company, insofar as these expenses can be determined. In this matter, average results only can be expected. The other has been called the problem of incidence. It has to do with methods of providing for the heavy initial expenses.

Equitable Distribution.—*General Principles.*—As stated above, there are some expenses which increase as the premium rate per unit of insurance increases, and others which do not so increase, but remain constant per unit of insurance. It would appear, therefore, that loadings should be greater on high-premium policies than on low ones, but not proportionally greater. Thus if the gross annual premium for one policy is 20 per 1000, and for another 40, the loading should be greater for the premium of 40, but not twice as great. Only that part of the loading which provides for those expenses which increase with the premium rate should be advanced in the same ratio. That portion of the loading which takes care of the expenses that

remain constant per unit of insurance, should not be so advanced. This general principle furnishes a basis for determining whether any particular method of loading is equitable as between classes and ages. If it were universally true, it could be used as the basis for a single method of loading which would take care of all different types of policies as well as differences in the ages of holders of the same kinds of policies; for once a constant has been found to provide general expenses, differences in premiums alone need be considered, regardless of whether the differences arise from the varying ages of policyholders or from the different types of policies held by them. The administration expenses are not the same for all types of policies, however, those incident to a 10-year endowment ordinarily cease at the end of 10 years, whereas those on a whole-life policy may continue for over three-quarters of a century. No one system of loading can be applied to all types of policies, therefore, without modifications of a somewhat arbitrary character and dependent upon the particular company's experience.

Methods of Loading.—*Constant-addition Loading.*—It would be possible to provide sufficient funds to meet all expenses by adding a fixed sum per annum to the net premium for each unit of insurance in force. Thus suppose a certain company having $1,000,000,000 of insurance in force estimates that $5,000,000 per annum will be ample provision for its total expenses. A constant of 5 per 1000 of insurance could be added to each net annual premium collected, and funds to meet the estimated expenses would be assured. This method, however, would violate the rule that loadings should be greater on higher-premium policies, discriminating against those persons whose premium rate per 1000 of insurance is low. For example, if the company operates on the basis of the American Experience table and 3 per cent, a person aged 35 who purchases a 10-year term policy of 1000 would have to pay a gross annual premium made up as follows:

Net annual premium for 1000................	9.42
Add loading for expenses....................	5.00
The annual gross or office premium...........	14.42

In this instance, the loading is more than 50 per cent of the net premium. On the other hand, a person aged 50 might purchase a 10-year endowment insurance policy of 1000 by paying a gross annual premium constructed in a similar manner:

Net annual premium for 1000................	92.73
The loading for expenses......................	5.00
The gross annual premium...................	97.73

In this instance, the loading is a little less than $5\frac{1}{2}$ per cent of the net premium. Obviously the policyholder in this case is not paying his full share of the expenses.

If this method of loading were used, the gross premium could be expressed as follows: Let P' represent the gross premium, and c the constant addition. The expression then becomes, $P' = P + c$, and for single premiums, $A' = A + c$.

Percentage-addition Loading.—Straight percentage loading consists in adding a fixed percentage of the net premium regardless of the age of the insured or the type of policy. Thus suppose a company collecting \$45,000,000 each year in net annual premiums estimates that \$15,000,000 per year will be sufficient to meet its expenses. If it loads each net annual premium $33\frac{1}{3}$ per cent of itself, the expense funds will be provided. This method, however, violates the rule that loadings should not increase in the same ratio as the premium. Using the same examples as above, the gross premium for the 10-year term policy would be derived, according to this method, as follows:

Net annual premium for 1000................	9.42
Add loading ($33\frac{1}{3}$ per cent of 9.42)...........	3.14
The gross annual premium...................	12.56

The gross premium for the 10-year endowment policy would consist of:

Net annual premium for 1000................	92.73
Add loading ($33\frac{1}{3}$ per cent of 92.73)..........	30.91
The gross annual premium..................	123.64

It at once appears that the higher-premium policies are loaded too heavily under this system. With the exception of those items of expense which are based on the premium, there is no appreciable difference in the expenses of the two policies.

Using the symbol k to represent the percentage addition, the gross premium, when this method is used, may be expressed,

$P' = P + kP = P\ (1 + k)$, and for single premiums, A', the gross single premium may be expressed, $A' = A + kA = A\ (1 + k)$.

Percentage and Constant.—Neither the constant nor the percentage method of loading satisfies the requirements of equity between high-premium and low-premium contracts. The one discriminates against the low-premium policy and the other places an unjust burden upon the high-premium contract. It would seem, therefore, that the two systems are sufficiently complementary to enable a combination of them to be made in such a way that good average results might be obtained. Thus suppose a company has estimated that its constant expenses, those which do not vary with the premium rate but which remain constant per unit of insurance in force, amount to 3 per annum per 1000 of insurance. Suppose now that its variable expenses, those which depend on the premium rate, are estimated to be about 20 per cent of the total net premiums collected each year. If the net premium is loaded 20 per cent of itself, and then the constant of 3 per 1000 of insurance is added, loadings will increase as the premium rate increases, but not in the same ratio. That part of the loading which provides for the expenses that are heavier on the higher-premium policies increases as the expenses increase, and that part which provides for expenses that do not increase with the premium rate remains the same. Again returning to the examples given above, the gross premium for the term policy would now be found as follows:

Net annual premium for 1000................	9.42
Add 20 per cent of net premium..............	1.88
Add constant of 3...........................	3.00
The gross annual premium...................	14.30

And for the endowment policy:

Net annual premium for 1000................	92.73
Add 20 per cent of the net premium..........	18.55
Add constant of.............................	3.00
The gross annual premium...................	114.28

This would seem to be the ideal method if the expenses which have been considered constant per unit of insurance were really constant. But, as explained above, this is not exactly true; hence there are

modifications in actual practice according to the experience of the individual company.

The formulas for the values of A' and P' when the above method of loading is used, are:

$$A' = A\,(1 + k) + c$$
$$P' = P\,(1 + k) + c$$

These formulas may be used for all three methods of loading, since when the straight percentage method is used, $c = 0$; and when the constant alone is desired, $k = 0$.

Modified Percentage.—The customary method of loading in this country is to add a fixed percentage to ordinary life policies regardless of the age at which they are issued. This is quite equitable, since the American table overstates the mortality at the younger ages and the resulting concealed loading is therefore greater for them. Then for limited-payment life and endowment policies, it is the rule to add one-half of the fixed percentage on the net premium of the policy issued, and one-half of the percentage on the net premium for an ordinary life policy as at the same age. For example, suppose the fixed percentage to be added to the net annual premiums of all ordinary life policies issued, is 33⅓ per cent. For limited-payment life and endowment insurance policies, then, gross premiums would be determined by adding 16.6 per cent of the net rate for the policy being issued, plus 16.6 per cent of the net rate for an ordinary life policy at the same age as the policy issued. Using these percentages and net annual premiums according to the American Experience table and 3 per cent, the following examples show the work:

Ordinary Life, Age 35

Net annual premium per 1000	21.08
Add 33⅓ per cent or ⅓, of net premium	7.03
Gross annual premium rate	28.11

20-payment Life, Age 40

Net annual premium per 1000	33.14
Add 16.6 per cent or ⅙, of net premium	5.52
Add ⅙ of 24.75 (the ordinary life rate at 40) ..	4.13
Gross annual premium per 1000	42.79

20-year Endowment Insurance, Age 45

Net annual premium per 1000................	44.90
Add 1/6 of 44.90............................	7.48
Add 1/6 of 29.67 (ordinary life rate at 45)......	4.95
Gross annual premium per 1000..............	57.33

While this system yields results that are satisfactory in practice, it clearly fails to provide for an exact adjustment of the gross premium rates as between old and young entrants. In participating companies this adjustment may be accomplished for the most part in the distribution of the surplus. The system makes an adjustment as between different classes of policies, however, and its results are somewhat similar to those of the percentage and constant method.

It must be understood that this method and these percentages are not ordinarily used by any company without modifications at some ages and for some types of policies. Some companies charge the rates derived in the above illustrations for those particular policies and at those ages. Others quote lower gross rates. Companies quoting high gross rates generally expect to return high dividends to policyholders, or else they grant policies having many liberal features. A few rates quoted by companies operating for the most part on the basis of the American Experience table and 3 per cent are herewith set forth to illustrate the variations in gross premium rates:

Company	Ordinary Life, Age 35	20-Payment Life, Age 40	20-Year Endowment, Age 45
Examples, above..	28 11	42.79	57 33
A................	28.11	42.79	57 34
B................	28 11	42.79	57 34
C................	28 11	42 79	56.69
D..............	27 00	41 00	54 80
E..............	26 88	41 10	55 15
F........	26 35	40 34	54 15

It may also be observed that this method is especially practicable in this country because most of the business is on the ordinary life, 20-payment life, or 20-year endowment, plans, and is paid for by periodic premiums. For other types of policies and for those paid

for by single premiums, modifications must be made. The changes may consist in varying the percentages, in using one of the other methods of loading, or in adding a somewhat arbitrary amount to cover the estimated expenses which the particular type of contract is likely to cause.

Modifications.—*Non-participating Business.*—Though the use of one or more of the methods of loading described above to arrive at the gross premium for non-participating policies is quite practicable, modifications in them will ordinarily have to be made. The modified percentage method, which is most generally used in this country where the great bulk of the business is on a participating basis, is not so well adapted to non-participating business, because an adjustment of equities in the distribution of the surplus is not feasible. Dr. Thomas Bond Sprague, a celebrated English actuary, worked out a formula on the constant and percentage basis which has been considered a good practical formula for ordinary classes of business. On ordinary life policies it provides for an initial commission to be spread over life, a constant on the amount insured to take care of general expenses, and a percentage calculated on the net premium plus the first two additions to provide for renewal commissions and fluctuations. The American Experience table furnishes such a conservative basis for insurance rates, however, that some companies do not find it necessary to load policies at certain ages at all, and others are loaded lightly. Thus one prominent company quotes the net annual premium rate on the basis of this table and 3½ per cent as the gross rate on ordinary life policies at ages below 31. At age 40, this company quotes a rate of 24.21 per 1000 of insurance for ordinary life, 31.37 for 20-payment life, and 42.62 for 20-year endowments. The net rates for these policies according to the American Experience table and 3½ per cent are 23.50, 30.75, and 41.18, respectively.

Semi-annual and Quarterly Premiums.—In the treatment of both net and gross periodic premiums, we have thus far proceeded on the assumption that periodic premiums would be paid annually at the beginning of the contract-year. Premiums payable semi-annually and quarterly, however, are by no means uncommon. To calculate semi-annual and quarterly net rates from annual net rates requires the consideration of two possibilities. First, in the absence of policy provisions to the contrary, part of the premium may be lost to the company in the year of death, and second, there is a loss of interest to the company because of the delay in paying part of the premium. It is shown on pages 75–76, that the value of an annuity payable semi-annually

may be expressed, $a_x^{(2)} = a_x + \frac{1}{4}$, and the value of an annuity payable quarterly is, $a_x^{(4)} = a_x + \frac{3}{8}$. It has also been demonstrated that the net annual premium may be looked upon as an annuity-due for the premium-paying period, and its value in terms of the single premium and corresponding annuity for an ordinary life insurance is,

$$P_x = \frac{A_x}{1 + a_x}.$$

Now if the premium is to be paid semi-annually, the corresponding annuity must be adjusted to a semi-annual basis. A whole-life annuity-due of 1 payable $\frac{1}{2}$ every six months, has a value greater by $\frac{1}{2}$ than the value of $a_x^{(2)}$. The single premium should therefore be divided by $\frac{1}{2} + a_x^{(2)}$. Using the symbol $P_x^{(2)}$ to represent the total amount each year when two semi-annual payments are provided, that is, when the premium is to be $\frac{1}{2}P_x^{(2)}$ every six months, its value in the case of a whole-life policy becomes:

$$P_x^{(2)} = \frac{A_x}{\frac{1}{2} + a_x^{(2)}} = \frac{A_x}{\frac{1}{2} + a_x + \frac{1}{4}} = \frac{A_x}{\frac{3}{4} + a_x}.$$

In the case of an annuity of 1 payable $\frac{1}{4}$ at the beginning instead of at the end of each quarter, the value would be greater by $\frac{1}{4}$ than the value of $a_x^{(4)}$, or $\frac{1}{4} + a_x^{(4)}$.

Hence,

$$P_x^{(4)} = \frac{A_x}{\frac{1}{4} + a_x^{(4)}} = \frac{A_x}{\frac{1}{4} + a_x + \frac{3}{8}} = \frac{A_x}{\frac{5}{8} + a_x}.$$

One-fourth of this value, then, is the amount to be paid each quarter, or the net quarterly premium.

Since the single premium is divided by $\frac{3}{4} + a_x$, and $\frac{5}{8} + a_x$, to arrive at twice the semi-annual and four times the quarterly rates, respectively, the results will be greater than the annual premium which is found by dividing the single premium by $1 + a_x$. In general it will be found that the addition to the annual premium necessary to obtain the semi-annual rate is about 2 per cent, and for the quarterly rate, 3 per cent.

Other problems present themselves, however, when the gross semi-annual and quarterly rates are attempted. There is the additional expense of collecting the premium twice or four times each year. In this country it has not been customary to use net semi-annual and quarterly rates in the determination of the corresponding gross rates. The common practice has been to consider semi-annual and quarterly

payments as being installments of the gross annual premium. The latter is considered due and payable at the beginning of the contract-year, but as an accommodation to the insured the payment of the premium may be made in installments during the year. In case of death, then, any unpaid installments due subsequently during the year of death are deducted from the amount of the claim paid by the company at settlement. This practice eliminates the possibility of the company's losing a part of the premium during the year of death. The interest factor and the extra expense of collecting two or four times each year instead of once, may be looked upon as a problem in loading.

The practice of companies varies as regards the percentages added. An examination of the methods of 20 companies that do the bulk of the business in this country reveals the fact that five of them add 4 per cent to the gross annual rate and divide by 2 to arrive at the semi-annual premium; and for the quarterly rate, they add 6 per cent and divide by 4. Four others multiply the gross annual premium by .52 for the semi-annual, and by .265 for the quarterly premium, which yields the same results. Four others use .51, and .2575, or equivalents; three use .51, and .26. The remaining companies use slightly different percentages. One company is unique in that it divides the gross annual premium by 2 for the semi-annual, and by 4 for the quarterly, but takes promissory notes for the deferred installments at 6 per cent per annum for the period of deferment. It thus appears that quite a number of companies approach the net addition of 2 per cent and 3 per cent, based, however, on gross instead of net annual rates, although nearly half of the more prominent companies retain the heavier additions of 4 per cent and 6 per cent.

Term Policies.—Short-term policies require special treatment in the matter of loading. In the case of these it is not customary to pay so high a percentage of the initial premium as a commission. If initial expenses are paid from surplus funds and then spread over the period the contract is to remain in force, because of the shortness of the period, the net rate must be increased considerably. If the modified percentage method were used, gross premiums would carry a heavier percentage of loading than if a straight percentage addition were employed. Thus the net annual premium for a 10-year term policy at age 35 by the American Experience table and 3 per cent is 9.42. If this is loaded 33⅓ per cent, the gross premium will be 12.56; whereas if it is loaded 16.6 per cent plus 16.6 per cent of the ordinary life rate at the same age, the gross premium will be 14.50.

It has been stated on good authority[1] that gross rates for short-term policies might well be calculated by adding a percentage and a constant to the net rates, but that such loadings must be considerably higher than corresponding whole-life loadings to provide proportionate commissions, because the business remains on the books only a short time, yet the expenses occasioned should be provided by the loadings during that period. Mortality experienced on this type of contract, too, has been heavier in general than on ordinary life, limited-payment life, and endowment contracts, and the company is exposed to adverse selection especially in the matter of renewal or conversion when these options are granted.

Long term policies covering the greater part of the income-producing period of life, say 20 or 30 years, may sometimes be desired. In general the problem of loading in connection with such contracts is not essentially different from that of ordinary life insurance. The mortality is likely to be higher than on ordinary life policies, however, and the loading should be a little greater than on ordinary-life non-participating contracts.

Double endowment insurance policies present problems quite opposite to those associated with term insurance. On such contracts a very favorable mortality may be expected. Even if within certain limits mortality should be heavier than normal, premiums might be diminished, since the premium for the endowment portion of such contracts is reduced in case of high mortality by a greater amount than the premium for the term insurance part is increased.

Continuous-installment Policies.—It has been observed above that the premium for a continuous-installment policy on the ordinary-life plan consists of two parts, the premium for the regular installment feature, and the premium for a deferred reversionary annuity to the beneficiary. The net annual premium for the guaranteed benefit may be loaded in the same manner as that of an ordinary life policy. The extra net premium for the additional benefit may then be loaded on a straight percentage basis to cover the extra expenses it involves, the two gross premiums combined forming the gross annual premium for the entire benefit.

Concerning the matter of an equitable distribution of expenses, it may be observed that each type of policy and each kind of business presents its own peculiar problems. Variations in method or in percentages or constants added, must be made for each particular class

[1] Moir, Henry, Office Premiums, in Transactions of the Faculty of Actuaries, Vol. II., No. 19, p. 216.

of business. In all such adjustments, the expenses incident to each particular class of business as determined by the company's experience must be the basis. Judgment must then be exercised in finally fixing the rates so they may be effectively used in competition.

Initial Expense.—The problem of incidence, or of meeting the heavy initial expenses, is a most serious one for a new company, or for a company that has no surplus funds on which it may draw for first-year expenses. The maintenance of a level office premium, the fulfillment of statutory requirements concerning reserve liabilities and the necessity of meeting competition, render this problem exceedingly difficult. For instance, the gross annual rates per 1000 for ordinary life, 20-payment life and 20-year endowment policies at age 35, according to the American Experience table, and 3 per cent, loaded by the modified percentage method, one-sixth, and one-sixth, are 28.11, 38.34, and 52.48, respectively. In Chapter IX, it is shown that the terminal net values of these policies at the end of one year is 12.88, 22.00, and 34.59. If the premium income from each of these three types of policies is fairly uniform throughout the year, it may be assumed that each has been in force approximately six months, on the average, on December 31 of the calendar year of issue. As explained in Chapter X, it is customary to make this assumption in reporting liabilities to state insurance departments. The legal reserves or policy-values which must be held against them, therefore, are 16.98, 25.93, and 40.76. It is also shown in Chapter XI, that the technical or tabular net cost of insurance which the company provides for on these policies is 8.83, 8.75, and 8.64, respectively. Agents' first-year commissions, salaries, and other initial expenses, which for the most part occur during the calendar year of issue, may be estimated as averaging at least 50 per cent of the initial gross premium. The sum of these items, even if the actual cost of insurance is but one-half of the tabular cost, exceeds the first premium. Thus the company that has no surplus on which to draw for initial expenses has to face a serious problem. A new company cannot charge a high initial premium without handicapping its agents in competing with older companies, and it probably will be obliged to pay high commissions in order to attract and hold competent agents. An older company, or one with ample surplus funds, may draw upon its surplus to meet those initial expenses which first-year loadings fail to provide, replenishing its funds later from the sources discussed in Chapter XI.

One important condition assists in meeting first-year expenses. The medical examination places new entrants in a select class regard-

ing mortality. Since premiums are determined on an ultimate table, a considerable saving may be expected during the first year because of a lower death rate than that used in premium calculation. It is quite usual for the actual net cost of insurance during the first year to be less than 50 per cent of the ultimate mortality cost expected and 20 per cent of that provided for in the premium calculation. This factor may also be used to vary the valuation basis as explained later.

Methods of Meeting Initial Expenses.—Speaking generally, there are three methods of providing for initial expenses. First, they may be paid from surplus funds, as explained above. Secondly, an initiation fee may be charged in addition to the regular premium for the first year. Thirdly, the company may be permitted to hold reserves lower than those resulting from full level-premium valuation. There are several different methods of valuation that serve to provide for the first-year expenses. This subject is discussed in Chapter X.

Surplus.—A most convenient and practical method, and the one most used by companies having sufficient surplus funds, consists in drawing upon the surplus for initial expenses. It is the method contemplated in Dr. Sprague's formula for non-participating business; i.e., the initial expense is spread over the life of the policy in the form of a regular addition to the net premium. This means that the company will draw upon the surplus, pay the initial expenses at the outset, and then gradually recover it as the policyholder continues to pay his annual premiums.

Since percentage expenses actually incurred are based on a percentage of the gross premium and not on the net premium, it is proper to formulate methods yielding such results that the percentage of loading based on the gross premium, and deducted from it, will leave the net premium. Thus if the net premium were 100 and the loading 10 per cent, if the 10 per cent means 10 per cent of the *gross* premium, the latter would be found by means of the proportion: $\frac{100}{90} = \frac{x}{100}$; whence $x = 111.11$. Thus 10 per cent of 111.11, the gross premium, deducted from itself will leave 100, the net premium. On this basis, the formulas previously given would be modified. Thus the formula for the constant and percentage method becomes: $P' = \frac{1}{1-k}(P + c)$. In level-premium calculation it was learned that any sum, such as a single premium, may be spread over life by dividing it by the present value of an annuity-due, or $1 + a_x$. In the case of an ordinary life policy, then,

once the initial expenses have been estimated, they may be spread over the premium-paying period by dividing the amount by $1 + a_x$. Then they may be added to the net premium. The same result will be accomplished if the initial expenses are added to the net single premium and the sum divided by $1 + a_x$. If the average expense is represented by the symbol K, the entire formula expressing the value of P' according to the constant and percentage method becomes:

$$P' = \frac{1}{1-k}\left(\frac{A_x + K}{1 + a_x} + c\right).$$

For limited-payment life policies, the expression is:

$${}_nP'_x = \frac{1}{1-k}\left(\frac{A_x + K}{1 + a_{x\overline{n|}}} + c\right).$$

When the expenses have been determined as a percentage of the net premium plus the constant, the above equations become,

$$P' = (1 + k)\left(\frac{A_x + K}{1 + a_x} + c\right);$$

and

$${}_nP'_x = (1 + k)\left(\frac{A_x + K}{1 + a_{x\overline{n|}}} + c\right).$$

Certain French companies, whose actuaries have devoted considerable thought to the subject of initial expenses, proceed along somewhat different lines. The amount of money required to meet the first-year expenses of a policy is first ascertained, and deducted from the net premium for the first year. The amount deducted is then spread over the premium-paying period by dividing it by the present value of an annuity-due of 1 for that period. The resulting annuity is then added to each net annual premium including the first. The cost of management may then be treated as a constant addition to the net premium. Having added to the net premium the annuity to recover the initial expenses, and the constant to provide the expenses of management, they increase the total by a percentage of the gross premium, to take care of renewal expenses. The sum of the three items, net premium, annuity, and constant, may be multiplied by $\frac{1}{1-k}$, where k is the percentage addition as illustrated above, to arrive at the gross premium.

Initiation Fee.—A cash initiation fee sufficient to meet new business expenses would render every policy self-sustaining from the outset. In place of cash, an interest-bearing note might be accepted, to

be paid off by means of dividends, or to be deducted from the amount of the insurance at settlement. This plan has never found much favor with companies. In the first place it constitutes a departure from the level-premium plan, and secondly it would be quite difficult to sell a policy carrying a premium of this nature in competition with level-premium policies. A few companies, however, have accomplished something in this direction with so-called "premium-reduction," or "guaranteed-dividend" policies, on which the premiums after the first are reduced to a stipulated rate. In selling such a policy, however, the reduction of the premium rate after the first payment, or the guaranteed "dividends," are usually stressed, and the reduced rate compared with the rate for policies of other companies after deducting dividends, so that there is no discussion, ordinarily, of the extra first premium as being in the nature of an initiation fee. Participating business with a high initial premium followed by large dividends is somewhat similar. This plan would not be practical for the collection of all of the initial expenses.

Single-premium Policies.—These present no problem so far as renewal expenses are concerned. An amount sufficient to cover administrative expenses during the life of the contract may be added to the net premium, as well as an amount sufficient to meet the agent's commission. The rate of commission on this type of contract does not ordinarily exceed 7½ per cent of the gross premium. An examination of the rates quoted by the leading companies reveals the fact that the amount of loading varies considerably on this kind of business, as it does on level-premium business. The following table may serve to illustrate:

SINGLE-PREMIUM RATES, AGE 35

	Ordinary Life	Twenty-year Endowment
N. S. P., Amer. Ex., 3 per cent	419.88	590 30
N. S. P., Amer. Ex., 3½ per cent	370 55	542.60
Highest gross rates quoted—participating company on 3 per cent basis	493 38	692 20
Lowest—non-participating 3½ per cent basis	370.55	542 60

It is thus seen that at this age the loading varies from nothing on the part of one non-participating company, to nearly 15 per cent of the gross premium on the part of a participating company. Divi-

dends paid by the participating company will of course reduce the cost to its policyholders.

Loading Return-premium Policies.—The calculation of the net premium required to provide for the return of net premiums without interest was discussed in Chapter VII. When the net return premium has been determined it must then be loaded for expenses. When return-premium policies are issued, it is the gross premiums that are returned, hence the loading above the net premium requires an additional premium, which must also be loaded. Suppose π represents the net annual premium for an increasing insurance of 1, ϕ the net extra premium to provide for the return of the entire gross premium, ϕ' the gross extra premium to provide for the return feature, and P'' the entire gross premium including the extra premium of ϕ' loaded k per cent of itself, Then, $P'' = P' + \phi'$. But P'' is an increasing insurance of $P' + \phi'$. increased by the percentage k. Hence:

$$P'' = P' + \phi' = P' + \pi(1 + k)(P' + \phi')$$

$$\phi' = \pi(1 + k)(P' + \phi')$$

$$= \pi P'(1 + k) + \pi\phi'(1 + k),$$

$$\phi'[1 - \pi(1 + k)] = P'(1 + k),$$

$$\phi' = P'\left(\frac{\pi(1 + k)}{1 - \pi(1 + k)}\right),$$

$$P'' = P' + P'\left(\frac{\pi(1 + k)}{1 - \pi(1 + k)}\right)$$

$$= P'\left(1 + \frac{\pi(1 + k)}{1 - \pi(1 + k)}\right)$$

$$= P'\frac{1}{1 - \pi(1 + k)}.$$

EXAMPLE.—Suppose the gross annual premium for a 10-year pure endowment is .09320, the net annual premium for a 10-year increasing insurance of 1 is .050377, and the loading is 16.6 per cent. Then:

$$P'' = P'\frac{1}{1 - \pi(1 + k)}$$

$$= \frac{.09320}{1 - (.050377 \times 1\frac{1}{6})}$$

$$= .09929, \text{ or } 99.29 \text{ per } 1000.$$

Some companies quote gross rates of 100 per 1000 regardless of age, for such contracts. This they can very well afford to do; for

whether the life survives or fails they have gained the interest, less expenses, on premiums that have been paid.

Conclusion.—Enough has been said to give the student a general idea of the principal problems associated with loading, especially as pertains to the more usual forms of insurance. When the larger premiums are charged for participating policies and when the company has a substantial surplus, the problem of initial expenses is of less importance, and the problem of equity as between different classes and ages predominates. The best way to construct gross or office rates so as to attain equity between classes is to form a series of fundamental gross premiums for the type of policy which constitutes the largest proportion of outstanding business. Ordinarily, whole-life policies will form the basis, although some companies have such a large proportion of endowment insurance on their books that this class might be used. With most American companies, ordinary life, limited-payment life, and endowment insurance, must be kept in mind when forming the fundamental schedule. When the basic schedule has been established, the rates for other classes may be fixed in relation to it, the actuary having in mind the peculiar nature of the expenses of each different type and modifying the loading accordingly. Considerations of a practical nature must then be given due weight, such as the necessity of meeting competition or the desirability of encouraging or discouraging this or that type of business. The matter of encouraging types of business may also be accomplished through instructions to the agency force, or by varying the scale of commissions to agents.

In forming the final schedule of rates which the company is to quote, care must be exercised to see that the various tables are consistent with one another. When but few types of contracts are issued, this is a comparatively simple matter. When, however, a score of variations of the regular forms are offered, and when additional features are added, such as double indemnity, and disability benefits, the problem becomes more complicated. A few of the inconsistencies which might arise, and which must be avoided, are herewith set forth:[2]

1. The whole-life non-participating rate should not exceed the rate for an endowment-insurance policy maturing at an advanced age.

[2] Moir, H., Office Premiums, in Transactions of the Faculty of Actuaries, Vol. II., No. 19, pp. 230-231.

2. Endowment-insurance rates should not decrease with an increase in age.
3. Loading with a view to providing for a heavy dividend schedule should not be carried to such extremes as to make the resulting gross rates appear absurd when compared with non-participating rates.
4. Survivorship-insurance rates should not be lower than the corresponding joint-life rates.
5. Joint-life and last-survivor rates together should agree approximately with the corresponding ordinary-life rates at the individual ages.
6. The premium during the first few years of a policy on the step-rate premium plan should not be lower than the corresponding short-term rate.

In most instances the net rates serve as a guide, though not in all, as in No. 6, above. Net rates loaded in any manner that results in inconsistencies in office rates, similar to those enumerated, might leave the impression that the company is not in all instances granting benefits consistent with the premiums charged.

CHAPTER IX

VALUATION

Nature and Origin of Policy-Values.—When policies are issued under the natural premium or "yearly-renewable term" plan, the net natural premium at the beginning of each year is the present value of the benefit. It is found by discounting the amount at risk both for the probability and for the interest element involved. Since the present value of the premium at the beginning of any year is the premium itself, and since this is equal to the present value of the benefit promised, the company's liability under the contract is balanced by the premium collected. The policy, therefore, is said to have no "value," or "reserve." When, however, policies are issued on the level-premium plan, the policyholder is charged more than current mortality requires during the early years, and less than this during the later years, as was also shown in Chapter VII. The overcharges during the early years, then, must be set aside and invested so as to prepare for the heavier mortality which the policy must share in the later years. These overcharges, set aside and improved at the net rate of interest assumed, with benefit of survivorship, are called the "reserve," the "value," or the "liability" of the policy. They cannot be returned to the policyholder if he continues his insurance, nor added to the surplus, nor paid to stockholders in the form of dividends. They may be returned to the policyholder after the policy has been in force a few years if he withdraws from the company, and so they represent the value of the policy to him or his assigns. Hence such accumulations have been called "policy-values." Since the funds constituting these values must be held by the company for the policyholder if he desires to withdraw, and against his policy if the contract remains on its books, an amount equal to them is set up as the company's "liability" under the contract. Finally, since these funds are held in reserve for the purposes mentioned, and since they may be a sufficient inducement to some other company to assume responsibility for the contract if turned over to it, the policyholder continuing to pay premiums to the re-insuring company, they have

been called "reinsurance reserves," or merely "reserves." From this last statement it may be inferred that the company's "liability" under a policy, or the "policy-value," or "reserve," is the difference between the present value of the benefits promised by the company and the present value of future premiums which the insured agrees to pay. That is the fact; so the problem of valuation; i.e., the determination of the "policy-value," "liability," or "reserve," may be looked upon in two ways. First, the policy-value may be considered as the excess of past net premiums over past tabular mortality requirements, the excess being improved at the net rate of interest; and secondly, it may be looked upon as the difference between the present value, at the age of valuation, of the benefits promised, and the present value, at that age, of future premiums payable by the insured.

Thus the origin of policy-values is due to progressively increasing mortality. Under the level premium plan the progressively increasing risk from age to age is paid for by an average premium; uniform throughout life in the case of an ordinary life policy, and throughout the premium-paying period, for other types. The level premium, then, must be greater than the risk requires during the early years, and the excess that remains after providing for the risk must be preserved and increased at the fundamental rate of interest, in order that it may provide for the years during which the value of the risk is greater than the premium. These excesses during the early years, improved at the fundamental rate of interest, create a fund which will in most instances be called the policy-value when mentioned in this volume. The word "reserve," much used in this country to denote the policy-value, appears to be a misleading term in that it is used in the accounts of other corporations to indicate a fund held for protection beyond the measure of what is necessary to meet actual, known liabilities.

Various Kinds of Values.—There are several different kinds of policy-values to be distinguished from one another, each determined upon a different basis. Thus policy-values may be either *net* or *gross*, i.e., future gross premiums may be used as the basis of valuation instead of the net premiums which have been considered thus far in this chapter. Gross premium valuation is treated below. Policy-values may be calculated on the full level-premium basis, or according to some other standard, such as the preliminary term, modified preliminary term, select and ultimate, or Illinois standard, as explained below. The "full level-premium" value, as the expression indicates, is the full

value of a policy which is paid for under the level-premium plan. The full level-premium basis of valuation is used by the companies doing the greater part of the business in this country, and it is the basis from which the other values are derived. Taking first the full level-premium basis, then, if the policy-value is determined at the date when a premium is due, and just before the premium is paid, it is called the *terminal value.* If the terminal value is calculated on the basis of net premiums, it is called the *terminal net value.* The terminal net value, then, is the net value at the end of the policy-year. If the net annual premium is added to the terminal net value, the result will be the *initial net value* at the beginning of the next year. Let ${}_nV_x$ represent the terminal net value of an ordinary life policy, issued at age x, at the end of n years. Then the initial value *after* n years is equal to ${}_nV_x + P_x$. The *mean value,* as used in practice, is the arithmetical average of the initial and terminal values of the policy-year of valuation.

Since net valuation is required by the valuation laws in this country, and since the terminal net value is the basis from which initial and mean net values are found, the determination of terminal net value requires much more attention than does the study of other values. Moreover, terminal net values are adopted as the basis for surrender and loan values, because such values are determined as of the date of renewal, which is also the end of the individual policy-year.

Calculation of Terminal Net Values: Prospective Method.—*Ordinary Life Insurance.*—Suppose the age attained by the policyholder at the time of valuation be called the age of valuation. The prospective method of valuation involves taking the particular policy at the age of valuation and looking entirely to the future. Thus the present value, at the age of valuation, of the benefits promised by the company are weighed against the present value of the remaining premiums payable by the insured person, and the difference is considered the terminal net value. It is the sum, which, together with the future premiums payable, will enable the company to pay its claims as they fall due, provided, of course, the company has enough risks to insure the operation of averages.

For example, suppose the problem is given to calculate the terminal net value of an ordinary life insurance of 1 issued at age 35 at the end of 10 years, according to the American Experience table and 3 per cent. By the American Experience table and 3 per cent, is meant that any premium or annuity-value used in the calculation will have been determined on that basis.

It was shown in Chapter VII that at the time of issue the present value of the benefit is equal to the present value of the premiums payable. The present value of the benefit was called the net single premium, and at age 35 it was found to be .41988. At age 45, the age of the insured at the date of valuation in the problem just proposed, the present value of the benefit under an ordinary life insurance will be greater, since the probability of its being paid in the near future is greater than it was at age 35. It has been determined as .504585, the net single premium at 45. The present value, however, of the remaining premiums payable decreases from age to age. Thus at age 45 the present value of the remaining premiums payable is less than the present value of premiums payable at age 35, since a life aged 45 will survive, on the average, for a shorter period than will one aged 35. Carried to the end of the mortality table, the present value of the benefit promised would be 1, and the present value of premiums payable would become 0, since no more are due, because according to the table the life will certainly have failed. At age 45 the person who took the insurance at 35 will pay the premium as of age 35, and will continue to pay premiums during survival. The net annual premium at age 35 has been determined as .021081. At age 45, then, the present value of future premiums payable on the insurance issued at 35, would be the present value of an annuity-due of .021081. The present value of an annuity-due of 1 at age 45 has been determined as 17.0093. Then, $17.0093 \times .021081 = .358573$, the present value of future premiums due. Hence, $.504585 - .358573 = .14601$, the policy-value. If the policy were for \$1000, \$146.01 would be the company's liability under the policy, in excess of the contingent asset, which consists of the present value of future premiums payable. It is the sum which the company must have accumulated from the excess of past premiums over past mortality requirements, the excess each year having been improved at 3 per cent compounded annually, with benefit of survivorship. This sum added to the present value of future premiums with which the company may be credited, will equal the present value of the benefit. So, if the company has this sum, it is solvent so far as this policy is concerned. If the insured should withdraw from the company, the company will be relieved of its liability and will not need the 146.01, which may be returned to the insured as his surrender value. The policy-value of an ordinary life contract may be found, then, by multiplying the net annual premium as at the age of issue by the present value of annuity-due of 1 as at the age of valuation, and subtracting the result

from the net single premium at the age of valuation. Stated as a formula:

$$ {}_nV_x = A_{x+n} - P_x(1 + a_{x+n}). $$

In terms of the commutation symbols, this becomes:

$$ {}_nV_x = A_{x+n} - P_x(1 + a_{x+n}) $$
$$ = \frac{M_{x+n}}{D_{x+n}} - \frac{P_xN_{x+n-1}}{D_{x+n}} = \frac{M_{x+n} - P_xN_{x+n-1}}{D_{x+n}}. $$

Hence the value of the policy in the above example may now be found from commutation columns, Appendix C, as follows:

$$ {}_nV_x = \frac{M_{x+n} - P_xN_{x+n-1}}{D_{x+n}} $$
$$ = \frac{M_{45} - (P_{35} \times N_{44})}{D_{45}} $$
$$ = \frac{9897.032 - (.021081 \times 333623.0)}{19614.2} $$
$$ = .14601, \text{ or } 146.01 \text{ per } 1000 \text{ of insurance.} $$

The terminal net value of the insurance in the above illustration may be derived from net annual premiums and the annuity-due as of the age of valuation without using the single premium. If the contract were issued at age 45, the net annual premium would be .029665. The present value of an annuity-due of this amount would of course equal the present value of the benefit. Since the contract was issued at age 35, however, the net annual premium which the company will continue to collect during the insured's survival is only .021081, the net annual premium as of age 35. The difference between these two premiums is $.029665 - .021081 = .008584$, the amount by which the company would be short each year if it issued an insurance of 1 at 45 for a premium as of age 35. At age 45, then, the company must have saved enough from past premiums on the insurance issued at 35, to cover this shortage. The present value of the shortage is the present value at age 45 of an annuity-due of .008584, or $.008584 \times 17.0093 = .14601$, as before. The formula is, ${}_nV_x = (P_{x+n} - P_x)(1 + a_{x+n})$. Stated verbally, the terminal net value at the end of any year is the present value at the age of valuation of an annuity-due of the difference between the net annual premium as of the age of valuation and that as of the age of issue.

The terminal net value may be found from annuities only. Thus it was shown in Chapter VI that $A_x = 1 - d(1 + a_x)$; whence,

$$A_{x+n} = 1 - d(1 + a_{x+n}).$$

And in Chapter VII it was demonstrated that $P_x = \frac{1}{1 + a_x} - d$.

Substituting these values for A_{x+n} and P_x in the general expression of the value of ${}_nV_x$:

$$\begin{aligned}
{}_nV_x &= A_{x+n} - P_x(1 + a_{x+n}) \\
&= 1 - d(1 + a_{x+n}) - \left(\frac{1}{1 + a_x} - d\right)(1 + a_{x+n}) \\
&= 1 - \frac{1 + a_{x+n}}{1 + a_x} = \frac{a_x - a_{x+n}}{1 + a_x}.
\end{aligned}$$

Stated verbally, the terminal net value, at the end of any number of years, of an ordinary life insurance of 1 issued at any age, is equal to the difference between the values of the corresponding annuities at the age of issue and the age of valuation, divided by the value of an annuity-due as of the age of issue.

Using the same example as before:

$$\begin{aligned}
{}_nV_x &= \frac{a_x - a_{x+n}}{1 + a_x}, \\
{}_{10}V_{35} &= \frac{a_{35} - a_{45}}{1 + a_{35}} \\
&= \frac{18.9174 - 16.0093}{19.9174} \\
&= .14601, \text{ or } 146.01 \text{ per } 1000.
\end{aligned}$$

In order to find the policy-value by this formula it is not necessary to know the mortality table nor the rate of interest by which the annuity-values were calculated. James Chisholm used this principle and prepared " Tables for finding the Values of Policies of all durations, according to any Table of Mortality or any Rate of Interest," published in 1885. They resemble the conversion tables described in Chapter VII.

If the values of the above annuities are not at hand, the policy-value may be found directly from the commutation columns. Thus:

$$\begin{aligned}
{}_nV_x &= 1 - \frac{1 + a_{x+n}}{1 + a_x} = 1 - \frac{D_x N_{x+n-1}}{D_{x+n} N_{x-1}}, \\
{}_{10}V_{35} &= 1 - \frac{29078.1 \times 333623}{19614.2 \times 579160.7} \\
&= 1 - .85399 \\
&= .14601, \text{ as previously found.}
\end{aligned}$$

The terminal net value may be ascertained from single premiums only. Substituting for a_x its value in terms of A_x, as found in Chapter VI, and simplifying, ${}_nV_x = \dfrac{A_{x+n} - A_x}{1 - A_x}$. From annual premiums only, ${}_nV_x = \dfrac{P_{x+n} - P_x}{P_{x+n} + vi}$. To illustrate the use of these two formulas, take the same example of an ordinary life insurance of 1 issued at age 35. To find the terminal net value at the end of ten years:

$$ {}_{10}V_{35} = \frac{.504585 - .41988}{1 - .41988} = .14601, $$

and

$$ {}_{10}V_{35} = \frac{P_{x+n} - P_x}{P_{x+n} + vi} = \frac{.029665 - .021081}{.029665 + (.97087379 \times .03)} $$
$$ = .14601. $$

Single-premium Values.—Before considering the valuation of limited-payment life, endowment-insurance policies, and deferred annuities, it may be well to discuss briefly the subject of single premium values. As regards whole-life insurance the company must have in its possession at the end of each policy-year throughout the duration of a policy, either the net single premium as at the age attained by the insured person, or its equivalent. When a policy is issued the company must receive the net single premium, or charge periodic premiums whose present value is equal to the net single premium. Ten years later, as shown above, the company must have a fund, called herein the terminal net value, which together with the present value of the remaining premiums to be collected in the future, equals the net single premium at the age then attained, called herein the age of valuation. The same is true at the end of any number of years after issue down to the end of the mortality table, when the present value of future premiums becomes 0 and the face of the policy is the policy-value. If the company collects the net single premium at once upon issuing the policy, the present value of future premiums will be 0, since all have been collected at the outset. The formula, ${}_nV_x = A_{x+n} - P_x(1 + a_{x+n})$, still applies, but since $P_x = 0$, the policy value n years hence will be A_{x+n}. Therefore a table of net single premiums is also a table of single premium policy-values. Thus the terminal net single premium value of an ordinary-life policy of 1000, issued at age 35, at the end of 10 years, is 504.59, the net single premium at age 45, according to the American Experience table and 3 per cent.

In the case of a whole-life annuity, the value of the contract at any age of valuation is likewise the net single premium at that age.

The calculation of these values is explained in Chapter V. It will be noted that such values decrease from age to age as the contract grows older. Thus the present value of a whole-life annuity of 1 is 18.9174 at age 35. At age 45 it is 16.0093. If such an annuity is issued at age 35, then, its value 10 years after issue will be 16.0093. At age 95, the value will be 0.

Contracts covering a shorter period than the duration of life present somewhat different problems. Thus in endowment insurances the company must have the net single premium, or its equivalent, at the age of valuation, of a similar contract but for the term of years from the age of valuation to the date of maturity. The net single premium for a 20-year endowment insurance policy of 1000 at age 35 (American Experience and 3 per cent) is 590.30. At the end of 10 years the terminal net single premium value of such a contract is 757.24, the net single premium for a 10-year endowment insurance covering the balance of the 20-year period from age 45.

The company collecting the single premium for a pure endowment must retain that premium and improve it at the fundamental rate of interest to the end of the contract period. Hence at any time during the continuance of the contract, the company must possess the single premium improved at interest to that date. The company is able to retain this fund, because it has no claims to pay until the end of the pure endowment period; that is, it carries no risk except that of survival. Moreover, on any date of valuation the company will have been able to accumulate more than the amount of the single premium improved at interest, on the average, if it has a sufficient number of contracts in force. This is because some of the holders of such contracts will have died, thus relieving the company of any liability to them. For valuation purposes, the gains which the company makes through deaths is added to the policy-values of the survivors' contracts. Looking to the future, on any date of valuation the company must possess the net single premium for a pure endowment from then on to the end of the period. Thus the terminal net single premium value (Amer. Ex., 3 per cent) of a 20-year pure endowment issued at age 35 at the end of 10 years is 647.69, the single premium for a 10-year pure endowment of 1000 as of age 45.

To illustrate the benefit of survivorship involved, let the net single premium for the 20-year pure endowment at 35, which is 436.87, be improved at 3 per cent compounded annually for 10 years. It will amount to only $436.87 \times 1.344 = 587.15$, approximately, whereas the policy-value has grown to 647.69. The difference is attributable to

the benefit of survivorship, in that this contract shares the sums contributed by lives that have failed from the group which started at 35.

In the case of a 20-year term insurance of 1000, the net single premium at 35 is 153.43. This improved at 3 per cent compound interest for 10 years, amounts to $153.43 \times 1.344 = 206.21$, approximately. Yet the terminal net single premium policy-value at the end of 10 years is only 109.55, the net single premium for a 10-year term policy as of age 45. The policy-value is less than the original single premium improved at interest in this case, because the policy has had to bear its share of death claims over the period. The net single premium value of the 20-year term insurance, or 109.55, plus the net single premium value of the 20-year pure endowment, or 647.69, equals the net single premium value of the 20-year endowment insurance policy, or 757.24, as found above.

The terminal net single premium value of any type of policy is the net single premium for the same type of policy as at the age of valuation and continuing to the end of the period during which the policy being valued is to run. This value will vary, of course, as the contract grows older and the age of valuation advances from year to year. This value or its equivalent must be in the company's possession at the end of each policy-year throughout the duration of any contract. If the policy is issued on the level premium plan, then, the full level premium value at the end of any policy-year will be the difference between the net single premium for a similar policy, as of the age of valuation and for the balance of the term to the end of the period the policy being valued is to run, and the present value, at the age of valuation, of premiums due in the future.

Limited-payment Life, Level-premium Values.—It is evident that at the end of the premium-paying period, the company must have a fund equal to the net single premium at that age for a similar contract and for the balance of the term. In limited-payment life insurance, the company must possess the net single premium for an ordinary life policy at the end of the premium-paying period, as of the age then attained by the insured. Thereafter the policy-value at the end of each year is the corresponding net single premium. At any anniversary of the policy prior to the end of the premium-paying period, however, the company need possess only such a sum as together with the present value of future premiums will equal the net single premium at that age. Taking a 20-payment life policy of 1000 issued at age 35 according to the American Experience table and 3 per cent,

what is its terminal net value at the end of 10 years, on the same basis? The net annual premium at 35 is 29.85. At age 45, ten more of these premiums will remain payable, one of them being due at once. The present value of a temporary annuity-due of 1 from 45 to 54, inclusive, corresponding to the remaining future premiums, is 8.334. Then, $29.85 \times 8.334 = 248.77$, the present value of the remaining premiums, and $504.59 - 248.77 = 255.82$, the policy-value.

Pure Endowments.—Following the same rule, the terminal net value of a 20-year pure endowment of 1000 on the full level premium basis issued at 35, at the end of 10 years, American Experience and 3 per cent, may be found as follows:

From,	647.69,	the net single premium at 45 for similar contract, balance of term;
deduct,	258.88,	the present value of future premiums;
which yields,	388.81,	the terminal net level premium value at age 45.

It may be observed that the value of the pure endowment is greater than the accumulation of an annuity-certain of the level premium over the period from issue to valuation. This is again due to the benefit of survivorship.

Deferred Annuities.—Deferred annuities issued on the annual-premium plan may be valued in a similar manner. Thus let us suppose an annuity is issued at 35, first payment to be made by the company at 65. Its terminal net value at 45, ten years after issue, is the single premium for an annuity at 45, first payment at 65, less the present value at 45 of premiums remaining to be collected, subject to survival.

Term Insurance.—The net annual premium for a 20-year term insurance of 1000, at age 35, is 10.91. At age 45 there remain 10 premiums, subject to survival, to be collected. Their value is $10.91 \times 8.3339 = 90.91$. The net single premium for a 10-year term policy of 1000 at age 45 is 109.55. Hence, $109.55 - 90.91 = 18.64$, the level premium value of a 20-year term policy of 1000 issued at 35, at the end of 10 years.

This value is less than the accumulation of an annuity-certain of the premium for 10 years at 3 per cent, because the policy is charged with its share of the claims that have arisen.

Endowment Insurance.—The level-premium value of an endowment insurance may be ascertained in the same manner. Using the 20-year endowment insurance referred to above, the single premium for a 10-year endowment insurance of 1000 at 45 is first found. It is 757.24. The present value of premiums remaining at 45 on an endowment insurance issued at 35 is, $41.97 \times 8.334 = 349.78$. Hence $757.235 - 349.78 = 407.45$, the terminal net value at age 45 of a 20-year endowment insurance of 1000 issued at age 35. It may be observed that 18.64, the value of the term insurance, plus 388.81, the value of the pure endowment found above, equals 407.45, the value of the endowment insurance policy.

The formulas mentioned in the discussion of net annual premium values of ordinary life insurance may also be applied to types of contracts other than these, if the values of A_{x+n}, P_{x+n}, and the annuity-due are modified so as to be for the balance of the term instead of for the whole of life. For example, the net annual premium for a 10-year endowment insurance of 1 at 45 is .09085. The net annual premium for a 20-year endowment insurance of 1 at age 35 is .04197. The difference is $.09085 - .04196 = .04889$. This multiplied by 8.334, the present value of a 10-year annuity-due of 1 at 45, equals .40745, the value of the 20-year endowment insurance at the end of 10 years, as before.

Terminal Net Values by the Retrospective Method.—The prospective method of valuation discussed above is much used, yet the retrospective method is equally important. This method involves taking the premiums that have been paid in the past, deducting the policy's share of claims paid, and improving the balance with interest, and survivorship, to the date of valuation, to arrive at the policy-value. On the same mortality and interest basis, the same policy-value will of course result.

Considering single premium values first, assume that 81,822 persons are insured for 1000 at age 35, each paying the single premium of 419.88. The total fund collected in the beginning would be $419.88 \times 81822 = 34355421.36$. This sum increased at 3 per cent interest for one year amounts to 35,386,084. From this must be deducted 732,000 to pay the 732 death claims occurring during the year according to the mortality table. The balance, 34,654,084, is divided by 81,090, the survivors at the end of one year, to arrive at 427.35, the single-premium value of each policy. If the value of each policy at the end of the second year is desired, the fund at the beginning of that year, or 34,654,084, should be improved at interest

for one year, the second year's claims deducted, and the balance divided by the survivors at age 37, to arrive at the value.

On the net annual premium basis, each of the 81,822 persons would pay 21.08, yielding 1724807.76 at the beginning of the first year. Improved at interest at 3 per cent, this amounts to 1776551.99 at the end of the year. Claims amounting to 732,000 must then be deducted, leaving 1044551.99 to be divided by 81,090, the number of survivors, to yield 12.88, the terminal net value.

427.35 — 12.88 = 414.47, the amount by which the level-premium value is short of the single-premium value. This shortage is the present value of future premiums with which the company may be credited. Proof: Multiply 19.6608, the present value of an annuity-due of 1 at age 36, by 21.08, the net annual premium as of age 35, to produce 414.47. To find the net annual premium value of this policy at the end of the second year, take 1044551.99, the fund left at the end of the first year after paying the 732 claims, and add to it, 81090 × 21.08, or 1709377.20, the amount collected from survivors at the beginning of the second year, which yields 2753929.19, the total amount on hand at the beginning of the second year. This is improved at 3 per cent for one year, amounting to 2836547.0657 at the end of that year. 737,000 must then be deducted on account of the second year's claims, leaving 2099547.07 to be divided by 80,353, the survivors at the end of that year, to produce 26.13, the terminal net value. Continuing to the end of the mortality table, a table of policy-values might be formed. The same process may be followed for other types of contracts.

While the above method serves in presenting to the beginner the fundamental principles of retrospective valuation, a somewhat less irksome process consists in using one individual policy instead of the group. This, however, requires an understanding of the tabular net cost, or death strain, as explained in Chapter XI. Again referring to an ordinary life policy of 1000 issued at age 35 (Amer. Ex. and 3 per cent), the net annual premium value at the end of one year may be found as follows:

From,	21.08 × 1.03 =	21.7124,	amount end 1st year;
	deduct,	8.83,	tabular net cost 1st year;
	which yields,	12.88,	policy-value end 1st year.

From, $(12.88 + 21.08) \times 1.03 =$		34.98,	amount end 2nd year;
	deduct,	8.85,	tabular net cost 2nd year;
	which yields,	26.13,	policy-value end 2nd year.

The process might be continued to the end of the mortality table, showing the value at the end of each year.

A slight variation of this method is sometimes used. The tabular net cost may be discounted one year and deducted from the premium, the balance being then improved at interest.

Thus:	21.08,	the net annual premium;
less,	8.57,	discounted net cost $(8.83 \div 1.03)$;
yields,	12.51,	fund at beginning of year after allowing for cost of insurance;
times,	1.03,	
equals,	12.88,	terminal net value.

The variation offers no advantages.

The principal advantage of the retrospective method is that the same process may be followed to derive the value of all sorts of policies. The prospective method requires variations for each type of contract, and for some of the unusual types, it becomes quite complicated.

Non-continuous Retrospective Valuation.—The above process of valuation has been expressed by formula, but this is of no advantage since the labor of finding the value by formula is as great as that of finding it by arithmetical process. The objection to the process is that the value of a policy at the end of any year cannot be ascertained without first determining the value at the end of the preceding year. This means that if the value at the end of 10 years is desired, the value at the end of one year, 2 years, 3 years, etc., up to 10 must be found, the last only being desired. The method of arriving at the desired value directly without first finding the values at the end of each previous policy-year, is called non-continuous retrospective valuation.

This method is based on the fact that the accumulated value of

past premiums collected, less the accumulated value of past claims paid, equals the sum left in the company's possession. The calculation of the accumulated value of past premiums involves a discussion of what has been called a forborne temporary annuity-due. Supposing each of 81,822 persons aged 35 pays 1 into a fund at once, each of the 81,090 persons surviving to age 36 pays 1 into the fund a year hence, each of the 80,353 survivors at age 37 pays 1, 2 years hence, etc., what fund will have been accumulated at the end of 10 years? Obviously 81,822 should be improved at compound interest for 10 years, 81,090 for 9 years, 80,353 for 8 years, etc., the last payment of 74,985 being improved for one year only. The following shows the work:

$$\begin{array}{rcl}
81822 \times 1.3439\ 1638 &=& 109961.93 \\
81090 \times 1.3047\ 7318 &=& 105805.36 \\
80353 \times 1.2667\ 7008 &=& 101788.78 \\
79611 \times 1.2298\ 7387 &=& 97911.49 \\
78862 \times 1.1940\ 5230 &=& 94165.35 \\
78106 \times 1.1592\ 7407 &=& 90546.26 \\
77341 \times 1.1255\ 0881 &=& 87047.98 \\
76567 \times 1.0927\ 2700 &=& 83666.83 \\
75782 \times 1.0609\ 0000 &=& 80397.02 \\
74985 \times 1.0300\ 0000 &=& 77234.55 \\
\hline
\text{Total} && 928525.55
\end{array}$$

Now, if this fund is divided by 74,173, the number of survivors at age 45, the value to each one will be 12.5183. It may be noted that the value to each survivor is greater than the accumulation of an annuity-due and certain of 1 over the period, because some of the group who have made contributions will receive nothing back because they failed to survive the period. Thus 812 of them will have died during the last year, according to table, and these will have made all of the payments of 1. In other words the survivors receive the benefit of survivorship. The value to each survivor has been called the value of a forborne temporary annuity-due, because it is the value to a person aged 45 who was entitled to receive 1 each year from 35 to 44 inclusive, subject to survival, but who forbore to receive the payments, choosing instead to leave them with the company to accumulate at interest with benefit of survivorship; it being understood that he must survive the period to receive anything at all.

The above process, however is entirely too laborious; nor is it necessary. In finding the accumulation of the entire fund explained above, l_{35} was multiplied by $(1 + i)^{10}$, l_{36} by $(1 + i)^9$, etc., and the sum of the results taken. Using the symbols, this process may be expressed:

$$l_x(1 + i)^n + l_{x+1}(1 + i)^{n-1} + l_{x+2}(1 + i)^{n-2} \ldots + l_{x+n-1}(1 + i).$$

Since $(1 + i)^n = \frac{1}{v^n}$, the value of the fund may now be expressed:

$$\frac{l_x}{v^n} + \frac{l_{x+1}}{v^{n-1}} + \frac{l_{x+2}}{v^{n-2}} + \ldots + \frac{l_{x+n-1}}{v}.$$

Since the numerator and denominator of a fraction may both be multiplied by the same value without changing the value of the fraction, suppose $\frac{l_x}{v^n}$ be multiplied by $\frac{v^x}{v^x}$, $\frac{l_{x+1}}{v^{n-1}}$ by $\frac{v^{x+1}}{v^{x+1}}$, $\frac{l_{x+2}}{v^{n-2}}$ by $\frac{v^{x+2}}{v^{x+2}}$, etc. The above expression then becomes:

$$\frac{v^x l_x}{v^{x+n}} + \frac{v^{x+1}l_{x+1}}{v^{x+n}} + \frac{v^{x+2}l_{x+2}}{v^{x+n}} + \ldots + \frac{v^{x+n-1}l_{x+n-1}}{v^{x+n}}.$$

From Chapter V., this in turn,

$$= \frac{D_x + D_{x+1} + D_{x+2} + \ldots + D_{x+n-1}}{v^{x+n}}$$

$$= \frac{N_{x-1} - N_{x+n-1}}{v^{x+n}}.$$

If this is the value of the entire fund accumulated, it may be divided by l_{x+n}, the survivors, to get each one's share. The accumulated value of a forborne temporary annuity-due then becomes:

$$\frac{N_{x-1} - N_{x+n-1}}{v^{x+n}l_{x+n}} = \frac{N_{x-1} - N_{x+n-1}}{D_{x+n}}.$$

Now from the commutation columns the accumulated value at 45 of an annuity-due granted at 35, with benefit of survivorship, becomes:

$$\frac{N_{x-1} - N_{x+n-1}}{D_{x+n}} = \frac{N_{34} - N_{44}}{D_{45}}$$

$$= \frac{579160.7 - 333623}{19614.2}$$

$= 12.518364$, as found above by the more tedious process.

If 12.518364 is the accumulated value at age 45 of an annuity-due of 1 per annum from ages 35 to 44, inclusive, what will be the accumu-

lated value at age 45 of an annuity-due of .02108, the net annual premium at age 35? Obviously 12.518364 × .021081 = .2638996.

Having learned the accumulated value of past premiums collected on an ordinary life policy issued at age 35 at the end of 10 years, the next step in non-continuous retrospective valuation is to find the accumulated value at age 45 of the policy's share of claims paid during the past 10 years. The method of procedure is very nearly the same as in finding the accumulation of past premiums. For example, if an insurance of 1 is granted to each of 81,822 persons aged 35, 732 claims will occur during the first year, according to the table. By assumption these claims will fall due at the end of that year. If they are not paid, but are allowed to accumulate at 3 per cent, they will amount to $732 \times (1.03)^9$, by the time age 45 is reached. The next year's claims will accumulate at interest for 8 years, etc. Total accumulations may be found as follows:

$$
\begin{aligned}
732 \times 1.3047732 &= 955.0939\\
737 \times 1.2667701 &= 933.6095\\
742 \times 1.2298739 &= 912.5664\\
749 \times 1.1940523 &= 894.3451\\
756 \times 1.1592741 &= 876.4112\\
765 \times 1.1255088 &= 861.0142\\
774 \times 1.0927270 &= 845.7707\\
785 \times 1.0609000 &= 832.8065\\
797 \times 1.0300000 &= 820.9100\\
812 \times 1.0000000 &= 812.0000\\
\hline
\text{Total} \ldots\ldots\ldots\ldots &\ \ 8744.5278
\end{aligned}
$$

This sum divided by 74,173, the survivors at age 45, yields .1178936, each survivor's share of the accumulated death claims that have occurred in the group during the past 10 years. Thus, suppose each of 81,822 persons aged 35 agrees to contribute enough to pay 1 to the estate of each member who happens to die within 10 years. Now suppose that for some reason these persons fail to keep the agreement over the period, but at its end the survivors decide to pay all past claims that have occurred with interest at 3 per cent compounded annually to date. Each one's share will be .1178935, according to the table. This sum may be called the accumulated value of a forborne temporary insurance of 1.

Again, the process of finding the accumulated value of claims by

this method is too irksome. Resorting to the symbols, the total amount of accumulated claims may be expressed:

$$d_x(1+i)^{n-1} + d_{x+1}(1+i)^{n-2} + \ldots + d_{x+n-1}$$

$$= \frac{d_x}{v^{n-1}} + \frac{d_{x+1}}{v^{n-2}} + \ldots + d_{x+n-1}$$

$$= \frac{v^{x+1}d_x}{v^{x+n}} + \frac{v^{x+2}d_{x+1}}{v^{x+n}} + \ldots + \frac{v^{x+n}d_{x+n-1}}{v^{x+n}}$$

$$= \frac{v^{x+1}d_x + v^{x+2}d_{x+1} + \ldots + v^{x+n}d_{x+n-1}}{v^{x+n}}$$

$$= \frac{C_x + C_{x+1} + C_{x+2} + \ldots + C_{x+n-1}}{v^{x+n}}$$

$$= \frac{M_x - M_{x+n}}{v^{x+n}}.$$

This accumulation of the group's claims divided by l_{x+n}, the survivors, yields:

$$\frac{M_x - M_{x+n}}{v^{x+n}l_{x+n}} = \frac{M_x - M_{x+n}}{D_{x+n}},$$

each surviving policyholder's share.

The determination of each policy's share of past claims accumulated to age $x + n$ now becomes a simple matter of performing certain operations in subtraction and division of numbers obtained from the commutation columns. Thus when $x = 35$ and $n = 10$:

$$\frac{M_x - M_{x+n}}{D_{x+n}} = \frac{M_{35} - M_{45}}{D_{45}}$$

$$= \frac{12209.42 - 9897.032}{19614.2}$$

$$= .11789357, \text{ as before.}$$

Now, .2638996, the accumulation of past premiums,
less .1178935, the policy's share in the accumulation of past claims,

yields .1460061, the policy-value, which is \$146.01 to the nearest cent for a \$1000 policy issued at age 35, at the end of 10 years.

The above expressions may now be combined into one formula to express the terminal net value by the retrospective method under the full level-premium plan, as follows:

$$_nV_x = \frac{P_x(N_{x-1} - N_{x+n-1}) - (M_x - M_{x+n})}{D_{x+n}}.$$

Substituting the values when $x = 35$ and $n = 10$,

$$
{}_{10}V_{35} = \frac{.021081(579160.7 - 333623) - (12209.42 - 9897.032)}{19614.2}
$$
$$
= .14601, \text{ as before.}
$$

The above expression of the value of an insurance at the end of n years is especially valuable because it is applicable to all types of level-premium insurance policies, the only necessary change in it being to use the net annual premium for the particular policy in question instead of the net level premium for an ordinary life policy. Thus the value of a 20-year endowment insurance policy of 1000 issued, at age 35, at the end of 10 years is:

$$
{}_nV_{\overline{x\rceil}} = \frac{P_{\overline{x\rceil}}(N_{x-1} - N_{x+n-1}) - (M_x - M_{x+n})}{D_{x+n}},
$$

$$
{}_{10}V_{\overline{35\ 20\rceil}} = \frac{.041966(597160.7 - 333623) - (12209.42 - 9897.032)}{19614.2}
$$
$$
= .40745, \text{ or } 407.45 \text{ per 1000 of insurance.}
$$

For joint-life insurance the formula takes the form:

$$
{}_nV_{xy} = \frac{P_{xy}(N_{x-1\,:\,y-1} - N_{x+n-1\,:\,y+n-1}) - (M_{xy} - M_{x+n\,:\,y+n})}{D_{x+n\,:\,y+n}}.
$$

In a pure endowment no death claims are payable; hence $(M_x - M_{x+n})$ may be omitted, the terminal net annual premium value of a pure endowment being the net annual premium multiplied by the value of $\dfrac{N_{x-1} - N_{x+n-1}}{D_{x+n}}$. The same is true of deferred annuities.

The principle on which the non-continuous retrospective valuation formula is based may be extended to the valuation of return premium policies. Thus, accumulated premiums less accumulated claims, equal the terminal net value of the policy. From Chapter VII and what has just been explained above, it is evident that the accumulated claims at the end of n years on a temporary increasing insurance of 1 issued at age x, may be expressed by the fraction,

$$
\frac{C_x + 2C_{x+1} + 3C_{x+2} + \ldots + nC_{x+n}}{D_{x+n}}
$$
$$
= \frac{R_x - R_{x+n} - nM_{x+n}}{D_{x+n}}.
$$

Allowing π to represent the net annual level premium for a return premium endowment insurance, the value becomes:

$$\frac{\pi(N_{x-1} - N_{x+n-1}) - (M_x - M_{x+n}) - \pi(R_x - R_{x+n} - nM_{x+n})}{D_{x+n}}.$$

For the value of pure endowments with return of net annual premiums without interest in case of death before the end of the endowment period, this fraction becomes:

$$\frac{\pi(N_{x-1} - N_{x+n-1}) - \pi(R_x - R_{x+n} - nM_{x+n})}{D_{x+n}},$$

in which π is the net annual level premium for such a contract.

For the terminal net value of policies with return of gross annual premiums, the gross annual level premium must be substituted for the net premium in that part of the formula which expresses the accumulation of claims, the net premium being retained in that part which expresses the accumulation of premiums. Thus the expression of the terminal net value of an endowment insurance with return of gross annual premiums without interest is:

$$\frac{\pi(N_{x-1} - N_{x+n-1}) - (M_x - M_{x+n}) - \pi'(R_x - R_{x+n} - nM_{x+n})}{D_{x+n}},$$

in which $\pi =$ the net annual, and π' the gross annual level premium for the particular policy under consideration.

A general survey of the foregoing formulas will bring out the fact that usually the prospective method will be more readily applicable to policies of a simple character; whereas for the more complex contracts, such as those with return of premiums, the retrospective method will generally offer the greatest facility of valuation.

It may be remarked that the terminal net level-premium value of an ordinary life policy increases from age to age, whereas the present value of future premiums diminishes, until the policy-value equals the face amount of the policy at age 96. An ordinary life policy, therefore, is an endowment insurance maturing at age 96. The value of a pure endowment or of an endowment insurance increases until it equals the face amount at the end of the endowment period when it is said to "mature." At the usual insuring ages the value of a term insurance increases until about the middle, or a little past the middle, of the period, and then diminishes to 0 at the end of the period, when the company's liability ceases. The terminal net single premium value of an ordinary life policy is much greater

than that of a level-premium policy, at the outset and during the early years. It increases slowly until it reaches the amount of the policy at age 96. The terminal net value of a limited-payment life policy is greater at the end of the first year than that of a similar policy with premiums payable throughout life, and it increases rapidly until it reaches the single premium value at the end of the premium-paying period. After that it becomes the single premium value, which, as explained above, is the net single premium itself at each successive age. The level-premium value of a deferred annuity increases gradually until the end of the premium-paying period, if premiums are payable during deferment, and then gradually diminishes, following the single premium annuity-values down to the end of the table.

Effects of Different Mortality and Interest Rates on Policy-values.—The student is likely to reason fallaciously that a mortality table showing higher rates of mortality than another requires larger policy-values. Such is not necessarily the case, and for the valuation of policies on sub-standard lives, i.e., lives subject to extra mortality, an understanding of the principles involved is essential.

Suppose an ordinary life policy is issued to a person in that class of risks on which an extra rate of mortality is expected at first, the extra rate afterwards diminishing until it finally coincides with the normal for the remainder of life. The net annual premium for such a contract will be higher than that for a normal risk. Suppose at age 35 it is 29.67 per 1000 instead of 21.08, the normal premium. Suppose also, that the age of valuation, age $x + n$, coincides with the age when the extra mortality disappears and the risk is considered normal. At that age the single premium would be the normal single premium. The present value of an annuity-due of 1 at the age of valuation would be the value of an annuity-due by the normal table. The present-value of an annuity-due of 1, however, is multiplied by the higher premium, 29.67, instead of 21.08, to ascertain the present value of future premiums. The present value of future premiums would therefore be greater in this case than under a normal policy, and the policy-value, smaller. At any age later than $x + n$, the policy-value, would likewise be less until the end of the table is reached, when it would equal the face amount of the policy. At any age earlier than $x + n$ the sub-standard values would also be less than the normal, because as the life progresses from age to age, above age x, an increasingly greater proportion of the single premium and the annuity-due at valuation is based on the normal rate of mortality.

Considering the matter from the viewpoint of the retrospective method, though the premiums are greater, fewer of them will have been paid by a group of sub-standard risks up to the date of valuation, because more, proportionally, will have died out of it than out of a standard group, and the accumulation of the greater number of claims to be deducted will leave a lower policy-value.

Following the same line of reasoning, if the policyholder belongs in that class of risks in which the extra mortality is only slightly higher than the normal at first, but gradually increases so that there is a constantly increasing difference between the normal and the sub-standard rate, greater policy-values may be expected than those under the normal rate. Furthermore, if the sub-standard mortality is greater than the normal by a constant difference throughout, lower policy-values than normal may be expected, since the constant addition does not provide for an increase in mortality rates at the older ages sufficiently great to give equal policy-values.[1] From this statement, and taking the first two ways explained above, in which extra mortality may occur, it may be inferred that the mortality rate might be increased in such a manner as to result in no change in policy-values. Such is the case.[2] "Policy-values do not at all depend upon the absolute rate of mortality exhibited, but only upon the progression that the rate of mortality exhibits. It is the table in which the mortality increases the more rapidly, that requires the larger policy-values."[3]

The above discussion has centered about ordinary life policies. It would be wrong to assume that because one mortality table shows higher policy-values than another for ordinary life policies, the same relation will obtain for endowment insurance policies, since a different set of functions is involved. For a mathematical explanation see Robertson and Ross, "Actuarial Theory," page 354.

In practice, rated-up policies are often valued as though they were normal policies issued as of the increased ages. They may also be valued at the true age exactly as if they were normal policies, each year's extra premium being considered sufficient to meet that year's extra claims.

Different Interest Rates.—If a higher interest rate is used in premium calculations, the discounted value of each year's risk will be less. Hence single and annual premiums will also be less. By the retrospective

[1] See Robertson and Ross, Actuarial Theory, pp. 251-356.

[2] See Institute of Actuaries' Text Book, Second Ed., Part II, pp. 331-339.

[3] Dr. Thomas Bond Sprague, "How Does an Increased Mortality Affect Policy-values," in Journal of the Institute of Actuaries, Vol. XXI, p. 109.

method, premiums will accumulate at the higher rate of interest, but the premiums themselves are lower, whereas the claims accumulate at the higher rate of interest. The result will be, ordinarily, lower policy-values than would obtain with a lower interest basis. Thus suppose an ordinary-life insurance of 1 issued at age 35 on the basis of the American Experience table and $3\frac{1}{2}$ per cent. The net annual premium for such a contract is .019907. At the end of ten years the accumulation of premiums at $3\frac{1}{2}$ per cent is $\frac{P_x(N_{x-1} - N_{x+n-1})}{D_{x+n}}$. From commutation columns on the American Experience and $3\frac{1}{2}$ per cent basis this becomes $\frac{.019907(456871.2 - 253745.5)}{15773.6} = .2563539$, which is less than .2638996, the corresponding accumulation found above on the 3 per cent basis. The accumulation of claims is $\frac{M_x - M_{x+n}}{D_{x+n}}$, or

$$\frac{9094\,955 - 7192\,809}{15773.6} = .1205905,$$

which is greater than .1178935, the accumulation on the 3 per cent basis. Hence $.2563539 - .1205905 = .13576$, the policy-value, which was found to be .14601 on a 3 per cent basis.

One exception to the statement that higher interest yields lower policy-values is found at the lower ages where the mortality rate decreases from year to year as the child grows older. It is proved in the "Institute of Actuaries' Text Book," Chapter XVIII, Art. 69 and 70, that an increase in the interest rate has the same effect on policy-values as increasing the mortality rate by a certain constant throughout the table would have.

Paid-up Policies.—As explained above, the value of a policy may be considered as being held by the company, for the benefit of the policyholder. After a policy has been in force for some years it is customary to pay the terminal net value to the policyholder if he elects to cancel his contract. The amount payable on surrender is called the surrender value. Instead of taking cash, however, the policyholder may elect to continue his insurance, without further premiums, as a term policy, for such a period as the surrender value will provide. The determination of the length of time a given sum used as a net single premium will carry a policy is discussed in Chapter VI. If, however, the insured elects to continue his insurance on the same plan as the original, but with the face value reduced to such an amount as the surrender value used as a net single premium will purchase, a much

simpler problem is presented. Thus if ${}_nV_x$ is available to purchase a paid-up whole-life policy of W amount, it appears that $W = \frac{{}_nV_x}{A_{x+n}}$; since A_{x+n} would purchase a whole-life insurance of 1 at age $x + n$. For example, it was found above that a whole-life policy of 1000 on the basis of the American Experience table and 3 per cent, has a value of 146.01 at the end of 10 years. The net single premium at age 45 is 504.59. Hence the person who takes such a policy at age 35 may surrender it at age 45 for a paid-up policy of, $\frac{146.01}{504.59}$ of 1000, or 289.36. It is customary for companies to offer such optional settlements at net single-premium rates, so that the insured is given as much protection as his surrender value used as a net single premium will purchase.

CHAPTER X

Valuation and Expenses: Legal Requirements

True Retrospective Values.—Terminal net values calculated as explained in the preceding chapter are the ones used in practice as the basis of surrender values and policy loans. They also serve as the basis for determining mid-year values or "mean reserves"; from which, in turn, is computed the total liability of the company on account of policies outstanding. Terminal net values are computed on the basis of net premiums; that is, the net premium is progressed at interest and the cost of insurance deducted to obtain the policy-value. It is assumed, therefore, that the loading will be adequate to cover expenses when and as they are incurred. As explained in Chapter VIII, however, without salvages on mortality or expense estimates, and interest earnings in excess of the net rate assumed, companies could not meet the heavy initial expenses and at the same time remain in possession of the policy-value at the end of the first year. The customary practice of the older and larger companies in providing for initial expenses, as also pointed out in Chapter VIII, is to draw on surplus, the latter to be replenished from loading on future premiums; thus each policy does not from the outset pay its own way. Now if the company were permitted to draw upon the policy-value for estimated initial expenses, or rather if it were permitted to pay the first year's estimated expenses from the first year's gross premium and then report the remainder as the policy-value at the end of the first year, the result would be called the true retrospective value of the contract. After the first year, the premium used in valuation would be the regular net premium plus an annuity-due to make up for the first year's deficiency in the full premium policy-value. Thus if F represents the amount to be provided for first-year expenses, and f the equivalent annual amount to be added to the remaining premiums, true terminal net values may be calculated by the retrospective method by simply deducting F from the first premium and using the result as the premium for valuation purposes the first year. After that, the regular net premium plus f may be used in valuation. Let ${}^1P_x$ represent the first premium, ${}^2P_x$ the premium after the first, and P_x the regular net premium. Then for retrospective valuation, the first premium

becomes ${}^1P_x$, which equals ${}^2P_x - F$, and subsequent premiums become ${}^2P_x$, which equals $P_x + f$. For prospective valuation, $P_x + f$ may be substituted for P_x in the formulas given in Chapter IX, and the true values will be expressed. Or, from the terminal net value deduct the present value of an annuity-due of f from the age of valuation to the end of the premium-paying period. Thus for whole-life policies, the true value may be expressed as, ${}_nV_x - f(1 + a_{x+n})$.

Gross Valuation.—True valuation, discussed above, is a variation of gross valuation. As the term suggests, gross valuation credits the company with the present value, as at the age of valuation, of future gross premiums; i.e., the company is credited with the present value of all of the loading; whereas in true valuation the company is credited with only that part of the loading which recompenses it for initial expenses. Let P'_x = the gross premium, and ${}_nV'_x$, the terminal gross value. Then the formula for gross valuation by the prospective method becomes ${}_nV'_x = A_{x+n} - P'_x(1 + a_{x+n})$. Now, since P'_x is greater than P_x, the resulting value of ${}_nV'_x$ will be less than the value of ${}_nV_x$. It will be lower by the present value of the loading, since the loading is the difference between the net and gross premiums. Hence the terminal net value less the present value, as at the age of valuation, of future loadings equals the gross value, by the prospective method. This method credits the company with the present value of future gross premiums but debits it only with the present value of future claims; not with that of future expenses. The laws of our states do not permit companies to use this method of valuation in determining liabilities to be reported to the state insurance departments.

Gross valuation by the retrospective method will yield higher policy-values than net valuation, since the premium accumulating is greater, claims accumulating at the same time as under the net-premium method. Thus, allowing ${}_nV''_x$ to represent the gross value by the retrospective method the non-continuous retrospective formula becomes:

$$\frac{{}_nV''_x = P'_x(N_{x-1} - N_{x+n-1}) - (M_x - M_{x+n})}{D_{x+n}}.$$

It thus appears that the gross value by the retrospective method, is greater than the net, by the accumulation of the loading.

The Hypothetical or Reinsurance Method.—A variation of the gross-premium method of valuation, known as the hypothetical or reinsurance method, was in vogue at one time with life insurance companies abroad. It consists in forming a table of annuity-values from the schedule of gross or office premiums according to the mortality table used in premium calculation, but according to a hypothetical rate

of interest, regardless of the rate of interest used in determining net premiums. The hypothetical rate of interest was usually 3 per cent, which was lower than the rate ordinarily used in premium calculations at that time. Allowing a' to represent the hypothetical annuity-value corresponding to the gross premium P',

$$a' = \frac{1}{P' + d} - 1, \text{ and } {}_nV'_x = A'_{x+n} - P'_x(1 + a'_{x+n}).$$

The policy-value by this method might be greater or less than the net value.[1] The value thus derived was supposed to be the sum which the company issuing the policy should pay in order to reinsure the risk with another company charging the same gross premium and earning the hypothetical rate of interest on its funds. It has thus been called the "reinsurance method." The use of it for purposes of making returns to the state insurance departments is not permitted in this country.

Preliminary Term Valuation.—One of the simplest variations of net valuation to provide for initial expenses, is one which is known as preliminary term valuation, used in the United States to a limited extent. It consists in treating the first year's insurance as a one-year term policy, regardless of the form of contract, for valuation purposes. The policy-value at the end of the first year, therefore, is *nil*. The gross premium charged, however, is the regular gross premium for the type of contract issued. After providing, then, for first-year claims, the balance of the first gross premium may be used to pay expenses. The policy-values commence one year later. Since, however, the deficiency in the policy-value at the end of the first year, must be made up over the premium-paying period, for valuation purposes an annuity-due whose present value equals this deficiency is added to the net level premium. The full level premium value (Amer. Ex. and 3 per cent) of an ordinary-life policy of 1000, issued at 35, at the end of one year, is 12.88. This being the deficiency to be made up, an annuity-due of .655 must be added to the net annual premium of 21.081 for valuation purposes. But $.655 + 21.081 = 21.736$, the net annual premium at age 36. Hence the net level premium used in calculating preliminary term values is the net premium as at one year older than the age of the applicant when the policy is issued, and for a premium-paying period one year shorter than would be the case under full level-premium valuation. For an ordinary life policy, the premium used in the valuation formulas is P_{x+1}. The single premium used in the prospective valuation formula is A_{x+n}, the same as in full level-premium valuation. The effect of this

[1] See Institute of Actuaries' Text Book, 2nd Ed., Vol. II, pp. 327 et seq.

method is to provide larger loadings the first year, and smaller loadings thereafter, than are provided when the company retains the full level-premium value. Since P_{x+1} is greater than P_x and since the other functions remain the same, the policy-values will be less until the end of the premium-paying period. When preliminary term valuation is to be employed, the contract is drawn so as to show that this is the intention of the parties.

The prospective method only is employed in preliminary term valuation. The retrospective method would yield policy-values higher than the corresponding full level-premium values; because P_{x+1}, will accumulate to a greater sum in a given period than will P_x, and the other functions remain the same. By the prospective method the preliminary term value of an ordinary life policy at the end of n years may be expressed, ${}_nV'_x = A_{x+n} - P_{x+1}(1 + a_{x+n})$. If $n = 10$, and $x = 35$,

$$\begin{aligned} {}_{10}V'_{35} &= A_{45} - P_{36}(1 + a_{45}) \\ &= .504585 - (.0217366 \times 17.0093) \\ &= .504585 - .369724 \\ &= .13486, \text{ or } 134.86 \text{ per } 1000 \text{ of insurance.} \end{aligned}$$

It will be observed that the terms in the above expression are the same as if the full level-premium value of an ordinary life policy issued at age 36 were being ascertained at the end of 9 years. Hence the preliminary term value of an ordinary life insurance issued at 35 is the same as the full level-premium value of the same contract if issued one year later and having run one year less. When a table of full level-premium values for ordinary life insurance has been prepared, the preliminary term value of any ordinary life contract may be ascertained from it by taking the full level-premium value of a contract issued as of one year older and having run one year less. Look down one line and one column to the left for the preliminary term value. (The value may be indicated by ${}_{n-1}V_{x+1}$.)

Suppose it were desired to find the preliminary term value of a 20-payment life policy of 1000 issued at age 35, at the end of 10 years. Again the premium for valuation purposes is as of an age one year older than the age of issue, and for a term one year shorter, since the deficiency in the full level-premium value at the end of the first year must be made up over the remainder of the premium-paying period. The net annual premium for a 19-payment life insurance of 1 at age 36 is .031474. The net single premium at age 45, the age of valuation, is .504585, and the present value of a 10-year

annuity-due of 1 at age 45, which covers the remainder of the premium-paying period, is 8.334. Hence: .504585 − (.031474 × 8.334) = .24228, or 242.28 per 1000 as the preliminary term value.

It should be noted that preliminary term values of 20-payment life insurances are not ascertainable by inspection of a table of full level-premium values. A table of full premium values of 19-payment life insurances, however, will show the values. Thus the 242.28 in the above example appears as the full level-premium value, at the end of 9 years, of a 19-payment life policy of 1000, issued at age 36.

The preliminary term value of a 20-year endowment insurance policy of 1000, issued at age 35, at the end of 10 years, may be found by deducting from 757.24 (the net single premium for a 10-year endowment insurance of 1000 as of age 45) the product of 8.3339 (present value, 10-year annuity-due, 45) and 44.52 (the net annual premium for a 19-year endowment insurance of 1000 as of age 36). The result is 386.22, which is the full level-premium value of a 19-year endowment insurance of 1000 at the end of 9 years, issued at age 36 on the basis of the American Experience table of mortality and 3 per cent.

Modified Preliminary Term Valuation.—Several objections have been offered to preliminary term valuation, the most serious being, (1) that the entire premium-paying period is taken to make up the deficiency in the full premium value at the end of the first year, and (2) on limited-payment life and endowment policies the expense allowances are unnecessarily heavy during the first year. To meet the second objection, the system of modified preliminary term valuation was introduced. It consists in taking the ordinary life premium at an age one year older than the age of issue as the basis upon which the value is determined. In the case of an ordinary life policy issued at age 35, 21.74, the ordinary life premium at age 36, is considered the preliminary term-rate. The modified preliminary term value of an ordinary life policy at the end of one year is therefore 0, the same as in preliminary term valuation, since 21.74 at age 36 and thereafter is sufficient to provide the benefit. Since this same premium is used for valuation purposes under the preliminary term plan, the modified preliminary term values are the same as the preliminary term values; i.e., the same as the full level-premium values of an ordinary life policy issued at an age one year older and having run one year less.

When limited-payment life policies are valued on the modified preliminary term plan, a policy-value is established at the end of the first year. The net annual premium for an ordinary life policy at

an age one year older than that of issue is again taken as the basis, but certain additional calculations must be made. Let us consider a 20-payment life policy issued at age 35. The net annual premium of 21.74, the rate for ordinary life insurance at 36, is sufficient to pay the claims arising from ages 36 to 55, and in addition, to establish a policy-value of 318.81 at the end of the nineteenth year. In other words, it is sufficient to pay claims for 20 years and establish the preliminary term value of 318.81 at the end of that period. On a 20-payment life policy, however, the value at the end of 20 years is 609.92, the net single premium for a whole-life insurance of 1000 at age 55. The difference of 291.11 must be made up over the premium-paying period. For valuation purposes only, an annuity-due must be added to 21.74 which will accumulate to 291.11 over the premium-paying period. From the preceding chapter, the accumulation of an annuity-due of 1 is ascertained from the commutation columns by the formula,

$$\frac{N_{x-1} - N_{x+n-1}}{D_{x+n}}.$$

Hence:

$$\frac{N_{34} - N_{54}}{D_{55}}$$
$$= \frac{579160.7 - 170140.0}{12703.88}$$
$$= 32.196518.$$

Then to accumulate the deficiency of 291.11, an annuity-due of $\frac{291.11}{32.196518}$, or 9.0416, must be added to 21.74, which yields 30.78 as the premium to be used for valuation purposes. Thus the present value of a 19-year annuity-due of 1 at age 36, covering the remainder of the premium-paying period, is 13.579. This multiplied by 30.78 yields 417.96 as the present value of future premiums, which is deducted from 427.36, the net single premium for an ordinary life policy at age 36, to arrive at 9.40, the modified preliminary term value at the end of the first year.

At the end of 10 years, the modified preliminary term value is:

$$504.585 - (8.334 \times 30.7816) = 248.05.$$

Let π represent the annuity-due which will accumulate the difference between the preliminary term value and the single premium at the end of the premium-paying period. Then,

$$\pi = \frac{A_{x+t} - {}_{t-1}V_{x+1}}{\dfrac{N_{x-1} - N_{x+t-1}}{D_{x+t}}} = \frac{D_{x+t}(A_{x+t} - {}_{t-1}V_{x+1})}{N_{x-1} - N_{x+t-1}},$$

in which t represents the premium-paying period. Hence the policy value at the end of n years by this method becomes,

$$_{n-1}V_{x+1} + \frac{\pi(N_{x-1} - N_{x+n-1})}{D_{x+n}},$$

which has been shown in the arithmetical demonstration above to be equal to,

$$A_{x+n} - (P_{x+1} + \pi)(1 + a_{x+n\,:\,\overline{t-n|}}).$$

It will be noted that not all of the full premium value of an ordinary life policy at the end of the first year will be used for the expenses of the 20-payment life contract. On first thought one might suppose that the modified preliminary term value of a 20-payment life policy, issued at age 35, at the end of one year, was the difference between the full premium value and that of an ordinary life contract; that is, one might suppose that 22.00 — 12.88 = 9.12, was the modified preliminary term value. As observed in the above calculation, however, this is not the case. In order to understand the reason for this discrepancy, let us consider the 20-payment life contract as two contracts, one an ordinary life insurance and the other a 20-year pure endowment of the difference between the full premium value of the ordinary life contract and the net single premium at age 55. The difference in the net annual premiums of the two contracts, or 29.85 — 21.08 = 8.77, is the net annual premium for the pure endowment element; i.e., an annuity-due of 8.77 from age 36 will accumulate with benefit of survivorship to an amount equal to the difference in the policy-values of the two contracts at the end of 19 years. If, under the modified preliminary term plan, the insurance element is looked upon as an ordinary life contract valued on the straight preliminary term basis, all of the terminal value of 12.88 at the end of the first year could be spent for new business. Then the preliminary term values would not begin until the end of the second year, and they would not catch up with the full premium values until the end of the mortality table is reached. At age 55, the deficiency is the difference between the full premium value and the preliminary term value, or 327.58 — 318.81 = 8.77. (It is merely a coincidence that the deficiency in this case happens to be 8.77, the same as the difference in the net premiums of 20-payment life and ordinary life contracts at 35). In other words, the net annual premium for the pure endowment element will provide 282.34 at the end of 20 years, which with the full premium value of the ordinary

life insurance, or 327.58, would establish the single premium of 609.92, required at age 55. If no value, however, for the insurance element were set up the first year, i.e., if the insurance element were valued by the preliminary term method, there would be a deficiency of 8.77 in the amount required as the single premium at the end of the premium-paying period. For meeting this deficiency, then, a value is set up at the end of each year; it is the accumulation of an annuity due which, beginning at age 36, will accumulate with benefit of survivorship to 8.77 in 19 years. From formulas given above, the annuity-due may be calculated, and found to be .28. This is the amount, then, by which the value at the end of the first year would be deficient if the entire 12.88 were spent and only the difference between 22.00, and 12.88, or 9.12 were held as the policy-value. Hence, only 12.88 — .28, or 12.60, of the full premium value of an ordinary life policy at the end of the first year may be spent. The full premium value of the 20-payment life contract, 22.00, less 12.60, equals 9.40, the modified preliminary term value as found above.

The valuation of endowment insurances under the modified preliminary term plan involves no new principles. Again the ordinary life policy as of an age one year older is made the basis. In the case of a 20-year endowment insurance of 1000 issued at age 35, however, the value must be the face of the policy at the end of 20 years. The net annual premium of 21.74 will provide for the claims during the period and set up a value of 318.81 at age 55 under the preliminary term plan. This leaves 1000 — 318.81, or 681.19 to be provided. It was found above that an annuity-due of 1 from age 35 will accumulate with benefit of survivorship, to 32.196518 at age 55. To accumulate 681.19, therefore, an annuity-due of, $681.19 \div 32.196518 = 21.15725$, is required. This must be added to 21.74, the ordinary-life premium at age 36; which yields 42.90 as the premium to be used for valuation purposes. At the end of 10 years, then, the company may be credited with the present value of future premiums, or $42.90 \times 8.334 = 357.528$, which is deducted from 757.24, the net single premium of a 10-year endowment insurance of 1000 as of age 45, the age of valuation, to yield 399.71, the modified preliminary term value.

The Illinois Standard.—As a result of the conference of the Committee of Fifteen in Chicago in 1907, various further modifications of the modified preliminary term standard have been adopted by state legislatures as the minimum standard for determining policy-

values. The principal one of these has become generally known as "Modified Preliminary Term—Illinois Standard." The object of this method is to allow lower policy-values during the early years of endowment insurances, and of life contracts with premiums limited to periods less than 20 years, than would be required under the straight modified preliminary term standard, but to require higher values than those ascertained by the preliminary term method. Life, and limited-payment life policies with premiums for 20 years or longer, are valued on the straight preliminary term plan. Limited-payment life insurances with premiums for less than 20 years, and endowment insurances, are valued on the straight preliminary term plan only to the extent of that portion of the first year's premium which is equal to the premium for a 20-payment life policy issued as of the same age. Funds sufficient to provide for the full level-premium value of a 20-payment life policy at the end of the first year may be spent for new business, the policy-value being deficient to that extent at the end of the first year.

Companies that avail themselves of this method usually adopt 3½ per cent, the maximum legal rate of interest when the American Experience table is used, as the basis of calculation. Consider a 20-year endowment insurance of 1000 issued at age 35 on the basis of the American Experience table and 3½ per cent. Suppose its value according to the Illinois standard at the end of one year is desired. The full level-premium value of such a contract at the end of one year is 32.86. The corresponding value of a 20-payment life contract is 19.58. Since that part of the first premium necessary to provide 19.58 of the policy-value may be used to meet initial expenses, the policy-value at the end of the first year will be just this much less than the full premium value, or $32.86 - 19.58 = 13.28$, the value according to the Illinois standard.

This deficiency of 19.58 at the end of the first year must be made up over the remainder of the premium-paying period. Therefore, an annuity-due whose present value is 19.58 must be added to the net annual premium of the 20-year endowment as of age 35 to arrive at the premium to be used in valuation. The present value of a 19-year annuity-due of 1 at age 36 is 13.08. $19.58 \div 13.08 = 1.4969$, the 19-year annuity-due whose present value is 19.58. The net annual premium of the 20-year endowment at age 35 is 40.12. $40.12 + 1.497 = 41.617$, the premium for valuation.

To find the value at the end of one year by the prospective method, $41.617 \times 13.08 = 544.35$, the present value of future premiums, which

is deducted from 557.63, the net single premium for a 19-year endowment insurance of 1000 at age 36, to yield 13.28, the value as found before.

At the end of 10 years, the value according to the Illinois standard is: $723.72 - (41.617 \times 8.169) = 383.75$, in which 723.72 is the net single premium for a 10-year endowment at age 45, and 8.169 is the present value of an annuity-due at 45, according to the American Experience table and 3½ per cent.

Select and Ultimate Valuation.—Select and ultimate valuation consists in anticipating the mortality savings during the early years, due to medical selection, and in allowing the company to use the present value of these savings to meet the initial expenses of the contract. This is accomplished by substituting both the net single premium and the present value of the annuity-due according to the select table, and retaining the usual ultimate net annual premium, in the prospective formula. During the first 5 years, the single premium used in valuation is less than the corresponding ultimate premium. After the 5-year period is past, it is, of course, the usual ultimate premium. The present value of an annuity-due at any age during the first 5 years is greater than the corresponding annuity-due by the ultimate table. This value multiplied by the usual net annual premium on the ultimate table yields a greater sum, with which the company may be credited, than the usual present value of future premiums. This sum deducted from the lower single premium gives a lower policy-value. After the 5-year period, all of the functions are again the usual ultimate values, so that after the benefit of medical selection has disappeared, the full level premium values are required. Since the single premium and the annuity-due gradually approach the corresponding ultimate values as the policy progresses from age to age during the 5-year period affected by selection, the policy-value gradually approaches the full premium value by the ultimate table during this period. The advantage of this system is that sufficient funds are released to meet most initial expenses, the deficiency in the full premium value being made up over the period affected by selection, from savings due to a lower mortality rate than that provided for by the premium rate charged. The select and ultimate method was adopted as the legal minimum standard of valuation by the State of New York in 1907, and abolished in 1923. The select table formerly used in valuation is the one described in Chapter IV. The maximum rate of interest allowed was 3½ per cent. New York still retains the select and ultimate standard for limiting the expenses of new business.

Suppose it were desired to find the value of an ordinary life insurance of 1, issued at age 35, at the end of one year according to the American Experience table, select as explained, and $3\frac{1}{2}$ per cent. Designating the net single premium at the age of valuation according to this select table by the symbol $A_{[x]+n}$, the annuity-due at the same age and on the same basis by the symbol $a_{[x]+n}$, and the select and ultimate value by ${}_nV'_x$, the formula becomes: ${}_nV'_x = A_{[x]+n} - P_x a_{[x]+n}$. From commutation columns formed on the basis of the select table and $3\frac{1}{2}$ per cent, $A_{[35]+1} = \dfrac{8719.18}{23328} = .37377$, and $a_{[35]+1} = \dfrac{432005}{23328} = 18.519$. P_{35}, on the ultimate table and $3\frac{1}{2}$ per cent, is .019907. Hence, $.37377 - (.019907 \times 18.519) = .00512$, or \$5.12 per \$1000 of insurance. At the end of the fifth year, the value becomes ${}_5V_x$, and is found by the usual methods as described in Chapter IX. Thus, when n equals 5 or is greater than 5, the prospective formula is

$$ {}_nV_x = A_{x+n} - P_x(1 + a_{x+n}). $$

Legal Valuation Requirements

Enough has been said in the preceding pages to indicate that the legislatures of the various states have established minimum standards of valuation. The object of such legislation is to prevent unscrupulous or incapable persons from conducting a life insurance business on an unsound basis. Carefully managed companies have always made adequate provision for increasing mortality rates at the older ages by voluntarily charging adequate premiums and by preserving sufficient policy-values to make certain their ability to meet their obligations. Nevertheless, before the days of strict supervision, a new concern would spring up and offer insurance at rates lower than those charged by the better-managed companies, yet high enough to allow some policy-values to be set aside. These values, however, were considered "profits," or were expended in such a manner that those in charge of the enterprise would profit. Then as the mortality rates of its policyholders advanced, the company failed.

It is for the protection of the public against such practices that state legislatures have seen fit to prescribe minimum valuation requirements. To carry out the regulations most states have created state insurance departments which operate under the supervision of a state executive officer known as the insurance commissioner, or superintendent of insurance. Each company must render an annual report

setting forth its financial condition and giving much additional information concerning itself to this department. In practically all instances the reports show the companies' standing as of December 31. Among the liabilities included in a company's report to the state insurance department is a statement of the amount of the "reserve." The "reserve," or "reserve liability," is an approximation of the total amount of all policy-values which the company should hold in order that it may have funds which together with future premiums will enable it to meet its claims. Instead of the company's being debited with the present value of all future claims and credited with the present value of future premiums, it is charged merely with the total of policy-values, which, because of the manner in which policy-values are determined, accomplishes the same end.

Aggregate Valuation.—It is quite possible for a company to report the sum of all of its policy-values. This sum may be approximated, however, by separating the policies into groups and ascertaining the approximate value of each group. Thus suppose the company wishes to report full level-premium values on the basis of the American Experience table and 3 per cent. If it has on its books 1200 ordinary life contracts that were issued at age 35 and that have now been in force one year, $1200 \times 12.88 = 15456$, the value of this group. If it has 700 similar policies outstanding for 2 years, $700 \times 26.13 = 18291$, the value of this group. This process might be continued until all of the policies on the books have been accounted for. The sum of the aggregate values of the different groups constitutes an accurate approximation of the aggregate reserve liability of the company.

At this point it should be noted that no allowance for the discontinuance of policies should be made, since the company is at once relieved of equivalent resources and liabilities when a policyholder withdraws and receives the value of his policy, less certain deductions which are usually made in case of early withdrawals.

The routine work of ascertaining the aggregate reserve liability of a company has been much facilitated by the computation and tabulation of the values of all ordinary policies. If all of the policies issued are reported to the actuarial department of the company at the close of each business day, or at the end of each week, that department may readily set down the sum which must be reported as reserve liability against those policies on December 31, the close of the current calendar year. Likewise, renewals and terminations, together with the manner of termination, of old policies may be reported and the

proper adjustments made. The actuarial department may then report to the state insurance department from time to time, stating the number and amount of the various kinds of policies in force, and the liability which should be reported because of them on December 31. Keeping such reports quite closely up to date, especially towards the end of the calendar year, will enable the state insurance department, as well as the company, to state the total reserve liability of the company as of December 31, within a short time afterwards.

For many years after state regulation was established, companies were required by the valuation laws to report values on the full level premium basis. In the early days, expenses were spread quite evenly over the duration of the contract, initial expenses ranging from 5 to 10 per cent of the first premium. Full premium valuation, therefore, worked no particular hardship on new companies. About the time of the Civil War, however, expenses began to be crowded more and more into the first year. It was then that net valuation laws requiring full level-premium values became detrimental to companies that had no surplus funds. The laws were not always enforced strictly, but when they were, many small new companies were forced out of business. Preliminary term valuation and its various modifications, now afford greater opportunities to new companies.

Mean or Mid-year Values.—Another problem arises because policies are issued on all business days during the calendar year, whereas, the company's reserve liability is reported only as of December 31. Terminal values are suitable as the basis for surrender values and policy-loans, as explained in Chapter IX, but on December 31, some of the policies written during the year will have been in force but a few days, some a few months, and others nearly a full year. If the premium income of the company is fairly uniform throughout the year, it is safe to assume that policies written during the current year have been in force for an average duration of six months; policies written during the previous year, for one year and six months, etc. It is customary to make this assumption in practice, the resulting reserve liability of the company not being thus made to differ greatly from what it would be if the terminal values of each policy for the exact number of years and days in force were calculated. Policy-values resulting from calculations based on this assumption are called mean, or mid-year values.

The net annual premium value of a policy at the beginning of the first year is, of course, the net annual premium collected. The value then diminishes during the year as the policy's share of claims

are paid, since the amount of its share of claims is greater during the year than the interest earned on the net annual premium. It is demonstrated in Chapter IX that the net annual premium, increased at the fundamental rate of interest for one year and diminished by the tabular cost of insurance, yields the value at the end of the year. At the beginning of the second year, however, another premium is paid, thus increasing the value at once by the amount of the net annual premium. This is the initial value the second year. The initial value again diminishes in that claims exceed interest earnings, until the second year's terminal value is reached. The value at the beginning of the year is, therefore, higher than at its close. In practice the arithmetical mean of these two values is taken as the mean or mid-year value. To find the mid-year value of a policy written during the current calendar year, add the net annual premium to the terminal value and divide the result by 2. To find the mid-year value of policies in force more than one year but less than two years, add the terminal value at the end of the first year, the net annual premium, and the terminal value at the end of the second year, and divide the result by 2. The process may be continued to find the mid-year value of policies in force for any number of years. It appears, then, that mid-year values are higher than the corresponding terminal values.

It will be readily appreciated that the above-described method is not quite accurate, but experience has shown it to be a safe basis. It is the method employed in this country to determine a company's reserve liability.

The mid-year value of a single-premium policy is lower than the following terminal value. The interest earnings are, of course, greater than on annual-premium policies and, as explained in Chapter XI, the cost of insurance is less. For example, the single-premium mid-year value of an ordinary-life insurance of 1000 during the first year if issued at age 35, on the basis of the American Experience table and 3 per cent, is, $\frac{419.88 + 427.36}{2}$, or 423.62. The initial and terminal values are 419.88 and 427.36, the single premiums at ages 35 and 36, respectively. The annual-premium mid-year value, on the other hand, is,

$$\frac{21.08 + 12.88}{2} = 16.98.$$

Mid-year values, according to the preliminary term, modified preliminary term, or select and ultimate standard, may be determined in

the same manner. The modified preliminary term mid-year value of a 20-payment life policy, issued at age 35 on the basis of the American Experience table and 3 per cent during the second year, may be found by adding 9.40 (terminal value, first year), 32.59 (terminal value, second year) and 30.78 (premium used in valuation)· and dividing the result by 2, to arrive at 36.39, the mid-year value.

Other *ad interim* values might be determined, and values of policies whose premiums are paid semi-annually, quarterly, or m times a year might be found, but they are not a practical necessity in this country. As explained in Chapter VIII, the annual premium is considered due and payable at the beginning of the policy-year, and any unpaid installments are deducted from the amount paid by the company at settlement. Accordingly, policies paid for by semi-annual and quarterly premiums may be valued as if the premiums were payable annually in advance.

Valuation to meet legal requirements is thus performed on the mean or mid-year basis. Where full level-premium valuation is required, it is customary to prescribe minimum net premium values on the basis of the American Experience table of mortality and interest at 3½ per cent, for policies of recent issue. Many states permit companies to value their policies on this basis, or to report values calculated on the same mortality and interest basis, but according to the preliminary term, modified preliminary term, Illinois standard, or some other slight modification of the modified preliminary term standard. Many companies elect to report full level-premium values on the basis of the American Experience table and 3 per cent. Lower values than those resulting from calculations on the prospective method are not permitted. As previously pointed out, the retrospective and prospective methods yield the same result when the same net premiums are used. If a legislative act should fail to prescribe minimum net premiums, it may prescribe minimum valuation standards and the prospective method, which will make the company as safe as it is possible to make it by valuation requirements. Companies will then be compelled to charge sufficient premiums to provide funds for expenses, claims, and the valuation requirements. The states differ as regards valuation requirements, but a company complying with any of them should have little trouble in reinsuring its risks if they have been properly selected. If, after providing for current claims and expenses, a company is found not to possess assets equal to the legal reserve liability, the state insurance commissioner must see that it takes proper measures to remedy the situation. If

recovery is out of the question, the proper steps are taken to wind up the company's business, and reinsure its risks in another company. States vary as regards the specific procedure, but the purpose of all is to compel each company to maintain assets closely approximating the legal reserve liability, in addition to funds for other liabilities, so that in the event of financial difficulty some other company could be induced to take over the assets as the consideration for assuming the liabilities.

CHAPTER XI

Cost of Insurance or Death Strain: Dividends and Surrender Values

Actual Insurance or Amount at Risk.—From what is said in the chapters on valuation, it will be realized that an insured person not only assists in paying the claims of those who die, but also makes contributions toward the payment of his own claim. This is necessarily so since premiums must be collected to pay all claims. Those members of an insured group who are fortunate enough to survive to extreme old age, must contribute much to pay the claims of those who die earlier, and also enough to pay the greater part of their own claims. If the holder of an ordinary life policy should survive to age 96, his policy-value would be the face value of his policy. This policy-value is accumulated from his own premium payments. To appreciate the amount paid in by a person who takes an ordinary life policy of \$1000 at age 35 and carries it to age 96, suppose his average annual premium payment was \$20, and that he could have received 3 per cent compounded annually if he had deposited the \$20 in a savings bank each year. The savings bank account at the end of 61 years would amount to $20 \times 174.01339 = \$3480.2678$. If he makes the payment to an insurance company his policy-value will be \$1000, leaving a difference of \$2480.27 as his contribution toward the claims of others and the expenses of the company.

Should the person holding an ordinary life policy die soon after having entered into the contract, the company would have to pay nearly all of the claim from its mortality fund, which it has collected from other policyholders. Whenever the death occurs, the company's mortality fund must contribute the difference between the policy-value and the face amount of the claim. Since a claim is assumed to fall due only at the end of the policy-year, the difference between the face of the policy and its terminal net value is said to be the net amount which the company has at risk. This difference is the amount at risk over and above what the policyholder has contributed in the way of a policy-value towards the payment of his own claim. It has therefore been called the "actual insurance."

The actual insurance decreases as the policy-value increases. In ordinary life, limited-payment life, and endowment insurances, the actual insurance decreases steadily as the policy-value increases, until the amount at risk becomes 0 when the policy-value becomes the face value. Agents might profit by bringing this fact to the attention of prospects who have been heavily insured for some time, when trying to persuade them to take additional protection. In the case of temporary insurance the actual insurance or amount at risk decreases until the middle, or a little past the middle, of the term, and then increases until it again approaches the face of the policy as the contract draws to a close. When the amount of insurance is 1, $1 - {}_nV_x$, is the general expression for the actual insurance. When the term is one year only, ${}_1V_x$ is 0, and the face of the policy is considered the amount at risk. When a whole-life policy is paid for by a single premium, ${}_nV_x$ becomes A_{x+n}, and $1 - A_{x+n}$ = the actual insurance. When the face of the policy is a sum other than 1, it may be represented by S. The amount at risk, therefore, may be expressed by $S - {}_nR_x$, in which R represents the value of the particular policy under consideration.

This subject may be presented in another manner, more intelligible to the beginner. In the event of a death the entire face of the policy is paid by the company. This, however, relieves the company of liability for the policy-value by taking the policy off the books. If the policy-value amounts to a considerable proportion of the disbursement made by the company, the company's net resources are only very slightly reduced. In endowment insurances, if death occurs a short time before maturity the encroachment on the company's resources is very slight, since the company is merely obliged to pay the claim a little sooner than would otherwise have been necessary. The pressure on the company's resources, occasioned by a death, is the difference between the actual disbursement of the face of the policy and the liability for the policy-value from which it is relieved. The amount at risk is, therefore, $S - {}_nR_x$, as stated above.

Tabular Net Cost.—The tabular net cost of insurance for any year, or death strain as it is called by English actuaries, is the value of the amount at risk, or the expected contribution to mortality costs, that year. It is explained in Chapter III that the value of any risk is ascertained by multiplying the amount at risk by the probability of its occurrence. The cost of insurance for the first year, therefore, is $\frac{d_x}{l_x}(S - {}_1R_x)$. And for the nth year, $\frac{d_{x+n-1}}{l_{x+n-1}}(S - {}_nR_x)$. But $\frac{d_x}{l_x} = q_x$, and $\frac{d_{x+n-1}}{l_{x+n-1}} = q_{x+n-1}$. Allowing ${}_nK_x$ to represent the cost of a whole

life insurance of 1 at age $x + n$, the formula is:

$$_nK_x = q_{x+n-1}(1 - {_nV_x}).$$

Example.—Calculate the tabular net cost of insurance, or death strain, of an ordinary life insurance of 1, issued at age 35, during the first year, according to the American Experience table of mortality with interest at 3 per cent.

Solution.—In Chapter IX it was learned that the terminal net value of such a contract is .01288, at the end of the first year. From the mortality table, $q_x = .008946$, when $x = 35$. Hence:

$$\begin{aligned} _nK_x &= q_{x+n-1}(1 - {_nV_x}), \\ _1K_x &= q_x(1 - {_1V_x}), \\ _1K_{35} &= q_{35}(1 - {_1V_{35}}) \\ &= .008946(1 - .01288) \\ &= .00883, \text{ or } 8.83 \text{ per } 1000 \text{ of insurance.} \end{aligned}$$

Stated verbally, if an ordinary life policy of 1000 issued at age 35 should become payable at the end of one year because of the insured's death during the first year, the company will pay 1000 to the beneficiary. 12.88 of the 1000, however, has been contributed by the insured, over and above his share of the tabular mortality costs. Therefore, $1000 - 12.88 = 987.12$, the net amount by which the company's mortality funds would be reduced by the claim. Hence, 987.12 is exposed to a risk measured by the fraction .008946. The amount at risk multiplied by this fraction yields the value of the risk, which is the tabular net cost of insurance issued at age 35, during the first year for an ordinary life insurance of 1000.

At age 45 the value of the above policy has been determined as 146.01. Then, $1000 - 146.01 = 853.99$, the amount at risk during the tenth year. The probability that death will occur that year is expressed by a fraction whose numerator is the number dying at age 44, and whose denominator is the number living at that age. From the mortality table this fraction is, $\frac{812}{74985} = .010829$. Hence, $853.99 \times .010829 = 9.25$.

Or from the formula:

$$\begin{aligned} _nK_x &= q_{x+n-1}(S - {_nV_x}) \\ &= q_{35+10-1}(1 - {_{10}V_{35}}) \\ &= q_{44}(1 - .14601) \\ &= .010829 \times .85399 \\ &= .009247, \text{ or } 9.25 \text{ per } 1000, \text{ the cost of insurance during the tenth year.} \end{aligned}$$

In one-year term insurance the policy-value at the end of the year is 0, and the whole amount of the policy is considered at risk. The tabular cost of insurance, therefore, is the face amount multiplied by the probability of death during the year, so that for 1 of insurance the tabular net cost at age x is q_x.

In determining the tabular net cost of limited-payment life and endowment insurances, the same probabilities are involved, but the amount at risk is less than in ordinary life insurances since the policy-values are greater. This makes the tabular net cost of insurance on the higher-premium policies less. Thus the cost of insurance during the tenth year of a 20-payment life policy of 1000 issued at age 35, according to the American Experience table and 3 per cent:

$$(1000 - 255.78) \times .010828 = 8.06.$$

For a 20-year endowment insurance the cost during the tenth year on the same basis, is: $(1000 - 407.45) \times .010829 = 6.42$. To find the tabular net cost of insurance of any policy during any year, deduct the terminal net value of the policy at the end of that year from the face value of the contract, and multiply the result (which is the amount at risk) by the probability of death during that year. For 1 of insurance, the formula, ${}_nK_x = (1 - {}_nR_x)\dfrac{d_{x+n-1}}{l_{x+n-1}}$, is absolutely general for all insurances of a level amount. It may be made to cover increasing and decreasing insurances, also, by using $(wd)_x$ for the whole amount payable at death, the expression then becoming:

$${}_nK_x = [(wd)_x - {}_nR_x]\frac{d_{x+n-1}}{l_{x+n-1}}.$$

We may reckon the tabular net cost of insurance by multiplying the initial policy-value by $1 + i$ and deducting the terminal net value from the result. For example, the tabular cost during the first year of an ordinary life insurance of 1 issued at age 35, may be found as follows:

$$.021081 \times 1.03 = .02171$$

$$.02171 - .01288 = .00883, \text{ or } 8.83 \text{ per } 1000 \text{ of insurance.}$$

For the tenth year, the terminal net value at the end of the ninth year (.12965) plus the premium (.021081) yields an initial value of .15073 at the beginning of the tenth year; this increased at 3 per cent yields .15525 at the end of the tenth year; from which the terminal net value of .14601 at the end of the tenth year is deducted, leaving .00925 as the cost of insurance.

Furthermore, the cost of insurance may be considered in this manner: If 81,822 persons aged 35 were insured for 1000 each, 732,000 must be paid in claims at the end of one year, according to the table. Each of the 732 persons will have contributed a policy-value of 12.88 towards the payment of his own claim. $12.88 \times 732 = 9428.16$, which deducted from 732,000 leaves 722571.84 as the aggregate cost of insurance for the group of 81,822 persons. The cost per individual, therefore, is $722571.84 \div 81822 = 8.83$, as previously found.

The tabular net costs of insurance for all of the more common policies have been computed and tabulated in convenient form. Such tables are perhaps more readily available in a work entitled "The Principles and Practices of Life Insurance," originally prepared by Nathan Willey, with explanations, additions and revisions by Henry Moir, published by The Spectator Company of New York.

Actual Cost of Insurance.—Since the American Experience table of mortality displays a higher mortality rate than is ordinarily experienced in practice, it will usually be found that the actual cost of insurance is less than the tabular cost. The tabular net cost is the "expected" net cost, i.e., the cost which would be experienced if the actual mortality rate of the company happened to coincide with that shown by the table. There is a direct relation between losses and costs. It is possible, therefore, to compute the actual losses, at the end of any year, compare them with the expected losses, and thus arrive at the ratio of actual to expected cost. Allowing ${}_nK'$ to represent the actual cost and ${}_nK$ the expected or tabular cost, ${}_nK' : {}_nK ::$ Actual losses : expected losses. Using the figures in the last example given, and assuming that the company experiences only 700 deaths, instead of the 732 expected according to the table, the company would actually be obliged to pay out 700,000 in death claims during the year. The total of the policy-values by which the company's liabilities are reduced is,

$$700 \times 12.88 = 9016.$$

Hence, $700000 - 9016 = 690984$, the actual cost of insuring 81,822 persons. $690984 \div 81822 = 8.44$, the cost per individual. Referring again to the proportion:

$${}_nK' : {}_nK :: \text{actual losses} : \text{expected losses}$$

$${}_nK' : 8.83 :: 700000 : 732000$$

$${}_nK' = \frac{8.83 \times 700}{732} = 8.44, \text{ as before.}$$

When the same amount of insurance is granted on each life, the ratio of actual to expected deaths may be used instead of actual to expected losses.

The usual method of computing the actual cost of insurance is to set down the proportion:

$${}_nK' : {}_nK :: \text{losses} - \text{policy-values} : \text{aggregate } K.$$

Thus in the above example:

$${}_nK' : 8.83 :: 690984 : 722571.84.$$

Whence,

$${}_nK' = 8.44.$$

Surplus, Its Sources and Distribution.—Any sum a company may have on hand after providing for current expenses and actual death claims, and after setting aside a sufficient amount to meet its reserve liability, may be looked upon as surplus. Many companies allot and retain certain of their surplus funds for specific purposes such as to provide for mortality fluctuations, variations in the value of assets, etc. Quite a number of our companies do not set forth in their published statements all of their surplus funds under the name of "surplus." They prefer to set aside "special funds," rather than show too great a "surplus." This is doubtless a prudent practice, since the showing of a large surplus might cause a demand for a higher dividend schedule, which might weaken the company's ability to meet an emergency such as the epidemic of influenza in 1918.

Sources of the Surplus.—*Mortality Salvages.*—From what has been said on the actual cost of insurance, it will be realized that if at the end of any year a company has experienced a lower rate of mortality than that shown by the table on which premium calculations are based, certain mortality salvages will ordinarily inure to the benefit of the company. On the other hand, if the company has a number of policies of large amount on its books, and happens to experience a high rate of mortality amongst the holders of these contracts, its actual losses may be greater than the expected, although the rate of mortality experienced on all lives exposed may be less than that expected. If 32 fewer deaths occur than were expected, and if each of these was insured for 1000, the company has been relieved of paying 32,000 which it provided for in premium computations. The entire 32,000 cannot be considered as saved, however, since the company has not been relieved from liability for the policy-values of these contracts. These policy-values together with premiums payable in the future will be sufficient for the company to pay the claims when they arise. The difference between

the 32,000 and the sum of the policy-values may be looked upon as a contribution to the surplus funds of the company. The salvage from mortality is $32000 - (32 \times 12.88) = 31587.84$, if all were issued at 35. There are several ways of computing this mortality salvage. The percentage of salvage may be computed by ascertaining the aggregate expected cost of insurance and comparing it with net losses; that is, actual losses less policy-values. If the probability of death for the year at age $x + n$ has been determined from the company's actual experience and represented by q'_{x+n}, the mortality salvage may be computed as being equal to, $(q_{x+n-1} - q'_{x+n-1})(1 - {}_nV_x)$. For example, let $x = 35$, $n = 1$, and $q'_x = \frac{700}{81822}$. Then,

$$\left(\frac{732}{81822} - \frac{700}{81822}\right) \times (1 - .01288) = \frac{32 \times .98712}{81822} = .000386,$$

or .386 per 1000 of insurance. Now, of course, if 81,822 policies of 1000 each were in force, the saving is $.386 \times 81822 = 31587.84$, the result obtained by deducting policy-values of 412.16 from 32000.

Excess Interest.—As previously noted, a conservatively low rate of interest is assumed in premium and valuation computations. Should the company assume that it will earn 3 per cent net, with annual rests, and should it actually earn 5 per cent, the excess interest of 2 per cent over the assumed rate constitutes another source from which surplus funds may be accumulated. Investment expenses, and losses on unfortunate investments, should ordinarily be deducted from actual interest earnings in determining the net rate earned. It may be maintained, however, that unusually heavy losses might be considered as such and not deducted from actual interest earnings. If these losses can be met from surplus funds, they may simply be written off, and not deducted from interest earnings. The net rate earned on sound investments may then be reported, thus making a more favorable impression on the insuring public than if it were stated that no interest, or only a very low rate, was earned on investments as a whole. Again, if the prices of securities owned by the company should increase, it might be asserted that the increases should be considered as investment gains and added to total interest earnings. Since changes in the market prices of the company's securities, however, often signify nothing as to the revenue derived or the ultimate return of the principal invested, such changes should be ignored except when the securities are sold and the gain or loss actually realized, or when the price changes clearly indicate that a

loss of part or all of the principal or interest payments may be expected. When investment expenses and losses, as well as profits realized and accrued, have been determined, the total contributions to, or deductions from, the investment earnings on account of them, becomes: total investment profits realized and accrued — (investment expenses + investment losses). The result, added to the total interest earnings and rentals received and accrued, yields the total return on investments during the year. This sum, divided by the mean of the funds invested during the year *less one-half of the investment earnings,* yields approximately the average rate of interest earned.

EXAMPLE.—Assume 100 invested at 4 per cent net. One year hence it will amount to 104, the mean amount invested during the year being 102. Now 102 − 2, one-half of the interest earning, equals 100, which when divided into 4, yields 4 per cent.

$$\text{Or, average rate} = \frac{I}{\frac{A+B}{2} - \frac{1}{2}I} = \frac{2I}{A+B-I},$$

in which A and B represent funds invested at the beginning and the end of the year, respectively, and I is the interest earned.[1]

The actual rate having been determined, the excess over the assumed rate multiplied by the initial policy-value yields the amount which may be taken as the policy's contribution towards the surplus as a result of earnings in excess of the rate assumed in premium calculations.

Loading.—Surplus funds may also arise when the total expenses of the company, exclusive of investment expenses, are less than the total loadings collected. During the first few years a policy is on the books, no gains may be expected from this source, as pointed out in the discussion of loading. If the policyholder, however, continues to pay the premium for a number of years, the loading on his policy will be sufficient to reimburse the company for initial expenses, pay the current expenses, and in addition contribute something toward the company's surplus funds. In determining total actual expenses for comparison with total loadings, investment expenses may be omitted since they are met by, and deducted from, excess interest earnings.

Other Sources.—The aforementioned three sources, namely, mortality salvages, excess interest earnings, and excess loadings or low expenses, constitute the principal sources of surplus. Other sources

[1] Moir, Henry. Life Assurance Primer, p. 58.

may be considered, although they are of minor importance. Of these, gains from forfeitures are most commonly mentioned. These are made when the policyholder lapses or surrenders his contract during the early years of its existence, in which case a part or all of the policy-value is retained by the company. The amount contributed by such discontinuances may be found by deducting aggregate surrender values paid, from the total amount of the policy-values of discontinued contracts. These contributions by forfeited policies do not constitute a real gain, however, since, for the most part, such a gain is simply a replacement of the loss to surplus incurred in placing the business on the books in the first place. Furthermore, most companies are now so liberal in allowing surrender values to a high percentage of the policy-values, after the policy has been in force for a short time, that such gains would be small even if no peculiar expense elements were involved in forfeitures. Profits from annuities may constitute another source of surplus, but such gains are of minor significance in this country.

Distribution of the Surplus.—*Contingency Reserves.*—Once surplus funds have been accumulated, the question arises, what shall be done with them? In the allotment of surplus funds, as in the calculation of premiums and in other matters, the first and foremost consideration should be the safety of the company. A portion of the earnings should, therefore, be retained by the company and placed in special funds to meet unforeseen contingencies. The amount to be set aside for such contingencies depends on several considerations: the size of the company, the nature and location of its risks, clauses in its policies, covering hazards concerning which statistical data are insufficient for an accurate estimate of the value of the risks, legal restrictions, as well as general business conditions, the advisability of paying high dividends on participating business to meet competition, and other practical consideration. The determination of the amount of contingency funds to be retained by the company is largely a matter of judgment, based on whatever experience is available, and an intelligent, sensible view of the general situation.

The Contribution Plan: Mortality Salvages.—After having set aside surplus funds estimated to be sufficient to meet unknown contingencies, the remainder of the surplus may be distributed to those entitled to it. If the company is a stock company writing non-participating business only, divisible surplus may be apportioned at so much to each share of stock. On the contrary, if the business is participating, the determination of the dividend which each policy-

holder should receive, presents a very interesting and intricate problem. Only a very general discussion of it can be attempted here.

The plan of distribution generally followed is known as the contribution plan. The fundamental principle upon which this plan is based, is that each policy should be credited with that proportion of the divisible surplus which it has contributed. There are several variations in the practical application of this principle. Indeed it is impossible to determine exactly what each individual policy has contributed. Various groupings are necessary, therefore, in order to arrive at a result approximating equity. For instance, it may be found that one type of policy contributes a greater proportion of the mortality salvages than another. Even among policies of the same type, those on lives at one age may contribute more to mortality salvages than those at another. Among policies, too, of the same type and on lives at the same age, those of recent issue may contribute more than those in force for some time. Hence, some sort of classification as regards type, age of insured, and age of policy may be essential to an equitable apportionment of the surplus arising from mortality salvages. Once the percentage of salvage has been determined for a group of policies, to find the amount of surplus to which any particular policy is entitled, multiply the percentage of salvage by the tabular cost of insurance.

Loading Salvages.—As regards surplus funds contributed when loadings prove to be more than sufficient to meet expenses, it is again desirable in some instances to separate policies into different classes, since one type may be more expensive than another. It was pointed out in the discussion of loading, that if loading is determined in such a manner as not to result in an equitable distribution of expenses, a correction, at least in part, is possible in the dividend formula. Thus one company recently allowed an expense saving of 27.5 per cent of the loading with 4.35 per cent interest to go toward making up the dividends of life and endowment policies, but only 12 per cent of the loading on term policies. Another company distinguished between policies of recent issue and those which had been in force for some time. It took the mean between the gross annual premium and the net ordinary life annual premium for the same age as the basis and considered 13½ per cent of this mean as the expense charge for the first policy-year. The expense charge was then decreased 4 per cent of itself each year to 60 per cent for the 11th year, then by 3 per cent to 30 per cent the 21st year, 2 per cent to the 26th

year, by 1 per cent until it became 15 per cent of the initial amount for the 31st year and thereafter, with the provision that it should cease, however, should the policy become paid-up.

Excess Interest.—The policy's share of excess interest earnings may be taken as the difference between the average rate allowed in dividend calculations above the rate assumed in premium and valuation computations, multiplied by the initial policy-value, without regard to the type of policy, its age, or the age of the insured, since these elements are considered in determining the initial policy-value. Funds, other than the initial policy-value, belonging to the policy, such as dividends left to accumulate, should be allowed the full rate of interest actually earned by the company, less a small contribution to contingency funds, instead of merely the excess of the actual over the assumed rate. For example, if the initial policy-value is 167.09, the assumed rate 3 per cent, the actual rate allowed 5 per cent, accumulated dividends 60, the policy's share of the company's investment earnings, is:

$$167.09 \times .02 = 3.34$$
$$60. \quad \times .05 = 3.00$$

Total share, 6.34

Occasionally a student will ask at this point, "Why not .05 on the initial value of 167.09?" He should recall, however, that in premium calculations, the company has given .03 in the form of a lower premium, since the value of each risk was discounted at 3 per cent in premium calculations, and the company must earn and retain 3 per cent to maintain the policy-value on which the additional .02 per cent is reckoned.

Some companies use the mean policy-value, others the terminal net value at the end of the preceding policy-year, instead of the initial value, for determining the sum contributed by the policy through excess interest earnings.

When these three sources of the surplus are used in determining the dividends, the process is called the "three factor" method. The determination of the percentages of loading, of mortality salvages, and of excess interest earnings, is largely a matter of bookkeeping and statistics, judiciously applied. Once these percentages have been determined for use in any particular year, the individual surplus account may be made up as follows:

Mortality salvage ...	....per cent of cost of insurance	$. ...	$ ....
Loading salvage	per cent of loading	....	....
Excess interest	per cent of $\left\{\begin{array}{l}\text{initial,}\\ \text{mean, or}\\ \text{preceding}\\ \text{terminal}\end{array}\right\}$ value	....	....
	Total = dividend		

Example.—Suppose the mortality salvage is 25 per cent of the cost of insurance, the loading salvage 30 per cent of loading, and excess interest 1.5 per cent above an assumed rate of 3 per cent, computed on the initial policy-value. Assuming further that for a particular policy the net annual level premium is 18.80 per 1000, the gross premium 24.00, the initial policy-value at the beginning of the dividend year 41.00, and the tabular cost of insurance that year, 8.45, what is the dividend?

Solution.—

$.25 \times 8.45$	$= 2.1125$	mortality salvage
$.30 \times (24. - 18.80)$	$= 1.56$	loading salvage
$.015 \times 41$	$= \quad 615$	excess interest
	4 29	the dividend

If interest on the loading is allowed at the full rate, $.045 \times 5.20 = .23$, should be added to 4.29, which makes the dividend 4.52.

The same result may be derived by debiting and crediting the policy in the following manner:

Dr.	Cr.
Policy's share of expenses.	Terminal value end previous year.
Actual cost of insurance.	Premium paid.
Terminal value end current year.	Actual net interest earned on those two items.
Balance = Dividend.	

Example.—Using the previous illustration, the policy's share of expenses equals 70 per cent of the loading of 5.20; 75 per cent of 8.45 is the actual cost of insurance; $(41. \times 1.03) - 8.45 =$ terminal value at the end of the current year.

The initial value, 41, less net annual premium, 18.80, yields 22.20, the terminal value at the end of the previous year; $24.00 =$ premium paid; $.045 \times 46.20 = 2.079$, the net interest earned on both. The statement now becomes:

Dr.		Cr.	
Expenses	$3 64	Value end previous year	$22 20
Actual cost of insurance	6 34	Premium paid	24 00
Value end year	33 78	Actual interest on both	2 08
	$43.76		$48 28

Balance = $4 52, the dividend, as before.

It will be noticed that this method allows interest at the rate actually earned, net; that is, gross interest less investment expenses and a contribution to surplus, on the full amount of the loading for the year.

The three-factor method is not used in all instances. Some companies use the three-factor method for a part of their business and the so-called two-factor method for other parts. One company at a recent date used the three-factor method in making up dividends on life and endowment policies, and the two-factor method for term policies. Excess interest was omitted on term insurances, as it might well be since the policy-values are so slight. The percentage of mortality salvage was less than for life and endowment policies. In general, however, the two-factor method involves dividing the surplus into two principal parts; viz., (1) profit from surplus interest, and (2) all other profits. The first is distributed according to the initial policy-value, and the balance in proportion to the loading paid by the policyholder during the year. This method is more commonly used by British companies than by our own for general apportionment. Thus at a recent date a certain Canadian company determined dividends on annual dividend policies as follows: Ordinary life, 40 per cent of loading first year, 45 per cent second and third, and 72½ per cent thereafter; 20-payment life and endowments, 37½ per cent of loading first year, 47½ per cent second and thereafter. Interest profit was 1.2 per cent on the terminal value at the end of the preceding year. Policy-values and net premiums were calculated on the $O^{m(5)}$ 3½ per cent basis, the excess of the gross premium over this net premium being the loading. Since the percentage of the loading covers all profits other than excess interest, it will be higher than the percentage of loading salvage under the three-factor method.

Fully Paid Policies.—So far in this discussion of dividend distribution, only annual-dividend and annual-premium policies have been considered. Expressing the three-factor method as a formula, Mr. Sheppard Homans, who with Mr. David P. Fackler, was its author, used *B*, followed by symbols showing the kind of policy, to

indicate the surplus which is contributed by the policy and with which it should be credited the nth year, as follows:

$$_nB_x = ({}_{n-1}V + P'_x - \text{expenses})\,(1 + i') - \frac{d'_{x+n-1}}{l'_{x+n-1}}[(wd) - {}_nV_x] - {}_nV_x.$$

This formula is general, since P'_x disappears in the case of single-premium policies and policies paid up, and ${}_{n-1}V_x$ disappears when dealing with the first year of insurance. Nevertheless, in practice companies may depend on excess interest to carry the renewal expenses of single-premium contracts, consequently no loading salvages enter into dividend calculations, and not all of the excess interest can be included in the dividend. The dividend, then, consists of (1) interest on the policy-value at a rate somewhat lower than the excess rate on premium-paying policies, and (2) mortality salvages. Thus while one company allows 1.35 per cent of initial values as excess interest toward dividends on its annual-premium policies, it only allows 1.1 per cent of initial values of single premium and paid-up policies, since $\frac{1}{4}$ of 1 per cent is assessed for expenses. Mortality salvages may also differ from those of premium-paying policies, according to the mortality experienced in either group.

Deferred Dividends.—Various kinds of deferred dividend contracts have been issued in this country. At one time, policies were issued which provided that in case of death the face value would be paid, but in case of discontinuance the policyholder forfeited all claims to any portion of the surplus and the policy-value. Those who survived to the end of a designated period gained the surplus contributed by those who died and the policy-values and surplus forfeited by those who failed to keep up their premium payments. These contracts were known as tontine policies. They could not now be issued under the laws of most states. Another type of contract that once attained some degree of popularity was known as the "tontine-dividend" policy. Such a contract provides that in case of death within a designated period the face of the contract only will be paid. In case of discontinuance, a portion or all of the policy-value is allowed as a surrender value, but no share in surplus funds is allowed unless the insured survives a designated dividend period. In other words, surplus accumulations are forfeited by those who die or discontinue their contracts before the end of a dividend period, and these forfeitures augment the surplus to be divided among the survivors who maintain their policies in force for the full period. There are quite a number of such contracts in force, most of which are

now about to the end of a 20-year dividend period. Of these contracts the companies have had few discontinuances.

The principle may be applied to shorter periods. Thus a dividend distribution may be provided at the end of every five years, any share in surplus accumulated during any one of these periods, being forfeited by a policy in case of death or discontinuance prior to the end of the full five years. Tontine dividend policies have often been referred to as deferred dividend contracts. Especially is this true when distributions are made at short intervals.

Still another arrangement may be made. It may be provided that in case of death during any dividend period, the face of the policy, plus a proper share in surplus accumulations since the last dividend date, will be paid by the company. Such a distribution of surplus is known as a post-mortem dividend. No part of the policy's share of the surplus is forfeited because of death during a dividend period. If the policy, however, is discontinued during any dividend period, the surplus accumulated to its credit since the last dividend date is forfeited to the company and thus augments the surplus of those who keep their policies in force. This type of contract has been called a deferred-dividend policy. The expression, "deferred-dividend policy," however, has been applied to so many different plans whereby dividends are not paid annually but are postponed for a period, that it is necessary for the person contemplating the purchase of such a contract to examine carefully the policy provisions governing dividend distributions. Other expressions have been variously used to indicate some sort of dividend postponement, such as "accumulation," "distribution," "semi-tontine," and "progressive participation" policies. Most of these imply accumulations from surplus sacrificed by deaths and discontinuances. Much confusion in the minds of the public, and legislation in many states compelling annual distributions, has resulted.

The principal argument against tontine and tontine-dividend policies, as defined above, is that the tontine principle is contrary to the very nature of life insurance. In life insurance, the heirs of those who die are recompensed from the payments made by those who survive; whereas in tontines those who survive profit at the expense of those who die. As regards deferred dividends which are sacrificed only in case of surrender or lapse, and which are not forfeited upon death, the anti-tontine argument does not seem to be so convincing. In favor of such deferred dividends, it may be urged that they (1) tend to induce the policyholder to make serious efforts

to keep the policy in force, (2) work no injustice in case of death, (3) tend to prevent adverse selection in surrenders and lapses, (4) lessen the strain on the company's resources due to heavy lapses in time of financial panic or business depression, (5) lessen the temptation on the part of the company to pay such high dividend rates in order to meet competition as to weaken the company's financial strength, (6) may be determined with greater accuracy since more time is available for observing the company's experience, and since unusual years, such as 1918, do not greatly disturb the average. In favor of annual dividends as contrasted with any form of deferred dividends, it may be stated that they (1) are better understood by the public, (2) lessen the temptation to extravagance by the company since funds held by it are smaller and extravagance is more quickly detected by the insuring public, (3) avoid extravagant estimates of dividends by agents, at least in some measure, (4) do not deprive discontinuances of a share in the surplus, (5) assist the insured in meeting his premium payment each year. It may also be remarked that annual dividend policies are easier to sell. The tendency in recent years has been toward annual distributions, some companies declaring an "extra" dividend every five years.

When deferred dividends are sacrificed by the policy only upon discontinuance, the dividend to a surviving policy which would have been paid each year on an annual basis may be increased at the full rate of interest each year, and also by the policy's share of the gains resulting from the surplus sacrificed by lapsed and surrendered policies, to the end of the deferred-dividend period. In tontine dividends, the gains from surplus forfeitures on the part of policies terminated by death, together with interest on them, should also be added. In the case of tontine policies, now obsolete, a surviving policy's share of the gains from forfeitures of policy-values due to discontinuances, plus interest, should constitute still another addition to the dividend. In other words, deferred dividends may be treated as though they were annual dividends calculated in the usual manner and then set aside in a separate fund which progresses at interest actually earned, net, and by accretions from lapses and surrenders, or deaths.

Another factor that enters into the practical determination of dividends is the necessity of meeting competition. Successful competition involves pleasing the public. High dividends one year followed by low dividends the next, is not satisfactory to a majority of policyholders. Many think that in years when high dividends

might be paid, it is better to allow the extra surplus accumulated to remain with the company, so that the regular dividend schedule may be maintained in lean years, such as the year 1918. Then if the surplus becomes unreasonably great over a period, the company may increase the general dividend schedule, or increase dividends on policies of that type, or at those ages, which the records show to have been the heaviest contributors to the surplus, or declare an extra dividend. If practical matters outweigh other considerations, it may raise the dividend rate on those types and at those ages where competition is keenest.

The importance attached to maintaining dividends is illustrated by the fact that one company at a recent date set forth a formula for determining dividends with the proviso that if the resulting dividend should be less than that of the preceding year, the dividend as of that year should again be declared.

Dividend Options.—Once the dividend to which a policyholder is entitled has been determined, several options are offered as to the form in which he may receive it. In the first place, he may withdraw it in cash or use it to pay a part of the current premium. A large proportion of dividends are used in this manner. In the second place, he may use dividends to purchase non-forfeitable, paid-up additions; i.e., the dividend is used as a single premium to purchase additional insurance of the same kind as the original policy, the amount of insurance per unit of premium depending on the age attained by the insured. Most policies now provide for no loading on the additions; the dividend is used as a *net* single premium. No medical examination is required for the addition in most policies of recent issue, and quite a number of them provide that paid-up additions shall be participating. In the third place, dividends may be left with the company and allowed to increase at interest. The rate of interest allowed by the company depends on the average earned, although the rate allowed may be slightly lower than the average rate earned net by the company. When dividends left to accumulate, plus the policy-value, amount to the net single premium at the age then attained by the insured, the policy becomes fully paid up. Dividends left to accumulate may be withdrawn by the policyholder at any time, and after a policy has been in force for a considerable period, the full policy-value may be withdrawn in cash on surrender. Therefore, when the policy-value together with dividend accumulations shall have amounted to the face value of the contract, the policy may be surrendered for its full face value. Also, when dividends

are left to accumulate, the accumulation, together with the face of the policy, is paid if the death of the insured should occur.

Another arrangement may be made. It may be agreed that in the event of death, dividend accumulations will not be paid. Under this agreement, the dividends may be used as premiums for pure endowments, thus eventually maturing the policy as an endowment, or they may be used to hasten the maturity of an endowment-insurance policy. They may also be used to purchase deferred annuities, or the accumulation may be used to purchase an immediate annuity. These arrangements are not often made, however, the three standard dividend options found in nearly all policies being (1) cash, (2) paid-up additions, (3) left to accumulate. Whether the policyholder elects to receive his dividend in cash or to use it in one of the other ways provided, makes no difference so far as the mathematical value of the dividend is concerned. If he elects paid-up additions, he is given such additions as are equivalent to the value of his dividend in cash. So it is with the other options. In the case of poor health on the part of the policyholder, and if no medical examination is required for paid-up additions, the latter are of greater value. Since paid-up additions are purchased at net single premium rates, without loading, this option is often considered especially valuable, although the company may deduct the expenses of the additions from excess interest earnings. For practical purposes, however, the dividend may be deducted from the gross premium to determine the average annual payment which the policyholder is required to make in order to keep the contract in force.

In Great Britain dividends are called bonuses, and it is quite customary to apply the policy's share in the surplus to the purchase of paid-up additions without evidence of good health and at a uniform rate for each premium paid, or for each year in force. The amount added to the face of the policy is called a reversionary bonus. If the face is increased uniformly at all ages; e.g., 15 per each 1000 of insurance each year, it is called a uniform reversionary bonus. If distributions take place every 5 years, as is quite common, 75 would be added at the end of each 5-year period. This is somewhat similar to our dividend option to purchase paid-up additions, although our additions vary with the age, type of policy, etc. The compound reversionary bonus system involves an addition calculated not only on the face of the original policy, but on the total of previous bonuses as well. It is somewhat similar in effects to our option when the paid-up additions are participating.

Surrender Values and Policy Loans.—As explained previously, a part or all of the policy-value may be returned to the insured in case he wishes to discontinue his contract. The amount returnable is called the "surrender value." It is customary to allow no surrender value during the first two or three years of the policy's existence, because of the heavy initial expenses. After the policy has been in force for this probationary period, surrender values are allowed, but for a further period they are less, ordinarily, than the full policy-value. The practice varies with different companies as regards the time when they will allow the full policy-value as a surrender value. One company allows the full terminal net value on the basis of the American Experience table and 3 per cent after the third premium. Some companies do not allow the full terminal net value until the eleventh year and thereafter, others not until after fifteen years, and still others not until the twentieth year and thereafter.[2]

Surrender Charges.—The difference between the cash value allowed by the company and the terminal net value of the policy is commonly called the "surrender charge." Provisions for surrender charges vary greatly with the different companies. One retains the full terminal net value as a surrender charge until after the third premium has been paid, and makes no surrender charge if the policy is discontinued thereafter. Another retains the full policy-value until after the third premium, then charges $12 per $1000 of insurance as a surrender charge the third year, the surrender charge being decreased $1 per year on each $1000 of insurance up to and including the tenth policy-year. Another states that the surrender charge after the third premium shall not exceed 2½ per cent of the amount insured up to the fifteenth year. The general practice is to retain the full terminal net value as a surrender charge if the policy is lapsed during the first two years; to retain a considerable proportion of the terminal value, varying from about 25 per cent to 50 per cent, if the policy is surrendered the third year; and so on, the surrender charge gradually diminishing until it becomes *nil,* the full terminal net value being given after a considerable period, such as ten, fifteen, or twenty years.

Surrender values are granted because experience has demonstrated that after a policy has been in force for some time and has accumulated a considerable policy-value, it is not necessary, in order to do justice to itself and to other policyholders, for the company to retain all of the value on surrender. Because of this fact, laws have been enacted in many states compelling companies to give something as a

[2] See A. J. Flitcraft's "Compend" for the practice of any particular company.

surrender value after the contract has been in force for a short period, such as three years. These are known as "non-forfeiture" laws, and were first passed by Massachusetts in 1861 upon the instigation of Elizur Wright.[3]

Nevertheless, experience has also demonstrated that the number and amount of lapsed policies is of such magnitude as to constitute one of the serious problems with which life companies have to contend. In most instances, lapses and surrenders should be discouraged. In the case of insurance taken for the protection of dependents, it may be stated as a general theorem that neither the company nor the insured gains when a policy is surrendered. Exceptions may be noted in particular instances, but if called upon for an expression of opinion, one is forced to conclude after careful study that lapses and surrenders of family-protection policies should be discouraged by every legitimate means. A great proportion of the total amount of insurance and number of policies terminated during the first few years of existence, are terminated by lapse or surrender. To discourage early lapses as well as to protect the company and remaining policyholders, no surrender value should be allowed on annual-premium policies during the first two years. After that a surrender charge may be made for several reasons. The reasons that have been most often advanced are that the surrender charge (1) reimburses the company at least in part for initial expenses, thus constituting some protection to it and remaining policyholders from the contingency that the withdrawing policyholder may prove to be a source of expense; (2) it discourages surrenders and protects the company against heavy withdrawals in times of business depression; (3) it compensates in part for adverse selection, the healthy members who withdraw being penalized to make up for the higher mortality among the unhealthy ones who remain on the books.

Options on Surrender.—Once the surrender value has been determined, the policyholder may choose the form in which he will receive it, as follows: He may, (1) surrender his contract for cash, (2) accept paid-up extended term insurance, the full face value of the policy being continued as far into the future as the surrender value used as a net single term insurance premium as of the age attained at surrender will carry it; or again, he may (3) accept paid-up fractional insurance, whereby the face value is reduced to such an amount of paid-up life or endowment insurance as the surrender value used as a net single premium will purchase. ~~The calculation of the~~

[3] See the author's History of Life Insurance, pp. 147, 148.

term is explained in Chapter VI, and option ~~three~~ is considered in Chapter IX. Other options are sometimes given, such as using the surrender value to purchase a life annuity, temporary life annuity or annuity certain. The surrender value may also be used as the basis for automatic premium loans, ~~discussed below~~. In case the insured exercises no option, policies usually provide for the application of option number two, paid-up extended term insurance. In some states, companies are compelled by law to include a clause to this effect in the policy. Whatever option the insured exercises, he receives the mathematical equivalent of the cash value, although a person in very poor health might do well to exercise option two, paid-up extended term, while one in good health but poor financial circumstances might find it advantageous to use the automatic premium-loan privilege. Usually the latter option must be exercised prior to default in premium payment.

Policy Loans.—The earliest plan in this country whereby a company granted loans to its policyholders with the policy as security, was known as the part-note premium plan. It was introduced by the New England Mutual Life Insurance Company which began business in 1844. The plan consisted in allowing the policyholder to pay a part of his premium in the form of an interest-bearing note. In case of death, the face of the notes with interest was deducted from the amount payable on the claim. By the time of the Civil War, premium notes were used by many companies. The increase in commission rates to agents during the decade from 1860 to 1870, however, rendered premium notes undesirable. For instance, if a company should accept a note for one-half of the first premium and allow the agent the other half as a commission (ready cash, rather than notes, is necessary to hold an agency force), the position of the company would be untenable. It would really be granting insurance on credit. Hence the premium-note system was abandoned.

At a later date, it became evident that loans to policyholders of a portion of the policy-value might serve not only to meet a temporary financial need of the insured, but also to prevent surrenders. Hence there arose the practice of allowing loans sufficient to pay a current premium, when the loan did not amount to too great a proportion of the policy-value. Again, if the insured has the privilege of surrendering for cash, why not permit him to borrow this cash and allow the policy to remain in force? It was thought that a considerable number of policies might be kept in force in this manner which otherwise would be surrendered. A loan is much more advan-

tageous than a surrender to one who can no longer pass the medical examination. The loan mây be repaid, or the policy may be continued with the loan outstanding against it, in which case the loan plus interest on it will be deducted from the amount payable on the claim. Loans, however, are very likely to result in lapses. The usual procedure is that the insured borrows all he can, fails to repay the loan, and also fails to keep up the premium payments. The security is unquestionable since the company is lending funds for which it will be relieved of liability if the policy is lapsed without repayment of the loan. Despite the fact that the company takes no chances on the security behind its loans on policies, such loans constitute almost as serious a problem as that of surrenders and lapses, since in so many instances the loan is in reality the first step towards surrender.

For some years prior to the outbreak of the World War, writers on insurance topics were prone to decry the unfavorable situation regarding policy-loans. With the general prosperity that accompanied the War, however, the ratio of policy-loans to total assets materially decreased. While the ratio has again increased, the problem has not assumed the same prominence in current insurance literature. Nevertheless, the treatment of policy-loans requires constant attention on the part of insurance men and those interested in public welfare.

In a discussion of policy-loans it would seem advisable to separate insurance contracts into two classes. First, policies which are taken primarily for business purposes, such as those taken by contractors and others to enhance their credit and for other business reasons, should not be hampered by any attempt on the part of the company to reduce loans. Loans are probably less likely to result in surrenders, in such instances, except in the event of bankruptcy, and the loan privilege is often one of the principal reasons for taking insurance. On the other hand, in times of financial panic, companies are certainly justified in delaying the granting of loans for a period of 60 or 90 days. This is now provided in many loan and surrender-value clauses. To do otherwise would render the cash or loan value a demand liability, and such is not contemplated in the management of the company's investments. The company should be careful to avoid the necessity of borrowing at a high rate in order to lend to policyholders at the rate of 5 or 6 per cent stipulated in the policy contract. Companies have been compelled to do this on a few occasions in the past, but the experience seems not to have impressed all companies with the thought of this possibility as a really serious contingency,

since a few are again issuing policies without reserving the right to withhold loans or surrender values for a period of time.

The second class into which policies should be separated in this discussion, is made up of those which are taken primarily for the protection of dependents. When a policy protecting a wife, children, or other dependents is surrendered, the beneficiaries formerly protected by it are again exposed to the hazard of the breadwinner's premature death. The community as a whole is interested in this matter, and it seems reasonable that life insurance companies could render still greater social service by reducing loans on, and surrenders of, such contracts. Agents' renewal commissions, of course, tend to induce the agent to convince the policyholder of the inadvisability of this action if he consults the agent about surrendering, or negotiating a loan. The agent may point out other means of securing the money to meet the insured's needs. The insured can be impressed with the fact that he is borrowing a sum which his dependents may be called upon to repay after his death, at which time they can ill afford to do so. A comparatively small sum borrowed now will probably result in a lapse, which may mean a very great loss to dependents in a short time. If the insured insists on securing cash, let it be by means of a loan rather than by means of surrender. The agent should then follow up the loan, urging that the premium payments be made, and that the loan be repaid. The provision for an installment loan which is to be repaid by regular monthly, quarterly, or annual payments might be advisable. Certainly the loan ought to have a definite due-date, at which time the policyholder should be urged to repay it or renew his note. Some companies request the signature of the beneficiary on an application for a loan. This has doubtless acted as a deterrent to some policyholders; but when the policyholder has reserved the right to change beneficiaries, and when he knows his rights, this is, of course, ineffectual. It has been suggested that the loan be made repayable at the insured's bank, the idea being that his banker might assist in bringing pressure to bear, or that the insured would look upon it after the manner of a note due the bank, but this is of very doubtful value. The best solution of the problem is to institute a campaign of education concerning the disadvantages of the policy-loan to the insured. He may be shown in this manner how a loan and a surrender will defeat his purposes and may prove very expensive to him and his dependents. Perhaps agents should not emphasize the loan and surrender feature so much

in canvassing for family-protection policies. Such emphasis as is sometimes made tends to cause the policyholder to look upon the cash value of his insurance as a mere bank account which may be easily drawn upon to meet some temporary desire. Also, the agent who while canvassing sets forth the liberal loan privileges of his contract, and the advantages thereof, cannot very well attempt to dissuade a policyholder from later negotiating a loan by pointing out the disadvantages when the latter appears before him for this purpose. Certainly, full instructions to all members of the agency force explaining how to meet the arguments of the insured and how to convince him of the disadvantages of loans and surrenders, can do no harm.

In all matters concerning policy-loans, judgment and commonsense should prevail. Some circumstances require promptness and efficiency in granting loans. Automatic premium loans, that is, loans equal to any unpaid premiums, in case the surrender value is sufficient, are desirable. The insured may then repay the "automatic loans" and continue the policy; whereas, if the surrender value were automatically used to purchase paid-up extended term insurance, the insured might outlive the extended term and be uninsurable.

Sometimes old policyholders are urged to surrender for cash, invest the cash received, and then take new policies. This practice should be stamped out, since the policyholder seldom gains by it.

CHAPTER XII

Costs (*Continued*): Cost to Policyholder: Return on "Investment" Element

Net Cost to the Policyholder.—The determination of the net cost to the policyholder of an insurance contract involves several considerations which vary with the point of view assumed and the purposes of the investigation. If the purpose is to find the average annual payment made over a past period, the matter is very simple, once the period has been agreed upon, provided the insured is to continue his policy, or provided it is terminated by death at the end of the period. If it be assumed that the insured surrenders at the end of the period, other elements enter into the problem. If the purpose is merely to compare participating companies as regards net cost to policyholders on the basis of past records, the problem is less complicated than if the results are to be used to forecast future costs. If it is to compare costs under participating with those under non-participating contracts, another type of problem is presented. Then the period of time the contracts to be compared have run, or are to run, is of especial importance.

Average Annual Payments.—Under a non-participating contract, the payment which the policyholder must make each year is definitely stated in the contract. As previously noted, the premium stipulated in a participating policy merely indicates the maximum payment which the policyholder may be required to make. In practice, returns are made to the policyholder at the end of the year, in the form of dividends, which constitute a refund of that part of the premium which was not needed in paying expenses and mortality costs, in creating proper contingency reserves, and in meeting the reserve liability. These dividends may be applied to reduce the succeeding year's premium payment, or their cash equivalent used in other ways, as indicated in the preceding chapter. On annual-dividend policies, the policyholder is required to pay the stipulated gross premium the

first year (and sometimes the second year) but thereafter he need only pay the gross premium less the dividend. If he elects to use dividends in some other manner, or if deferred dividends only are allowed, the cash equivalent in terms of annual dividends may be deducted from the gross premium to determine the cost to the policyholder. Then a period of years may be assumed, and the question stated in some such terms as the following: Suppose a man took an ordinary life policy of $1000 at the age of 35, ten years ago, in Company X on the participating plan, and has maintained his policy in force. What has been the amount of his payment each year? This question may be answered as follows:

GROSS PREMIUM, $26.88

Gross Premium	Annual Dividend				
26 88	0	=	$26 88	first	premium payment
26 88	5 01	=	21 87	second	premium payment
26 88	5 22	=	21 66	third	premium payment
26 88	5 43	=	21 45	fourth	premium payment
26 88	5 66	=	21 22	fifth	premium payment
26 88	5 89	=	20 99	sixth	premium payment
26 88	6 13	=	20 75	seventh	premium payment
26 88	6.38	=	20 50	eighth	premium payment
26 88	6 84	=	20 04	ninth	premium payment
26 88	7.09	=	19 79	tenth	premium payment
Total..	....	=	$215.15		

Now if the person is to continue his insurance, the tenth year's dividend of, say, $7.34, should be deducted from the above total, leaving $207.81 as the total cost, or an average annual payment of $20.78. If death occurs at the end of this period and if the dividend for the year of death is not payable, the average annual cost would be $21.52. If death occurs at some other time, then the average annual cost till death may be found by taking a different number of years. If it be assumed that the insurance is to be continued, or that the dividend of the year of death, or of the year of surrender, will be allowed, the average annual payment may be found in the above example in this manner:

GROSS PREMIUM, $26.88

Year	Dividend at End of Year	Cost for Each Year
1	$5.01	$21 87
2	5 22	21 66
3	5 43	21 45
4	5 66	21 22
5	5 89	20 99
6	6 13	20 75
7	6 38	20 50
8	6 84	20 04
9	7 09	19 79
10	7.34	19 54
Total..	$60 99	$207 81

Average cost for 10 years, $20.78, the same average.

If such figures as the above are to be used as a basis for predicting probable future costs, several possibilities must be considered. In fact, is not customary for companies to make estimates of probable future cost; and in some states it is illegal for an agent or a company to publish such estimates. If the surplus accumulations of the company under consideration are high and have been steadily increasing over a period of years, the prospects for an increase in the dividend schedule are greater than if the surplus is low and has been declining in recent years. The latter may indicate a probable reduction of dividends in the near future. Again, the management of the company may become more efficient, more skill may be exercised in medical selection thus resulting in a lower rate of mortality, expenses may be cut, and investments more skillfully managed. On the contrary the reverse may take place, with respect to all of these particulars, thus making useless the most careful estimate. The better companies have given an exhibition of such regular and steady performance in the matter of dividends over such a long period of years, that the elements of uncertainty mentioned above may be greatly minimized, due regard being given to the trend of surplus accumulations.

The same methods may be employed for different ages, different periods of time, and different types of policies. Several insurance

services publish such figures as given above each year. They are available at the office of most life insurance agencies.

Cost when the Insured Surrenders.—When the insured person surrenders his policy for cash or its equivalent at the end of a definite period of, say, 10 years, what has been the average annual cost to him of the protection he has received? The usual answer in insurance services is found by deducting the cash surrender value from the total of annual premiums paid, less dividends, if any, and dividing the result by the number of years. In the previous example, if $207.81 be taken as the total of annual premiums paid, after dividends are deducted, the cash surrender value of $141.01 allowed by Company X at the end of the tenth year which the surrendering policyholder receives back from the company, leaves, 207.81 − 141.01 = $66.80, as the total cost, the average annual cost of the policy being taken as $6.68 over the 10-year period. For non-participating policies, the surrender value is deducted from 10 times the annual premium and the result divided by 10, or one-tenth of the surrender value is subtracted from the annual premium, which gives the same result.

This is a popular and easy method, and if the purpose is to compare costs in different companies it will serve quite well, for the assumptions made will be the same for each company considered. If an approximation of the true cost to the policyholder, however, is desired, several corrections must be made by taking into account the important factors left out by this method. For instance, the "protection" or insurance element is only the difference between the face value and the surrender value, since the latter may be looked upon as the "investment" element.

Students are sometimes puzzled on observing that the cost to a surrendering policyholder at the end of 10 years is stated at, say, $6.68 per annum for ordinary life, $4.39 for 20-payment life, and $1.24 for 20-year endowment insurance policies of $1000 face value. A partial explanation is that the cost of protection should be less under the high-premium contracts, because there is less of it. The other important factor is the ignored interest element, which is more important in the higher-premium policies.

The problem may be looked upon as one involving the cost of keeping the policy in force as distinct from the cost of protection. If such is the case, then the questions in order are: How much paid to the company? How much received back on surrender? Difference? Difference spread over the intervening period equals how much per

annum? If the answer to the last question is desired, it is not necessary to make adjustments between 3 and $3\frac{1}{2}$ per cent companies.

Other adjustments are necessary, however, to arrive at an approximation of the cost to the policyholder each year. For instance, the usual method as described above makes no allowance for the interest the policyholder might have received had he placed his premiums elsewhere than with the insurance company. Thus suppose a man takes a policy on which the non-participating rate is \$20.11 per annum per \$1000 of insurance, carries it for 10 years and surrenders for \$128.97. It is customary to consider \$201.10 as the total amount paid in to the company. But the man might have placed the \$20.11 in a savings bank each year at, say, $3\frac{1}{2}$ per cent compounded annually, \$20.11 per annum on this basis amounts to $20.11 \times 12.14199192 = \$244.18$, the sum contributed by him during the 10 years plus the interest which he might have received from the bank had he chosen to put his money there. Now if he surrenders for \$128.97, the difference between \$244.18 and \$128 97, or \$115.21, more truly represents the total cost to him, interest sacrifices on premiums paid in being considered. The \$115.21, however, cannot be spread over the period scientifically by dividing it by 10. It should be divided by the amount of 1 per annum, or 12.14199192, to arrive at the average annual cost. Hence the average annual cost would be $115.21 \div 12.142 = \$9.49$. This is quite different from the cost arrived at by the erroneous method in vogue to-day whereby the cost would be determined as, $\dfrac{201.10 - 128.97}{10} = \$7.213$. $9.49 - 7.21 = \$2.28$ per \$1000 per annum; the amount by which the erroneous method underestimates the cost to the insured, when interest is at $3\frac{1}{2}$ per cent and the other conditions are as stated.

The approximation of the true net cost to surrendering policyholders in participating insurance may be made by taking the amount of the average annual premium at the end of the period the policy has run. To attain greater accuracy, however, the amount of each annual payment (premium less dividend) might be found by increasing each payment at some conservative rate such as $3\frac{1}{2}$ per cent, to the end of the period the policy has run. The total of these amounts may then be considered the total cost to the policyholder, from which the surrender value may be deducted and the difference spread over the period as an annuity-certain in the manner just explained.

In all of the above discussions concerning cost to surrendering policyholders, only past performance has been considered. It was

assumed that the policyholder had lived for the period, kept up his premium payments, and then surrendered. Accuracy in any estimate of probable future costs over a period, demands the consideration of the probability that the policyholder will live to the end of whatever period is chosen. Of greater utility to a prospective purchaser of insurance is the statement that if the experience of the company is as favorable in the future as it has been the past few years, his average annual payment will be approximately $——.— over a period of 15 years; and if he surrenders, the cost of his policy less its surrender value will be about $——.— per year. The well-known services are accurate enough to fill in the first blank, the calculation of interest on premiums paid will correct the figures of the services for the second.

Participating vs. Non-Participating Costs.—One of the difficulties in comparing the cost of a participating with a non-participating policy is that the two contracts may not be alike in all particulars. This difficulty is not insurmountable, however, as the essential provisions of different contracts are quite similar and the costs of extra features in one of the policies may be estimated separately.

Assuming that two policies are sufficiently alike to make one as desirable as the other, the next point to be decided is the period of time over which the comparison is to be made. Since the participating premium is in all instances higher the first year, and in most cases, the participating gross premium less the dividend, is higher than the corresponding non-participating rate for several years, given a short period, the non-participating premium rate will be lower than the average of annual payments under the participating plan. If a long period is chosen, however, the average annual payments made under the participating contract will in many instances be less than the corresponding non-participating rate. So far as cost to the policyholder is concerned, the stereotyped arguments for the non-participating plan are low initial premium, cheaper protection during the early years, and a definitely fixed premium rate throughout the term of the contract, the lower cost during the early years counterbalancing the lower participating rates in the later years. For the participating plan: The high initial rate is soon reduced by dividends, so that after a few years the cost is less than the non-participating rate, the lower premium payments in the later years counterbalancing the high initial premium and giving cheaper protection, the average of annual payments over a considerable period being taken. The participating policy shows up to greater advantage when a consider-

able period is taken, whereas the non-participating plan is portrayed as more desirable from the cost viewpoint over a short period of years. One method of comparison consists in setting down the gross participating premium less the current annual dividend for several years in one column, and the non-participating premium each year in another, and then commenting on the number of years that pass before the participating cost becomes lower than the non-participating. This is generally less satisfactory than to compare the average annual payment under the participating contract with the non-participating rate.

Other methods of comparison may be brought out by this example: A graduating class of considerable size had decided to make such annual contributions to a fund for the benefit of its alma mater that at the end of 20 years following graduation, the amount of the fund would be $50,000. A certain reliable bank had offered to receive the contributions, improve them at interest at 4 per cent compounded with annual rests to the date marking the end of the 20-year period, and then turn over the accumulation to the trustees of the university. The class decided, instead, to take a 20-year endowment insurance policy of $10,000 on each of five of its members, whose insuring ages happened to be 21. Furthermore, it had narrowed its choice to two contracts, one participating and the other non-participating, and had decided that one of these would meet the needs of the class as well as the other, so far as policy provisions were concerned. Final decision as to which of these two policies would be chosen as the one for which the five men should make application was to depend solely on the estimated cost of the two contracts. The non-participating premium rate was $40.25 per $1000 of insurance. The participating company submitted the following statement:

.... MUTUAL LIFE INSURANCE COMPANY

. .. . AGENCY

Prospectus prepared for Chairman Insurance Endowment Committee, Class of 19 .

Age 21. Premium per $1000, $47.62 { with waiver of premium $47 82
with waiver of premium and annuity $48.23

Dividends payable annually and not contingent upon the payment of any future premium.

The dividends quoted below are on the basis of our present dividend scale which has been in force since 19

The . . . Company has never decreased its dividend scale a single penny in the entire history of the company (years). It was one of four companies that did not find it necessary to reduce during the war, "flu," or post-war periods.

The dividends herewith quoted while not guaranteed can be considered as a certainty in-so-far as past performance counts for anything.

1st dividend	$5 50	Net cost	$42.12	3% reserve company cash surrender and loan options after second premium.
2nd dividend	6 13		41 49	
3rd dividend	6 79		40 83	
4th dividend	7 47		40 15	
5th dividend	8 18		39 44	Average net cost for 20 years, $34.61.
6th dividend	8 92		38 70	
7th dividend	9 68		37 94	
8th dividend	10 47		37 15	Full reserve available after the third premium.
9th dividend	11 29		36 33	
10th dividend	12 14		35 48	
11th dividend	13 02		34 60	
12th dividend	13 94		33 68	
13th dividend	14 89		32.73	
14th dividend	15 87		31 75	
15th dividend	16 85		30.77	
16th dividend	17 83		29 79	
17th dividend	18 81		28 81	
18th dividend	19 79		27 83	
19th dividend	20 77		26 85	
20th dividend	21 75		25 87	
Totals.....	$260.09		$692 31	

The above figures are not estimated but are based upon policies actually on the books and receiving the above dividends this year.

Payment of a full year post mortem dividend in the event of death.

The specific question was framed in this manner: If there is no surrender or lapse in either case, and if the participating company's experience is such during the next twenty years as to enable it to maintain its present dividends on this policy, which is the cheaper of the two contracts as shown by comparing the present value of premiums payable according to the American Experience table with interest at 4 per cent?

The figures submitted by the participating company are quite striking in many ways. In the first place the premium payments of the participating contract are decreased by dividends so that after the third year they are actually less than the non-participating rate. In the second place, the cost the last year is so low as to favorably impress the layman. If the insured fails to survive the intervening years, however, he will not make that saving on the last premium. In other words the saving on the last year should be discounted for the probability involved as well as the 4 per cent rate which the bank offered. Thus it is observed that a scientific determination of the cost of these two contracts involves discounting each premium payment

both for the interest and for the probability involved. The American Men's Select table of mortality would probably yield more accurate results, but the American Experience table will serve sufficiently well. The present value of the payments under the non-participating policy may be found by multiplying the annual premium by the present value of a 20-year annuity-due of 1 according to the American Experience table and 4 per cent. The following shows the work:

The present value of a temporary annuity-due corresponding to the premium-paying period and calculated from commutation columns based on the American Experience table and 4 per cent, is:

$$\frac{N_{x-1} - N_{x+n-1}}{D_x} = \frac{78459.577 - 25123.735}{4033.496} = 13.2331406.$$

Hence, $13.233141 \times 40.25 = \$532.63$, the present value of premiums payable for $1000 of insurance under the non-participating contract.

If the participating policy is selected, the first premium payment will be for $47.62. If the insured should die during the year, the company will refund the dividend of $5.50 under the terms of the contract, in addition to paying the face amount of $1000. If he survives, however, and uses this dividend toward paying the next year's premium, the second premium payment will be, $47.62 - 5.50 = 42.12$. If he survives still another year, the third premium payment will be 41.49, etc., the last one being 26.85. The present value of these payments may then be found as follows:

$$47.62 = \text{present value, 1st premium payment;}$$

$$\frac{l_{22}}{l_{21}} \times 42.12 \div 1.04 = \text{present value, 2d premium payment;}$$

$$\frac{l_{23}}{l_{21}} \times 41.49 \div (1.04)^2 = \text{present value, 3d premium payment, etc.}$$

The sum of the 20 separate values is the present value of payments due.

It may be observed that the premium payments after the first are pure endowments whose present values may be found by using the commutation columns as explained in Chapter V. So the present value of the second payment becomes,

$$\frac{D_{22}}{D_{21}} \times 42.12 = \frac{3847.894}{4033.496} \times 42.12,$$

the commutation figures being based on the American Experience table with interest at 4 per cent. The present value of the third payment is

$$\frac{D_{23}}{D_{21}} \times 41.49 = \frac{3670.647}{4033.496} \times 41.49, \text{ etc.}$$

Since 4033.496 is the denominator in each of the 19 operations, the sum of the products of $42.12D_{x+1}$, $41.49D_{x+2}$, etc., may be first found, and the result divided by 4033.496 to arrive at the present value of all the payments.

The following shows the work:

$$
\begin{aligned}
3847.894 \times 42.12 &= 162073.29528\\
3670.647 \times 41.49 &= 152295.14403\\
3501.380 \times 40.83 &= 142961.34540\\
3339.739 \times 40.15 &= 134090.52085\\
3185.391 \times 39.44 &= 125631.82104\\
3037.974 \times 38.70 &= 117569.59380\\
2897.185 \times 37.94 &= 109919.19890\\
2762.733 \times 37.15 &= 102635.53095\\
2634.305 \times 36.33 &= 95704.30065\\
2511.641 \times 35.48 &= 89113.02268\\
2394.487 \times 34.60 &= 82849.25020\\
2282.574 \times 33.68 &= 76877.09232\\
2175.649 \times 32.73 &= 71208.99177\\
2073.496 \times 31.75 &= 65833.49800\\
1975.910 \times 30.77 &= 60798.75070\\
1882.645 \times 29.79 &= 56083.99455\\
1793.520 \times 28.81 &= 51671.31120\\
1708.314 \times 27.83 &= 47542.37862\\
1626.862 \times 26.85 &= 43681.24470
\end{aligned}
$$

Total.......... 1788540.28564

1788540.28564 ÷ 4033.496 = \$443.42186, the present value of the 19 payments after the first, to which the first payment of \$47.62 may be added, which yields \$491.04, the present value of premium payments under the participating contract.

If the policyholder should die during the last year, he would receive the twentieth dividend. In fact, each dividend may be looked upon as so much additional insurance in force for its particular dividend-year. Also, if the twentieth dividend were payable in case of survival so that the policy would mature for \$1021.75, the last dividend divided by 1.04 should be deducted from 26.85 in the work above. But since the contract matures for \$1000 only, the post-mortem dividends constitute additional insurances, the total of the net single premiums for which should be deducted from the present value of premiums as found above to compare with the present value

of payments under the non-participating contract. The amount to be deducted, however, is so slight that it is scarcely worth calculating. Even if the dividends were $20 each year, the post-mortem feature would be worth but $5.04 as a net single premium, on the American 4 per cent basis.

Since the present value of the premiums payable under the participating contract are so much less than the present value of those payable under the non-participating policy, a considerable margin may be allowed and still the answer to the question as stated would be that this participating contract is the cheaper. In other words, this participating policy is at least, 532.63 — 491.04 = $41.59 cheaper.

The question may be asked in this manner: If the insured lives to the maturity of the contract, how much will each contract have cost, with interest at 4 per cent?

For the non-participating contract:

$$40.25 \times 30.96920172 = \$1246.51$$

For the participating policy:

47.62 ×	2.1911231	=	$104.341282
42.12 ×	2.1068492	=	88.740488
41.49 ×	2.0258165	=	84.051126
40.83 ×	1.9479005	=	79.532777
40.15 ×	1.8729813	=	75.200199
39.44 ×	1.8009435	=	71.029212
38.70 ×	1.7316765	=	67.015881
37.94 ×	1.6650735	=	63.172889
37.15 ×	1.6010322	=	59.478346
36.33 ×	1.5394541	=	55.928367
35.48 ×	1.4802443	=	52.519068
34 60 ×	1.4233118	=	49.246796
33.68 ×	1.3685691	=	46.093407
32.73 ×	1.3159318	=	43.070448
31.75 ×	1.2653190	=	40.173878
30.77 ×	1.2166529	=	37.436410
29.79 ×	1.1698586	=	34.850088
28.81 ×	1.124864	=	32.407332
27.83 ×	1.0816	=	30.100928
26.85 ×	1.04	=	27.924000
		Total.......	$1142.312922

The above example may be extended to show the importance of the term of years over which the costs are to be compared. In the previous problem it was assumed that no probability of lapse need be considered. It has often been stated (but seldom demonstrated) that policies remain in force approximately ten years, on the average. Assume that *A* took the participating contract, *B* the non-participating policy, and that both died at the end of the tenth year. Which received his protection at the lower average annual cost?

On account of the post-mortem dividend, the sum of the first ten payments may be taken as set forth in the company's statement, and divided by 10, to yield $38.96 compared to $40.25, for the non-participating contract, and $34.61 for the participating policy on the twenty-year basis.

Again, in case of surrender at the end of ten years, the determination of the respective costs constitutes another problem, the solution of which is indicated in the preceding pages.

The fact should be emphasized that the above illustration, taken from experience, should be viewed merely as an isolated illustration of the factors involved in determining net cost to the policyholder and in comparing the cost of one policy with that of another. In most instances the average annual payment is a fair basis for comparison. To show the importance of viewing the above example as an isolated illustration and not one from which any definite conclusions may be drawn concerning the relative cost of participating and non-participating policies, it may be remarked that the same participating contract by the same company, but at age 25, showed, according to the previous year's dividend schedule, an average annual payment over a 15-year period of $41.56, as compared with $40.71, the non-participating gross premium. There were few participating policies presented to the chairman of the insurance committee, whose cost was less than that of the non-participating contract, the average taken over a period of 15 or fewer years. In comparing two companies as regards cost to the policyholder, it will be found that on one type of policy at one age the first company may be cheaper, whereas on another type and at a different age the second may be less expensive even though the policies are essentially the same so far as liberality is concerned.

Determination of the Rate of Return on "Investment" Policies. —The inducement to purchase life insurance contracts being so often the facilities they afford for convenient and systematic saving, and the so-called "investment" element being so often emphasized, an

examination of this feature may be of interest to the prospective purchaser as well as to persons seeking knowledge of the various phases of the life insurance business. Life companies have not stressed any particular claims to being investment institutions, their chief service consisting in furnishing protection. Incidentally, however, they may be considered as our greatest savings institutions. They not only make savings possible by furnishing protection against the contingency of saving periods being cut short by premature death, but also supply the facilities for saving. The chief competitors of life companies in this line are savings banks and building-and-loan associations. The relative merits of these institutions may be shown more clearly if the essential features of a satisfactory method of saving are kept in mind. Before adopting any such method one should take into consideration the amount to be saved, the time in which it may be saved, the safety of the savings fund, and the rate of interest on the savings deposit. The plan should provide for regular, periodic payments sufficiently large to create the fund within the desired time. The higher the rate of interest, the shorter the period which will be required to create a given sum. It is also essential that some sort of compulsion be exercised upon the individual in order to stimulate the will power to save, and that some obstacle be put in the way of his withdrawing from the plan before the object has been accomplished. Compared with life insurance companies, savings banks are subject to the following objections: (1) In this form of saving there is likely to be no definite relation between the deposits and the amount desired at a given future time; (2) there is no penalty if deposits are not made on time; (3) the entire savings fund may be easily withdrawn; (4) the rate of return is in many instances not so great; (5) death may cut short the savings period; (6) savings banks, in most instances, are not so carefully regulated and supervised as are the life companies.

Building-and-loan associations usually yield a higher rate of return than savings banks, and provide a penalty for failure to make the specified deposits on time, and for withdrawal before the series matures. They, too, are not always carefully supervised, however, and they are unable to spread their investments over a wide field. It seems reasonable to conclude, therefore, that even if no returns were available on premium payments made to life companies, the protective and savings features would still be sufficient to warrant the statement that life companies are among our greatest social service institutions as well as our safest financial corporations.

Not only do life companies supply an excellent method of saving, but their policies may also be taken as a means of making investments. In an investment one is interested primarily in safety and rate of return. The savings and investment feature in insurance consists in and is brought about by regular periodic payments sufficient to meet claims and expenses and create necessary policy-values. The policy-values give rise to this feature. For example, it has been shown that the policy-value of a 20-year endowment insurance becomes the face value at the end of the twentieth year. Now suppose a person whose income is sufficient to do so resolves to save \$20,000 in 20 years. Even if his income remains sufficient, three things may happen to prevent him from accomplishing the desired result if he does not avail himself of insurance. First, he may die before the end of the period. Second, his will power to save may fail. Third, he may lose a part or all of his savings by way of speculation or unfortunate investments. If he takes a 20-year endowment insurance of \$20,000, death during the savings period cannot defeat his plans; the agent's efforts and inducements of various kinds may stimulate him to keep up the premium payments; and the safety of his funds in the possession of his insurance company can rarely be duplicated. In case of his death, then \$20,000 will be immediately available, and if he survives the period the \$20,000 is more likely to be available by means of insurance than by any other method of saving this amount.

As to the probability of loss from the failure of any well-established company, the safety of the investment element of life insurance policies is practically unquestionable. Since the insured is certain to receive \$20,000 whether he lives or dies within the period, many are inclined to view such a contract as one on which the insured cannot lose. Nevertheless, it will be realized that it is not the desire of the company that the holder of such a policy should die soon after it has been issued. It prefers that the insured survive to the end of the period and pay up the contract. The loss to the company, and consequently the gain to the insured's estate, will be greatest if the death of the policyholder takes place soon after issue. In the case of a 20-year endowment insurance, the loss to the company will be less, and the gain to the insured's estate will be less, in the event of death soon after insuring than it would be in the case of a term insurance policy covering the same period. (This would not be so if the insured were old enough for the term to extend to the end of the mortality table. The premium for the term would then be the same as for the endowment, but in practice policies are not

granted at extreme ages.) Though in the estimation of many, the endowment is very profitable to the estate of the insured if the latter dies within the period, it is not so profitable as a term policy, with the difference in the premiums of the two placed in a savings bank. Viewed in this light, an endowment insurance is not a profitable "investment" unless the insured survives to the end of the period. The danger of loss is not from the possibility of failure on the part of the company, but from the possibility that the insured may fail to survive the period, in which case his estate will receive no greater benefit than it would have from a lower-premium term contract.

Many opinions to the effect that the insured gains on an endowment insurance whether he lives or dies have come to the writer's notice. Probably most individuals do so gain, because without the endowment they would not have saved the premiums or could not have invested the small premium payments successfully. This has to do, however, with insurance as a means of saving, rather than a safe and profitable investment. On further reflection the beginner realizes that the company gains if the insured survives and pays the premiums and that the insured's estate gains by his early death, but confusion results when it is shown that the insured gains on survival. A little clear thinking, however, is sufficient to bring about a realization of the fact that in an endowment insurance the insured will win on the insurance element and lose on the pure endowment feature if he dies within the period, whereas he will win on the pure endowment element and lose on the insurance if he survives. Taking all factors into consideration, costs and benefits are mathematically balanced.

At this point it is necessary to analyze the so-called "investment" element. Using the 20-year endowment insurance as an illustration, it is customary to refer to the surrender value as the "investment" element. The surrender value depends on the terminal net value, which at the end of any policy-year is made up of the value of the term insurance plus the value of the pure endowment. At the end of the period the value of the term insurance is 0, and the value of the pure endowment is the face of the policy. Hence the "investment" is said to be slight at the beginning but increases steadily over the period, whereas the term insurance is great at the beginning and decreases steadily. At any time during the period the insurance is said to be the difference between the surrender value and the face amount of the contract. A demonstration occasionally appears in life insurance literature, of the fact that the annual premium for a

decreasing insurance plus the annual deposit necessary to create the investment is equal to the annual premium for a corresponding endowment insurance.

Since an endowment insurance may be conceived of as a term insurance plus a pure endowment, the benefit payable at maturity may be looked upon as being made up of two parts, a speculative element and an investment element. For example, suppose a man has \$647.69, which is the net single premium for a 10-year pure endowment at a certain age. If he places this in a savings bank at 3 per cent it will amount to \$870.43 in 10 years. If he purchases a pure endowment with it and survives the 10 years, he will receive \$1000. The gain of \$129.57 is sometimes referred to as a "speculative gain," to distinguish the benefit of survivorship from the investment element proper. Since any gain from the so-called investment element in any insurance policy is in part a "speculative gain" insofar as the benefit of survivorship is obtained by the insured, the rate of return on the so-called investment policies should not be considered as a rate of interest.

For practical purposes, however, the sum to which the insured is entitled on surviving any period, or in other words the surrender value, may be considered as his investment, and the difference between this sum and the face of the policy as the amount of his protection, since he may withdraw the investment at any time by surrendering the policy.

Types of Investment Policies.—So far in this discussion, attention has been directed primarily to endowment insurances. Term insurance policies may be excluded from the list of "investment" policies. It is true that the "investment" feature is not entirely lacking in term policies, but it is negligible. In an ordinary life insurance policy, the "investment" feature is more prominent, especially after the policy has been in force for some years. In long-term endowment policies the "investment" element is greater than in ordinary life policies, but in the former it may perhaps be looked upon more as a savings feature or as a means of protection against the hazard of dependent old age. In limited-payment life insurance, when the premium-paying period is short, and in short-term endowment and double endowment policies, the "investment" element is greatest.

Approximation of the Rate of Return.—A question that often arises in practice may be stated as follows: "If I choose an endowment policy at my age, and live to the end of the endowment period, what rate of return will I have received on my 'investment'?"

Most important in determining the rate of return is the elimination of the protection, or the insurance element. Term policies, since they embody such a small "investment" element, and their cost may thus be considered the cost of protection only, may be used to distinguish the insurance from the "investment" part of a policy. The entire process may be illustrated by a specific example. Let us suppose the problem to be a definite one. *A*, a prospective purchaser of life insurance, asks the agent of company *X* what the rate of return will be if he selects a 10-year endowment policy at his present age, say 35, and lives to the end of the period, or to age 45. Let us suppose the gross premium for the endowment is \$92.37 per annum, non-participating. Now, it is at once evident that a large part of the premium goes to make up the "investment" part of the contract. How much of it goes toward the "investment" feature? The agent consults his rate book and finds that \$10.30 is the gross annual premium for a 10-year term policy. Since \$10.30 is sufficient to provide protection against death over the period, or in other words to provide for the insurance feature, the difference between \$10.30 and \$92.37, or \$82.07, may be considered the part of the premium that goes to the "investment" feature. At the end of 10 years the endowment policy provides for the payment of \$1000 to the insured, whereas the term policy provides nothing. The problem thus becomes one in elementary arithmetic; namely, if a man pays \$82.07 per annum in advance for 10 years, and receives back \$1000 at the end of the tenth year, what is the rate of return on his investment if interest be compounded annually?

Solution.—If \$82.07 per year amounts to \$1000 at the end of 10 years, \$1 per year amounts to $\frac{1000}{82.07}$, or, \$12.18 at the end of 10 years.

Consulting compound interest table III it is found by glancing down the first column to the end of 10 years and then out to the right, that \$12.14 most nearly corresponds to the \$12.18 found above. And, since \$12.14 is the figure found in the column headed $3\frac{1}{2}$ per cent, that is the approximate rate received by the person who survives the period.

If the man fails to survive the period, the extra premiums are sacrificed to the company, since \$10.30 per annum is sufficient for the term policy which would pay \$1000 in case of death within the 10 years. To make this clearer, suppose the insured had chosen the term policy at \$10.30 and had placed the \$82.07 in a savings bank each year at $3\frac{1}{2}$ per cent interest, instead of paying \$92.37 for the

endowment policy. If he should die the day before his policy expires, his estate would receive $1000 from the insurance company, and in addition, the day after his death, his savings account would amount to $1000 also.

The various steps in the process may now be summarized in the following manner:

NON-PARTICIPATING POLICIES, $1000, Age 35

Item	10-Year Term	10-Year Endowment
Gross annual premium	$10.30	$92 37
Amount by which the endowment exceeds the term premium	...	82 07
Cash value at the end of 10 years	0	1000 00
Amount by which the cash value of the endowment exceeds that of the term	...	1000 00

$1000 \div 82.07 = 12.18$, the amount (approximately) of $1 per annum at the end of 10 years at 3½ per cent, as determined by referring to a compound interest table.

Other Examples.—Suppose the problem is changed to a question more frequently asked, but a little more complicated; namely, if a man aged 35 selects a 20-year endowment with a certain company instead of an ordinary life policy, how much extra will he have saved at the end of 20 years, and what has been the rate on the additional savings or "investment"? The only additional fact which must be ascertained to solve this problem is the cash value of the ordinary life policy at the end of 20 years. This is readily found by consulting the ordinary rate book which an agent usually carries, or by looking it up in any one of numerous vest-pocket editions containing information concerning life insurance companies.

SOLUTION, NON-PARTICIPATING POLICIES, $1000, AGE 35

Item	Ordinary Life	20-Year Endowment
Gross annual premium	$23.20	$43.55
Excess of endowment over ordinary life premium	...	20 35
Cash value at end of 20 years	327.58	1000.00
Investment difference	...	672.42

By paying an extra premium of \$20.35 over the ordinary life premium, the insured accumulates \$672.42 more to his credit in 20 years under the endowment policy. To find the rate of return on the extra premiums,

$$672.42 \div 20.35 = \$33.04$$

Again looking down the first column, table III, to 20 years and out to the right, 33.04 is found to be a little more than half way between 30.96, the figure in the 4 per cent column, and 34.72, the figure in the 5 per cent column. Therefore the return is a little more than 4½ per cent, compounded annually, and this will be sufficiently accurate for most purposes.

A third type of problem is even more likely to arise than the two just considered. Suppose a man aged 35 says, "I am thinking of taking a 20-year endowment policy with a participating company, instead of an ordinary life policy. If I choose the former, what rate of interest will I receive on the extra premiums I will pay for the endowment if I survive the period"? This problem differs from the one just preceding in that the annual dividends must be estimated and deducted from the premiums of both the endowment and the ordinary life policy.

SOLUTION, PARTICIPATING POLICIES, \$1000, AGE 35

Item	Ordinary Life	20-Year Endowment
Gross annual premium	\$27 00	\$50 00
Estimated annual dividend	6 65	10 56
Average annual cost to insured	20 35	39 44
Annual cost of endowment in excess of ordinary life	. .	19 09
Cash value, end 20 years	327 58	1000 00
Investment difference	. . . .	672 42

As before, $672.42 \div 19.09 = 35.20$, which, from the table, indicates that the rate is about 5⅛ per cent compounded annually.

Rule.—We may now give a somewhat rough but practical rule for determining the rate of return on "investment" policies: Compare the "investment" policy with one covering a similar period, which involves the "investment" element to the least extent, as regards premiums and cash values. Divide the difference in cash values at the end of the period by the difference in annual costs to the policy-

holder, and from the result determine the rate by consulting a compound interest table.

It must be emphasized that the rate of return depends on the types of policies and companies compared; as well as upon the length of time the policies are to run to maturity, and the age of the insured. The above examples are to be taken as *examples only.* They are not given to indicate any tendency as regards types of policies or companies. After determining the type of policy he desires, each individual must calculate the rate of interest in his own case for each company he is considering, in order to ascertain which company will produce the highest rate of return on the "investment" element in his policy. The rate of interest assumed by the company in determining net premiums and reserves is of no practical use in determining the rate which the policyholder will receive on his "investment." It is important to know this, because, when some are questioned concerning the rate of return, they may reply "The company guarantees 3 per cent, or 3½ per cent." This merely has to do with the rate the *company* is reasonably assured of earning on *its* investments, as previously explained. It is true, however, that the participating company which earns a high rate of interest on its investments is likely to yield the policyholder a high rate on the investment element in his policy, although the high rate on the company's investments may be overbalanced by other factors, such as the age of the person and the length of time his policy has to run, the specific application of the method used for distributing the surplus, etc. Also, it will be readily observed that there are several minor errors involved in this method. These, however, are of little practical significance. The lowest rate the writer ever found was 1¼ per cent, and the highest happened to be 7⅛ per cent on a 27-year endowment compared with ordinary life. The prospective purchaser in the latter instance was 38 years of age and in a quandary as to which policy to take.

The advantage of compound over simple interest may be illustrated by the fact that a sum invested at 5 per cent compounded annually for 15 years amounts to more than if 7 per cent simple interest were earned on it each year of this period.

CHAPTER XIII

Insurance on Sub-Standard Lives

Though as early as 1824 the attempt was made in England to issue insurance on sub-standard lives, it was not until the period of the Civil War that this was done in the United States. One of the original purposes of the founders of the Universal Life Insurance Company of New York, formed in 1865, was to insure persons rejected by other companies because of physical impairments. It was thought possible to cover the increased mortality on the impaired group by charging advanced rates, and to secure an agency force without difficulty, agents being anxious to make connections with a company in which they could place applicants rejected by their own companies. This company, however, soon abandoned its sub-standard business. Others appeared from time to time with announcements to the effect that they would accept impaired risks at increased rates, but this type of business met with no considerable degree of success until it was taken up by the New York Life Insurance Company in 1896. Even as late as 1905, Mr. Emory McClintock testified before the Armstrong Committee that in his judgment such a business was at that time of doubtful feasibility.[1] Mr. McClintock pointed out that rates on sub-standard lives were largely a matter of judgment, and that judgment rates are not likely to be high enough when competition is keen.

The first attempt to develop a scientific basis for sub-standard business was in the Specialized Mortality Investigation, completed in 1903. With the completion of the Medico-Actuarial Investigation in 1915, which dealt in a comprehensive manner with various impairments, many companies began to grant insurance on lives that could not meet the regular medical requirements.[2] Since then the business has grown to substantial proportions, and a majority of the more prominent companies are now writing sub-standard business in some form.

[1] Testimony, Legislative Insurance Investigating Committee, Vol. III, pp. 2259-2261.

[2] It should be remarked in passing that Mr. McClintock was a prime mover in the early investigations of the rate of mortality among sub-standard lives.

Definition.—*Factors Affecting the Risk.*—A sub-standard risk is one on which a policy will be issued, but not on the same basis as those issued on standard lives. Some of the provisions contained in standard policies will be omitted, a higher premium rate will be charged, or a form other than that applied for will be issued, because of defects revealed by the medical examination. Quite frequently, an additional application blank must be filled out and made a part of the policy contract. Standard risks are those which are normal in all respects, or those on which the regular provisions of the policy applied for will be issued at regular rates. Super-standard risks are those above the standard in respect to prospects of longevity. These are usually considered standard and given no special consideration. One company, however, favors this class by a distinction in its disability clause, and another by a special policy on which higher dividends are anticipated than are paid on its regular contracts. There are, of course, risks that will not be accepted by any company. A border-line risk is one that is doubtful. It has been defined as one that is clearly sub-standard according to the practice of the companies exercising the most rigid medical selection, and yet one that would be accepted as standard by the company with the lowest medical requirement. This is a very unsatisfactory definition, since judgment plays a part in determining whether or not a risk will be accepted by a particular company. The most rigid company occasionally accepts a poor risk. Also, it is evident that within certain practical limits a company may secure a relatively low mortality by adhering to high standards in medical selection and inspection; that if all companies did so, this definition of border-line risks would have a very limited application.

The Principal Impairments.—Many investigations into the effects of various impairments have been made. The extra mortality likely to result in a large group suffering from a specific impairment may be learned with a fair degree of accuracy. When two or more impairments are found in one risk, however, the estimation of the extra mortality expected is quite a difficult matter. Several considerations enter into the determination of whether a risk is standard or sub-standard. Persons engaged in certain occupations could not be accepted at standard rates. Race, domicile, family history, and the plan of insurance applied for, all have a bearing on the desirability of the risk. Physical conditions, also, such as heart murmur, overweight, albumin in urine, consumptive family history, irregular, intermittent, or rapid pulse, high blood pressure, unsatisfactory habits

concerning alcohol, history of syphilis, gastric and duodenal ulcers, or gall stones, may render a risk sub-standard, or lead to its being rejected altogether.

A considerable number of occupational hazards have been quite thoroughly analyzed and their effects on mortality have been established with some degree of accuracy. The hazards, however, are changed from time to time by changes in industrial processes, conditions of sanitation, and by the installation of accident-prevention devices, etc. There are several elements to consider when viewing occupational hazards from a life insurance point of view. For instance, those engaged in occupations that foster irregularity of living or over-indulgence in intoxicating liquors, such as persons in the liquor business, must be given careful attention. Sanitary conditions must be given due weight, those engaged in dark, dusty, or poorly ventilated places being considered as inferior life risks. Those engaged in occupations involving the possibility of poisoning, and those in occupations producing high accident rates, such as miners, railroad men, etc., must be treated as sub-standard risks.

Even in risks considered standard, there is a difference in hazard. A group of clergymen or teachers will usually display a lower mortality rate than a group of mechanics, though both groups are considered standard.

Race and domicile must also be given due weight. Orientals and Negroes especially make a poor showing as regards longevity. As regards domicile, it is important to know whether the risk resides in a tropical or semi-tropical climate, whether in the northern or southern part of this country, whether in a rural or urban district. Some states show a higher mortality rate than others, but it is not practicable to quote different rates in the various states of the Union. Domicile must be considered, however, in passing judgment as to whether a risk should be classed as standard or sub-standard.

Family history is of importance, and records of mortality, according to information given in the application, have been quite carefully kept. It has been found that light weight, coupled with a family history of tuberculosis, is a serious matter at the earlier ages. In the case of robust persons above middle age, a family history of tuberculosis is of little consequence, and becomes less so as age advances. Weight and build are of importance, however, even if the risk is middle-aged. It has been found that a history of consumption in a member of the immediate family results in an increased probability of its appearance in the applicant, consumption in a brother or sister

being as important as in a parent, persons under average weight also being more susceptible than those well developed. While the practice of companies differs, it has generally been considered that an applicant whose father, mother, brother or sister died of tuberculosis, should be treated as a sub-standard risk if light in weight and younger than middle age. If tuberculosis has been the cause of a death in the immediate family, a person slightly overweight is generally a more desirable risk at an early age than a light-weight person.

When a company issues a short-term endowment insurance, the reserve accumulates and the amount at risk diminishes rapidly; hence the company is soon off the risk. A company is therefore safer in accepting a slightly impaired risk for endowments than for term or life insurances. A person will not, indeed, apply for endowment insurance unless he knows his chances of surviving the period are very good. Exceptions are those who know they will not be accepted for term or life insurance at standard rates, and therefore apply for an endowment. The plan of insurance applied for may not be the deciding factor in determining whether the risk is standard or sub-standard, but it sometimes happens that when an impaired life applies for term or life insurance, the company will decline to issue this, substituting for it endowment insurance.

The specific physical impairments met with in sub-standard risks are quite numerous, the ones most often encountered being those enumerated above. Heart murmurs, overweight, albumin in urine, and consumptive family history constitute the causes of impairment in over half of the total number of sub-standard risks accepted by one prominent company. Heart murmurs may be functional or organic. Functional heart murmurs are occasioned by a variety of causes. They are rarely found in elderly persons, but not infrequently in young persons in run-down condition. They are likely to disappear as the general condition improves. Organic heart murmurs may indicate a number of conditions of varying degrees of seriousness, some of which render the risk uninsurable, while the others are given a special rating.

In an application, an overweight applicant is seldom described as flabby or fat, but as large-boned, heavily muscled, and active; or as enjoying good health, feeling well at all times and, although a little heavy, yet quite "solid." Nevertheless, overweights as a class produce a heavier mortality rate than those of average weight. At the earlier ages, this condition is not so serious as later on in life. Tall men

among the overweights show a higher mortality than short men of the same percentage of overweight. Overweight persons are especially susceptible to heart disease, apoplexy, Bright's disease, and diabetes. At age 30, provision should be made for 35 per cent extra mortality among men of medium height and 30 per cent overweight, according to the results of the Medico-Actuarial investigation. At ages 40 and 50, the increased mortality is 50 per cent. Underweight, on the other hand, is of more importance at the earlier ages. After middle age, a moderate degree of underweight is desirable. The best mortality seems to be experienced on elderly persons who are distinctly underweight, middle-aged persons who are about or slightly below the averages, and young persons who are slightly above the normal weight.

Other causes, numerous and varied, produce higher death rates than normal. Alcohol produces extra mortality both among steady users who do not become intoxicated and among those who occasionally drink to excess. Among those who have been free from symptoms of syphilis for one year following a two-year period of continuous treatment for it, the expected extra mortality has been estimated at from 35 to 80 per cent, depending largely on the care exercised by the company in selection. Appendicitis, stomach, duodenal and other ulcers, removed by successful operation, render the risk sub-standard for a year or two, after which no additional mortality from this source is expected, provided all signs of trouble have disappeared.

As indicated in Chapter IX, sub-standard risks may be divided for premium and reserve purposes, into three classes: first, those in which the extra risk increases with the passing of time; second, those in which the extra risk diminishes with the lapse of years; and third, those in which the added risk is constant. These are commonly referred to as increasing, decreasing and constant hazards.

Numerical Rating.—Until recent years, the extra rates charged were the result of the combined judgment of the medical, actuarial and other officials. The completion of the Medico-Actuarial and other investigations, as well as the careful preparation of statistical data by individual companies, has led to the expression of extra mortality as a percentage of the normal. Some of the largest companies have definitely established tables based on their own experience. It is customary to say that a certain impairment is expected to result in an average mortality of, say, 175 per cent, or in some instances it is expressed as 75 per cent, in excess of the standard. The use of such ratios to express extra mortality is known as the "Numerical System."

Mr. Arthur Hunter and Dr. Oscar H. Rogers are the pioneers in the development of this method. The fundamentals of the system were first set forth in Mr. Hunter's paper on "Selection of Risks from the Actuarial Standpoint," published in the Transactions of the Actuarial Society of America, Vol. XII.

In explaining the numerical method of rating, Mr. Hunter and Dr. Rogers in ~~another paper before the~~ Actuarial Society~~, published in~~ Vol. XX, state that every risk is made up of the following factors:

1. Build (weight in relation to height).
2. Family record.
3. Occupation.
4. Personal history.
5. Habits.
6. Physical condition.
7. Habitat or residence.
8. Moral hazard.
9. Plan of insurance applied for.

If a value of 100 per cent be assigned to the average risk accepted by the company and each factor expressed in terms of 100 per cent, the summation of the values of the factors yields the value of the risk in relation to the standard or average risk. Proper allowance is attempted when two factors are inter-dependent. Border-line cases, and cases involving peculiarities such as moral hazard, applications for large amounts of insurance, etc., are submitted to the actuarial or medical department, or to both.

The practical application of the numerical system may be illustrated as follows:[3] Suppose a jeweler aged 35 applies for insurance, and furnishes the following information: Height 5 ft. 10 in., weight 198 (20 per cent overweight); family history slightly better than average; contracted syphilis 14 years ago, had mild secondaries for six months, and was under treatment eighteen months after disappearance of all symptoms; was never intoxicated but used alcohol rather freely until 5 years ago when he married,—practical abstainer ever since. From statistical tables covering these points, suppose that it is learned that he should be given a mortality rating of 165, or 65 in excess of normal, which is arrived at in this ~~matter:~~ manner:

[3] Adapted from illustration of Dr. Oscar H. Rogers and Arthur Hunter in paper on the Numerical Method of Determining the Value of Risks for Insurance, T.A.S.A., XX, 297.

Basic rating (20 per cent overweight)	115
Allowance for height	5
Allowance for family history	−5
Occupation	0
Personal history	40
Habits	10
Total	165

The advantages of using the numerical rating system are similar to the advantages of schedule rating in fire insurance. The disadvantages of personal judgment are overcome to a considerable degree by using this method, but of course judgment must be exercised in assessing the individual factors, especially when more than one impairment is present, or when two or more characteristics of the risk are inter-related. The numerical method enables the actuarial department to make an accurate estimate of extra mortality expected according to the company's own experience. It is also of practical value for use in explaining to agents and their prospects the reasons why the company must deviate from standard rates or the plan of insurance applied for. It results in a more uniform treatment of risks, reduces the liability to error, lessens the work of the medical expert, makes possible a more careful analysis of doubtful cases, and also the handling of a larger volume of business, since much of the latter may be rated by experienced clerks without the advice of the company's medical experts.

Many theoretical objections to the method have been offered, but it is being used more and more, and proves satisfactory in practice. The ratings are changed from time to time as experience dictates. The ratings of any one company are not likely to correspond with those of another, because of differences in selection. For the same reason the averages of a large number of companies may not fit the needs of any particular company for purposes of premium and reserve calculations.

In connection with reserve calculations the principal problem that confronts one who wishes to arrive at accurate results is the incidence of sub-standard mortality; i.e., whether the extra mortality is constant throughout, or falls heavier during the early, or the later, policy years. Mr. James E. Hoskins presented a paper on this subject in the Transactions of the Actuarial Society of America, Vol. XXIII, pp. 114 et seq. From the discussions that follow this paper, it is

evident that opinions differ, and on quite a number of points statistics are not conclusive. The principles involved are treated in Chapter IX.

Methods of Treating Sub-Standard Risks.—Several different plans are used by life companies in treating sub-standard lives, in order to meet different needs in handling various types of risks. Just as there are great differences between standard and sub-standard lives, so there are great differences within the group of sub-standard lives. To be equitable to all persons insured in the company, these groups should be treated differently. However, to handle the smaller groups individually would necessitate great expense and cause needless trouble and complications. The majority of companies which insure under-average lives, therefore, have adopted two or three plans. The general object of insurance on sub-standard lives is to deal equitably with each risk and to render the greatest good to the greatest number. It must be admitted, however, that none of the plans enumerated below can deal equitably with each and every case. Each company must adopt some policy which will enable it to make successful as a whole its under-average business.

In adopting satisfactory plans, the company must observe the following cardinal rules: (1) The plan must be equitable and fair to the under-average policyholder, yet must not place an unfair burden upon the standard policyholders. (2) The plan should be equitable as far as possible between the different types of sub-standard lives. (3) The premiums must be adequate to cover the risk and the added hazard. (4) Proper reserves must be maintained. (5) The policy must be made attractive to the sub-standard risk.

The sub-standard life must be shown that the raising of the premium or the reduction of the coverage merely provides for the additional hazard occasioned by physical disability, occupational hazard, or whatever is the cause of his being classified as sub-standard. Under some plans an improvement in his physical condition or a change of occupation will result in a reduction of the extra charge or the extra rating, or in an increase in the protection afforded.

The usual plans for meeting the extra mortality are: (1) to charge an extra premium or to advance the age of the applicant; (2) to change the plan of insurance to one requiring a higher premium rate than that applied for; and (3) to place a lien against the policy.

Extra Premium.—The extra-premium plan provides a flat extra premium covering a period of years, to defray the cost of the extra mortality. It may be conveniently used: (1) When the hazard is a permanent one and bears no direct relation to age (as blindness or

the loss of a limb). (2) When the hazard is a temporary one and will disappear entirely after a few years (for example, the extra premium usually imposed for 3 to 5 years after an operation for ulcer, appendicitis, etc.). (3) When the policy issued is on the endowment form, where the rated-up age would be beyond reasonable bounds, and would make the task of the salesman much more difficult. It may readily be seen that this plan is the best for a temporary hazard, inasmuch as the additional premium is automatically eliminated when the extra hazard is terminated. When the hazard is constant, this plan, as has been pointed out, is most favorable. Still, the net amount at risk on an individual life becomes less and less each year, while the extra premium is constant.

The extra-premium plan gives rise to the problem of the relation of loading to this extra premium and the question of participation in surplus. Inasmuch as this extra premium is only intended to cover the additional mortality occasioned by the extra hazard, it should not participate in surplus, except in the surplus contributed by it. When the hazard is temporary, the usual practice is to have the extra premium cover the hazard only. When the extra premium covers a permanent hazard, or is attached to an endowment policy, companies usually pay dividends (if earned) and add the regular loading to cover the agent's commission and the expenses incidental to writing the policy. It is true that the agent has a harder task in selling a sub-standard policy than a standard contract, and this may entitle him to a commission on the extra premium for his additional labor. It should not entitle the company to an additional loading for underwriting expense, inasmuch as there is none. If the extra premium is to participate, it is evident that the premium must have an additional loading. If the percentage of dividend increases with the duration of insurance, an extra premium should be so loaded as to provide for this increase. The extra premium is computed by arriving at the relative rate of mortality by the numerical system, or by judgment. The advantages of the extra premium plan are that it is equitable in cases of permanent hazard; it is equitable and convenient in cases of decreasing hazard; it is simple and readily explained to the applicant. The disadvantages of the plan are that it may give an excess to the company as the net amount at risk decreases from year to year, and it is not applicable to an increasing hazard.

Advance in Age.—This plan involves rating up the risk a certain number of years to cover the extra hazard. For instance, if at age 25, the ordinary life premium is $16.11, and the additional mortality

premium is $4.17, the total premium amounts to $20.28, which is nearer to the premium for an ordinary life policy at age 34 (premium $20.40) than at any other age. Consequently the risk is rated up 9 years. On this plan, it is customary to allow the insured the cash, loan, paid-up and extended insurance values at the increased age, and also to grant the dividends as of the rated-up age.

Unless the company adopts measures to prevent it, the insured may be in a position to take advantage of the advanced values of the rated-up age by surrendering his policy in the early years, thus getting back a substantial part of the extra payments he has made for the additional hazard. If the additional hazard expected by the company is to continue for many years as a percentage of the regular mortality, the advance in age is considered by many as the best plan to follow. Others differ with this view.[4] The general advantages of the plan are that it is simple and does not complicate the office system, it may be easily explained to the insured, and it shows good points in increased surrender values and the participation by the extra premium in profits of the company.

The defects and disadvantages of the advance-in-age plan are: (1) The insured is always willing to drop the policy upon getting better terms from some other company. This may result in an adverse selection for the company. (2) If the rating-up method adequately provides for the extra mortality in the early policy-years, it will provide for a much heavier mortality than it is reasonable to expect in the later policy-years. (3) Rating up short-term endowments does not materially affect the premium, hence it would not provide for much of an increase in the rate of mortality unless the rated-up age be fixed unreasonably high.

The Lien System.—The lien system of handling sub-standard lives, while never extensively applied in this country, has been in use for many years. It consists in placing a lien against the policy, the amount of which shall be deducted from the amount payable at death. It may be an amount constant throughout the term of the policy, in which case it is equivalent to charging an extra premium; but this is not the usual arrangement. This system is particularly applicable to risks in which the extra mortality decreases with the passing of time. When it is used, therefore, it is customary to reduce the amount of the lien each year the policy remains in force. The lien may be reduced by a definitely stated sum each year, but it is quite usual to reduce it by the amount of the annual premium. This method of

[4] See discussion by Messrs. Gore and Moir, T.A.S.A., XVI, 338, 346.

reduction is by no means scientific, since the decrease in the extra mortality bears no relation to the premium, but it is simple to operate and has some advantages for the salesman. The appeal to the prospect is, "Every time you pay a premium, you increase your protection by just that much." The agent of a participating company may add that the protection is increased by the gross premium. The liens imposed vary with the expected extra mortality; the poorer the risk the heavier the lien.

The system has certain other advantages for the insurance salesman. For instance, the applicant is generally optimistic regarding his physical condition, and inclined to think himself a good risk in spite of the medical examiner's findings. The life agent is in position, therefore, to make the statement, "In that case, you may prove yourself a good risk by living so many years, after which there is no lien." Also, the agent may induce an applicant to take additional insurance to cover the lien.

The lien is open to the objection that it may exist without the insured's knowledge of it, or at least without his full comprehension of it. Legislation has been passed in some states restricting the application of this system. After the Armstrong investigation, the laws of New York required that the form be approved by the Superintendent of Insurance, and the Superintendent for several years did not favor the lien system. Policies might have been issued which provided for steadily increasing insurance for a period of years, but this was not deemed practicable. Mr. Arthur Hunter shows [5] that it requires a very heavy lien to cover any considerable increase in mortality, and gives this as the reason why the lien system is so little used. It deprives the policyholder of a part of his protection, at a time when he probably needs it most, since this method is particularly applicable to impaired risks on which the extra mortality decreases as time passes.

Change of Plan.—This plan consists merely in issuing a higher-premium policy in place of the one applied for. When an ordinary life or limited-payment life policy is requested, the endowment is the one usually issued, the object being to write a policy which will allow the company to get off the risk before the extra hazard becomes serious. It is not a safe or proper form to use when a decided impairment exists. The premiums on this plan are no higher than for healthy risks and an abnormally heavy death rate results in loss to the company. This loss is lowered by changes in plan only by an

[5] T.A.S.A., XVI, 69.

amount equal to the difference in reserves on the endowment plan and those on the policies originally applied for. The plan merely provides for a rapid reduction of the insurance element in the policy, and fails to provide for the increase in the cost of mortality on this insurance element due to the fact that the risk is sub-standard.

The method is well adapted to cases in which the risk is desirable in every way except in this, that the family history shows deaths occurring mostly at ages from 60 to 70. It is open to the objection that, in participating insurance, it results in discrimination against those who of their own volition seek endowment insurance. But it is fairly easy to apply in practice since it involves no complications as to home office records, nor as regards reserves. Moreover, the agent who is skilled in presenting the advantages of endowment insurance over the life plan generally has little trouble in persuading the applicant to take the higher-premium policy. In fact the applicant will sometimes be best fitted under the endowment plan.

Special Dividend Classes.—In a participating company, sub-standard risks may be segregated from standard risks and placed in a separate class as regards dividends, on the theory that dividends will be sufficient to cover the extra mortality. One of our companies issues a special type of policy on which higher dividends are anticipated than on ordinary policies, these policies being segregated for dividend distribution purposes. To attempt sub-standard business, however, on the theory that contributions of sub-standard lives to the surplus will be sufficient to meet sub-standard mortality, is somewhat hazardous unless the insurance is limited to those who are only slightly impaired. At least it must be limited to lives which, by occupation or impairment, are sub-standard only to such a degree that the extra mortality can be met by the usual dividend declared on standard policies. Again, were this plan extended to the more serious impairments, the sub-standard risks should be subdivided into several groups for purposes of dividend distribution, if the spirit of the contribution plan is to be carried out even approximately, and this involves having enough lives in each group to insure the operation of averages.

The Double Endowment Plan.—A modification of the change of plan method as described above consists in issuing a policy for a short period of years with the provision that only one-half of the face amount shall be payable in the event of the insured's death while the policy is in force, the full amount being payable if the insured survives the period. Within reasonable limits, this policy may be written

at the same premium rate, regardless of the age of the insured. It may be used in cases where a considerable impairment exists, but it is not usual in insuring sub-standard lives. It is especially unattractive to the insured person in that if death should occur in the later policy years, the amount payable by the company is much less than the amount paid in by the insured person.

Use of Impaired-life Tables.—Certain British companies are said to have used special tables as the bases of premiums and values, but this has not been done in the United States. The use of a separate sub-standard mortality table for each of the principal impairments, with the necessary auxiliary tables, would introduce mechanical difficulties which the numerical method obviates, and that without in any way lessening the difficulty of rating a risk with a combination of two or more impairments. It would be unfair to use only one sub-standard table and thus put all under-average lives into one large group for premium, valuation, and dividend calculations.

Improvement in the Risk.—It may happen that after a rated-up policy, or a policy with an additional premium, has been issued by a company to an impaired risk, another company may issue a policy on the same risk at a standard rate or with a rating or premium lower than that issued by the original company. The insured then naturally appeals to his first company to reduce or remove the rating, extra premium, or lien. If the company does not do so, he often withdraws entirely. It is advisable, therefore, for a company to make some provision for improvement in the risk or for the remission of the extra premium if the impairment lasts only for a few years, as would be the case with an operation for appendicitis, for instance. Where an advance in age is about to be taken off, the insured having had a maximum loan, the policy at the true age would not provide adequate security. The payment of another premium is then required. Likewise, when the insured makes a change from a hazardous to a non-hazardous occupation the company may require a probationary period of one year. It may then change the policy to date from the exact time when the insured engaged in the less hazardous occupation and make a refund of the extra premium paid since that date, provided that the insured proves to be a standard risk at that time. This is to overcome the contingency of the insured's having become a poorer risk through the effects of the occupation which he may have had to give up.

In closing it may be said that insurance on sub-standard lives in this country has had a successful beginning and attained significant

proportions. Two fundamental difficulties which prevailed in the past —lack of accurate data on which to base rates, and inability to induce under-average lives to accept policies at other than standard rates— have been overcome, at least in part. The first important developments of a scientific nature have come within the past twenty years. The problems presented by under-average lives, however, are vital and important. Sub-standard insurance means the widening of the insurance field for the companies; it gives a truer meaning to life insurance by greatly extending the limits within which protection may be granted for impaired risks; it means an enlarged field for the agents, with the possibilities of additional compensation for additional labor. The companies must realize that all persons of good moral character, not actually ill, should be insurable on some basis, whatever the nature and extent of their disability. It is true that at present an insurance company cannot safely accept risks that are impaired beyond certain limits; yet a further development of sub-standard insurance and a widening of experience will eventually permit the companies to offer some form of protection, within practical limits, for all who need it.

CHAPTER XIV

Total-Disability and Accidental-Death Benefits

Disability Insurance.—*Nature and Development.*—Disability insurance is issued in connection with life insurance by including in the life insurance policy a clause stating that the company will waive premium payments, or waive premiums and pay certain additional benefits in case of the total and permanent disability of the policyholder. This may be provided in ordinary, industrial, fraternal, or group policies. The subject to be treated here, however, is disability insurance as offered in the life policies of old-line or legal-reserve companies.

It has long been recognized that life insurance alone is not sufficient to cover the contingencies to which human beings are exposed. The income of the breadwinner may be cut short by disability as well as by death. For several generations the friendly societies, fraternal orders, and accident and health companies have been granting protection against loss of earning capacity due to accident or ill health. Until recently, however, accident and health insurance was of the cancellable variety. It was quite expensive, since it protected against temporary disability. The majority of human beings become totally and permanently disabled for a considerable period preceding death. The average period is about one year and five months. So as far as the family budget is concerned, a protracted illness resulting in death is more serious than a sudden demise of the breadwinner. Instances are found of lingering disease that renders the latter incapable of earning anything for many months, sometimes for several years. During this time heavy expenses are incurred for professional services, and in other ways. Under such circumstances, life insurance policies may be lapsed, or great sacrifices necessitated to keep them in force. Since illness for a considerable period preceding death is so usual, in order to render life insurance a means of further protection, provision was made against allowing it to lapse during this period. Hence early disability clauses provided for waiver of premium following total and permanent disability; i.e., after the

occurrence of total and permanent disability, no further premium payments were necessary to keep the policy in force.

The disability clause first appeared in the life contracts of certain German companies as early as 1876. The first American company to adopt it was the Fidelity Mutual Life Insurance Company of Philadelphia in 1896. The Travelers followed in 1902. Since that time the growth in the popularity of this feature has been such that, with a few exceptions, our important companies are issuing policies containing some sort of provision for total and permanent disability benefits. At a recent date, 222 clauses were being offered, over 80 per cent of which were for both the waiver and annuity.

Essential modifications in the disability clause came with the increase in its popularity. It was soon realized that the preservation of the insurance by mere waiver of premiums was not the only important benefit that might be granted to a disabled policyholder. The clause, therefore, was so changed as to provide for the maturity of the contract in installments, on the occurrence of disability. This did not meet the situation, however, since a disabled person could not be certain that the payments would continue until death, and each payment reduced the protection to dependents for which the insured originally intended to provide. Hence companies have adopted the so-called waiver-and-annuity clause which provides disability benefits without reducing the amount payable when death finally results. This, of course, requires a substantial extra charge.

The disability clause has proved to be a very effective talking point in selling life insurance, the idea being that it "insures the insurance." Thus by competition has been brought about an ever-increasing liberality with reference to the definition of disability, the benefits granted, and the interpretation by the company of its own disability clause.

Reasons for Disability Insurance.—For the company, the most important practical reason for the incorporation of a disability clause in life contracts is that this renders the policy more attractive to the insuring public, thus increasing the volume of insurance sales. The company which issues a policy containing a good disability clause places a powerful competitive weapon in the hands of its salesmen. Each agent is therefore anxious that his company shall be in advance of other companies in the matter of liberality in the clause and in the interpretation of it. Also, since the disability benefit results in an increase in the gross premium on which commissions are based, the agent receives through it a slight additional commission. Policy-

holders likewise prefer a liberal disability clause because it guarantees the permanence of their insurance and covers a risk not contemplated in the other portion of the policy contract, at an extra cost so slight as not to be objectionable. That the hazard insured against must be given serious consideration is demonstrated by statistics which indicate that the probability of disability within the expectation of life is .0604 at age 25, .1080 at age 35, .1977 at age 45, and .3573 at age 55.[1] It is noticeable also that a considerable proportion of claims are caused by paralysis, insanity and paresis; and it is a well-known fact that in many types of insanity and paralysis, the insured may live for years. Also, blindness, though causing but about 4 per cent of total claims, does not materially affect longevity.[2]

Until recently, too, the disability clause supplied the only means of providing an income during the whole period of permanent disability when that period extended over a considerable number of years. Another reason sometimes given for the almost universal adoption of the disability clause is that it involves very little expense. In fact the principal advantages in writing total disability insurance in connection with life policies, rather than under separate contracts, are the saving in expense of selling, the lower clerical cost, and the better claim experience. The sale of ordinary accident and health policies involves heavy expenditures in the way of agents' commissions and other outlays each year the policy is renewed. When disability insurance is joined with life insurance, agents' first-year commissions are high since they are based on the gross premium which is greater because of the disability provision, but renewals are much lower than in the case of ordinary, cancellable accident and health insurance. Furthermore, life insurance is issued on the basis of a medical examination and an inspection report. A much more careful selection of risks for disability insurance in connection with life contracts is thus possible than in ordinary cancellable accident and health insurance. The purchase of an appreciable amount of life insurance along with the disability benefit has also been considered as an evidence of good faith tending to improve the moral hazard.[3] This, however, does not relieve the companies from the dangers of over-insurance, as explained below. Ordinary sickness insurance involves great difficulty and expense in the adjustment of a multitude of small

1 Mudgett, Bruce D., in S. S. Huebner's "Life Insurance," p. 288.

2 Marshall, E. W., and Talbot, Walter LeMar, in "Insurance Research and Review Service," Lesson 11, p. 7.

3 Woodward, J. H., in "Proceedings, Casualty Actuarial and Statistical Society of America," Vol. VII, Part I, p. 11.

claims from which a life company granting protection against long-term disablements is relieved.

Contingencies Insured Against.—The disability clause protects against total, permanent disability only. The definition of what constitutes total, permanent disability has been greatly modified and liberalized within the past few years, especially in the time since certain companies began writing non-cancellable accident and health insurance. The most liberal definition makes it the complete loss of business time for a period of three months.

The United States Bureau of War-Risk Insurance has defined total disability as "any impairment of mind or body which continuously renders it impossible for the disabled person to follow any substantially gainful occupation and which is founded upon conditions which render it reasonably certain that it will continue throughout the life of the person suffering from it."

The usual definition of total and permanent disability as applied by commercial companies is: If the insured, after the payment of premiums for the first policy-year, while the policy is in full force, and before attaining a certain age such as 60 or 65, shall furnish due proof that he has become wholly and permanently disabled so that he is and will be permanently, continuously, and wholly prevented thereby from performing any work, or engaging in any occupation, for compensation or profit, and that the disability has existed continuously for a specified period (varying from no waiting period, to sixty or ninety days, or six months, depending on the practice of the company), the company will waive future premiums and pay the stipulated benefits. Some clauses state that the benefits are payable if the insured is totally, and "presumably" will be permanently, disabled.

In addition, it is stated that the results of certain misfortunes will be considered total, permanent disability. Thus it is usually stipulated that the entire and irrecoverable loss of the sight of both eyes, the severance of both hands or both feet, or of one hand and one foot, will be considered total and permanent disability. Some of the more liberal clauses provide that "loss of use of" both hands or both feet, or of one hand and one foot, will entitle the policyholder to the disability benefits. Several companies stipulate that if the insured is wholly and continuously disabled from pursuing any occupation for wage or profit, and has been thus disabled for a period of ninety days, the company will presume permanent disability and pay the benefit.

Concerning waiver of premium for joint-life insurances, it may be arranged that the premium will be waived upon the disability of either life, or that half the premium will be waived upon the disability of one and the entire premium waived if both of the joint lives become disabled.

Limitations.—*Risks Not Covered.*—There are many hazards not covered by the disability clause. Such are partial and temporary disablements. Loss of hearing, which in some professions, such as teaching, is more serious than loss of sight, is considered partial disability. Some companies do not include disability benefits in term policies. Companies generally will not extend the disability provision to more than a certain maximum of insurance, usually $25,000, or $250 per month. Over-insurance is not entirely obviated by this limitation, however, since a person might take $25,000 in each of several companies and thus secure enough disability protection to render disability quite profitable to him, although information as to existing protection is usually requested in the application. The companies could, however, take steps to avoid granting disability insurance in amounts disproportionate to the applicant's income. With the consent of the state insurance department, a provision might be included to the effect that if the insured is covered by more than one policy, each company will pay only such proportion of the income insured by its policy as its disability benefit bears to the aggregate disability income. Such clauses are used in accident insurance, and so-called "contribution" or pro-rata liability clauses have long been incorporated in fire policies, but their use in life contracts has not yet been found necessary. A check-up on statements in the application through the Retail Credit Company of America, or by other means, is more practicable.

Again, all persons who live long enough become disabled by old age. The clause does not contemplate protection against this contingency, hence most clauses provide that only upon disability before age 60, or in some instances 65, will the insured be entitled to disability benefits.

At present, partial and temporary disability is not covered by disability clauses, yet cases may arise in which total and temporary disability may be compensated. For instance, the company may accept proof of disability, at some future date requiring proof of its continuance. Should the insured fail to furnish this proof the disability benefits cease. So long, however, as the insured can do this,

the company must make the payments, unless they are limited to the term of the policy, as is quite often the case.

Sometimes companies refuse to include the disability clause in policies issued on female risks. They usually deny it in the case of certain sub-standard lives and persons engaged in certain types of hazardous occupations. Regarding those partially impaired, the practice of some companies is to modify the clause, if the benefit is granted at all, when the applicant has lost an eye or a member, so that the loss of another eye or member will not constitute a valid disability claim. Sometimes the additional hazard is covered by an increase of 50 per cent in the disability premium. Medical directors are usually given discretionary power in the matter of excluding the clause from doubtful policies submitted to them prior to issue. Quite a number of companies do not cover disability resulting from military or naval service in time of war; and most of them do not apply the clause to extended insurance granted under non-forfeiture provisions.

Tuberculosis is the most prevalent cause of disability. Companies may wisely hesitate to grant the disability benefit at the younger ages to those whose family history shows the disease. In the case of tubercular family history, the ratio of the disability rate to the normal disability rate is much greater than the ratio of the death rate to the normal mortality rate.[4] Insanity and paralysis are also frequent causes of disability, consequently risks with a personal history of mental or nervous diseases might be rated up for the disability provision.

Interpretation of the Definition.—The efficacy of total and permanent disability benefits, and the sort of contingencies insured against, depend in part on the general policy of the company in interpreting the definition. In whatever way the clause may be worded, border-line cases are sure to arise in which there is uncertainty as to whether the person claiming disability benefits is entitled to them. Absolute helplessness of the insured is not necessary for disability to be construed as total. It has been held that the clause entitles the insured to recover the benefits if he is totally and permanently disabled from performing any kind of manual labor on which he depends for a livelihood. Loss of earning power seems to be the real test, total disability being considered a relative matter that depends on the nature of the occupation, the capabilities of the insured, and other circumstances peculiar to each case. Changes in the wording of clauses make legal decisions of little value by way of establishing

[4] Hunter, R. G., in Record, American Institute of Actuaries, Vol. IX.

precedents. The most liberal interpretation would result in paying the benefits when there is clearly no attempt at fraud, and when it is reasonably certain that the disability is of such a nature as permanently to render the insured incapable of producing a substantial portion of the income he was capable of earning when in health. Upon investigation, most of the causes of total, permanent disability prove to be such as to prevent a person from engaging, not only in his usual occupation, but in any other as well. Competition, together with a sincere desire on the part of companies to render the greatest possible service to policyholders, has led to a very liberal interpretation of the disability clause. When a person who holds policies in more than one company providing disability benefits becomes impaired and receives the benefits from one of them, it is difficult for the others to refuse the benefits and still retain the good will of the policyholder or of his community. In fact, it is often cheaper to pay the benefits than to stand on technicalities and deny them. So great is the present tendency towards liberality in interpretation that Mr. Henry Moir recently took occasion to point out the dangers incident to exercising it. The principal danger he cites is that when the company once goes beyond what it is required to do by the letter of the contract in respect to a certain provision, one can never be sure that the courts will not construe some other part in favor of a plaintiff.[5] Those companies which provide that total disability for a period such as ninety days shall be considered as permanent, largely remove the exercise of judgment in the interpretation of the definition. When doubt arises concerning the application of any part of the disability clause, the insured is usually given the benefit of the doubt.

Benefits Payable.—Just as the definition of disability, and the interpretation of it, have been very much liberalized, so the benefits granted have been greatly extended in recent years. As indicated above, the benefit originally consisted in waiver of premiums to keep the policy in force. The second step in the development of the benefit was to mature the policy in some manner, the usual arrangement being to waive premiums and pay 10 per cent of the face annually until the policy was exhausted, or if death or maturity occurred prior to the exhaustion of the policy, to deduct the annual payments from the amount of insurance payable. The third step consisted in waiver of premiums and the payment during disability of an annuity equal to 10 per cent of the face of the policy annually, or 1 per cent monthly, without deduction from the amount of insurance payable at death

[5] See Transactions, Actuarial Society of America, Vol. XXII, p. 464.

or maturity. In many companies, not only is the premium waived, but the same annual dividend is allowed, and the non-forfeiture provisions applied, as if premiums were being paid. This is the most usual benefit found in policies of current and very recent issue. Occasionally the face of a policy on the life of a business executive is made payable upon the happening of total and permanent disability. Regarding the waiting period, it was originally provided that the company could not be held liable for disability payments until one year after due proof of disability had been rendered. This was later changed to six months, which was the usual period for several years. Next, a few companies adopted a waiting period of only three months. At the present time, several companies require no waiting period at all, the benefits being payable on due proof of disability within the meaning of the clause. Regarding the waiver of premium benefit, it is customary to provide that the company will waive the payment of all premiums becoming due under the policy after the expiration of the current policy-year during which proof of disability is rendered.

Instead of stipulating that the disability annuity shall consist of 1 per cent per month, many clauses provide monthly payments of $10 per $1000 of insurance, with the same effect. Thus a $10,000 policy will provide an income of $100 per month, in addition to waiving premiums and in some instances continuing to apportion the usual dividend each year, to the end of the policy's term.

Causes of Disability.—Except in cases where total continuous disability for a stipulated period is considered permanent disability, few claims would be allowed if the insured were obliged to prove that disability is undoubtedly total and permanent. For instance, tuberculosis does not always result in permanent total disability. As explained above, however, many companies have gone on the assumption that the insured may be presumed under certain conditions to be permanently disabled. Mr. Arthur Hunter, Actuary of the New York Life Insurance Company, made an investigation of the causes of disability under policies of his company, omitting claims under the clause which provided that continuous total disability for three months entitled the policyholder to the benefits, and set forth his results in a paper published in the Transactions of the Actuarial Society of America, Vol. XXII. Of the total approved claims, Mr. Hunter found that 41.6 per cent were due to tuberculosis of the lungs, 2.1 per cent to other forms of tuberculosis, 14.7 per cent to insanity, 6.3 per cent to accident, 5.3 per cent to paresis, and 4.6 per cent to paralysis of different forms, including infantile. Thus approximately

75 per cent of claims arise from these causes. Mr. Hunter stated that in the published experience of companies tuberculosis caused from 30 per cent to 45 per cent of disability claims. When a company is sure a policyholder has tuberculosis, it appears wise to adopt a liberal policy in allowing benefits, as this enables the insured to improve his chances for recovery by taking the disease in hand early.

Premiums and Reserves.—If provision is made for waiver of the premium for an ordinary life policy in the event of disability before age 60, the benefit is an annuity of the gross annual premium beginning at disablement and terminating at the death of the disabled person. The extra annual premium for the benefit is payable to age 60, or until the policyholder dies or becomes disabled. Hence the present value of the benefit divided by the present value of an annuity-due of 1 payable to age 60, subject to survival as an active life, yields the net annual premium for the benefit. If it were desired to pay the sum of 120 per annum in addition to waiving premiums, the present value of this additional benefit added to the present value of the waiver benefit, and the sum divided by the value of the annuity-due for the premium-paying period, subject to survival as an active life, would yield the net extra premium for the combined waiver-and-annuity benefit. It will be observed that the 120 per annum is an annuity similar to the waiver benefit. The terminal net value of the extra benefit may be computed by the prospective formula in the usual manner. The net single premium for the extra benefit as of age of valuation, less the present value at that time of the remaining premiums payable subject to survival as an active life, yields the terminal net value if the insured is still an active life. If the insured is disabled, the value of the disability benefit is the present value of an annuity-due on a disabled life for the amount of the benefit.

The above calculations involve no principle with which the reader of the previous chapters is not familiar. Statistical data are of course necessary to determine the value of the various elements. It is necessary to know the rate of disability among active lives, and the rate of mortality among disabled lives, as well as the rate of mortality among active lives. In the absence of other statistics, Mr. Arthur Hunter used the experience of several large fraternal societies which had been granting disability benefits for many years, to compile a disability table. It is known as Hunter's Disability Table, and is now in general use in this country for the determination of premiums and for the valuation of the extra benefit. Some companies state in the disability clause that the "premium and reserves for this provision

are based upon Hunter's Disability Table and interest at the rate of 3 per cent per annum."

The rate of mortality among disabled lives displays a tendency to decrease for a period following the occurrence of disability, although this differs with different causes of the latter. After a certain point is reached, following the occurrence of disability, the rate of mortality tends to increase with age. Mr. Hunter eliminated the experience of the first policy-year following the happening of disability, and arrived at a practical solution of this difficulty. A very high rate of mortality prevails among disabled lives at the younger ages, probably due to tuberculosis, which is more prevalent at those ages. As a result, disability benefits continue but a short time before death. This is one reason why the premium for the extra benefit is low at the younger ages. A higher rate of disability will result from a liberal definition of it. The modern tendency toward this greater liberality may be expected to result in heavier losses in the future.

The gross premium charged for the disability benefit varies with different companies, largely because of variation in the clauses. The premiums also vary with the age of the insured. While the disability benefit may not in itself be participating, if it contributes to the general surplus of the company, the policyholder's dividend may be increased. Unless the extra premium is quite high, however, there is little likelihood of the disability provisions contributing to the surplus funds. Any statement of the cost of the disability benefit to the policyholder is likely to be of small usefulness, especially since the clauses of the various companies are being changed quite frequently. Of little value, also, is a comparison of the cost and coverage under the disability clause with the cost and coverage under the ordinary cancellable accident and health policy. A comparison, however, of cost and coverage under the better non-cancellable accident and health contracts is very valuable to the prospective purchaser of protection. He might find that at his age he would do better to take a life policy without the disability benefit, and at the same time secure a non-cancellable accident and health policy. On the other hand, he might find it advantageous to take the disability benefit in connection with the life policy. Cost and coverage are the essentials to be compared.

Administration of the Benefit.—The administration of the disability benefit involves the solution of problems not previously presented to life companies. So far as selling the benefit is concerned, this costs the agent little or no effort beyond what he would exert in selling a policy without the benefit. In fact, as noted above, it is in

many instances a positive help in making a sale. Nevertheless the agent must train himself, or be trained, to present clearly and forcibly the merits of his company's disability clause, and he must also be familiar with the clauses of his competitors. The expenses of selling consist in the usual commissions for selling life insurance.

The accounting problems arising out of the disability clause are best solved by keeping the accounts of the disability provision entirely separate as regards the extra premiums received because of the benefit, the claims paid, the necessary disability reserve liability, and any surplus accumulations which may arise from this cause. Disability reserve liabilities are reported separately in the annual statements of life companies. Complete separation is necessary to determine whether more liberal clauses may be drafted, whether the premiums are sufficient, whether extra dividends may be paid on participating policies as a result of experience, or whether premiums should be increased because of heavier losses than those anticipated. When a company modifies its disability clause from time to time, or when it offers more than one clause to the public, or has different clauses in force, the accounts under each clause should be kept separately in order that the statistician may arrive at accurate conclusions regarding the rates for the different benefits.[6]

As regards the claim department, new problems have presented themselves as a result of the benefit. Usually the fact of death and identity of the deceased may be determined with comparative ease. To ascertain whether a claimant is entitled to disability benefits is a very different matter. For instance, a considerable number of insured persons do not understand the disability clause, with a result that claims are presented which are withdrawn after the insured has been informed on the subject. Mr. Arthur Hunter states that a number of cases arise which, from the information first submitted, the company might assume to be legitimate. When the matter is followed up, however, by issuing forms for proof, or by further inquiry, the insured either withdraws the claim or fails to answer letters urging him to complete proof. Taking 100 of such claims, alleged causes of illness presumably temporary in nature, such as appendicitis, rheumatism, sciatica, malaria, neurasthenia, pneumonia, etc., accounted for 46 of those dropped. Tuberculosis accounted for 28, illness which might lead to permanent total disability, such as heart disease,

[6] For a detailed statement of problems and methods of accounting in connection with disability benefits, see J. S. Elston, Treatment of Disability Claims in the Annual Statistics, T.A.S.A., XXI, 134 et seq.

Bright's disease, diabetes, etc., 22, and accidents causing partial or temporary disability, 4.

In addition, there are a number of claims which are pressed by the insured but which are clearly not contemplated in the clause. Of 100 claims of this nature which were disallowed, 35 were for disability of a presumably temporary nature such as sciatica, rheumatism, neuritis, appendicitis, indigestion, anaemia, insomnia and jaundice. Twenty-eight were for illnesses which did not then, but might later, result in total and permanent disability, the principal ones being Bright's disease, heart disease, diabetes, and empyema. Accidents causing partial or temporary disability accounted for 23, and tuberculosis from which the insured had recovered or of which he failed to submit satisfactory proof, 14. It is thus seen that the claim adjuster has problems formerly confined to casualty companies. He should see to it that the claimant gets the full amount to which he is entitled, regardless of whether he is aware of his legal rights or not, and yet he must see that the company is not imposed upon by way of fraudulent or unreasonable claims. In some cases, such as death from insanity, it is the practice of at least one company to ascertain whether the insured or his representative should not have filed a disability claim, and if it is decided that he would have been entitled to the latter, the benefit is paid. On the claim department falls the duty and responsibility of putting the company's definition of disability into effect, and of safeguarding the interests of both policyholder and company.[7]

Accidental-Death Benefits.—A considerable proportion of our life companies have entered a part of the field formerly occupied entirely by casualty companies. A clause has been incorporated in the life policy which provides that double the face of the policy will be paid if the policyholder dies by accidental means. The wording takes the form commonly found in accident policies, the usual provision being that double the face of the policy will be paid upon due proof that death resulted within sixty days from the date of bodily injury and directly from such injury effected solely through external, violent and accidental means. It is not customary to pay the benefit if death results from military or naval service in time of war, nor from suicide,

[7] Numerous citations to sources of information on disablement insurance are to be found in Mr. Wm. A. Hutcheson's paper, published in Transactions, Actuarial Society of America, Vol. XXII., pp. 336, 339-341. A paper by Mr. E. H. Hazlett was presented at the May, 1923, meeting of the Society. The reader is also referred to Dr. Bruce D. Mudgett's Thesis on Disability Insurance, published in Annals, American Academy of Political and Social Science, supplement, May, 1915.

riot, insurrection, participation in aeronautics or submarine operations, violation of law by the insured, police duty in any military, naval, or police organization, or disease of any sort. The double indemnity benefit, like the disability benefit, is not included in the paid-up fractional or extended term insurance which may be elected on surrender or lapse. Some companies do not pay the double indemnity benefit if payments have been made under the disability clause, while others pay double the face whether the latter has become operative or not. Sometimes three or four times the face of the policy is payable if death is caused by certain types of accidents.

As there are certain industries in which the liability to diseases such as tuberculosis is greatly above the average, thus rendering total and permanent disability benefits inadvisable, so there are occupations in which general-accident double indemnity would not be granted by most companies. Travel accident double indemnity, however, might be granted in some of these instances. One company grades the various occupations in this manner:

	Disability Benefits	Travel-Accident Double Indemnity Benefits	General-Accident Double Indemnity Benefits
A	Yes	Yes	Yes
B	Yes	Yes	Yes
C	No	No	No
D	Depends on merits	of case	
E	Yes	No	No
F	No	No	Yes
G	No	Yes	No
R N.A = Risk not acceptable			

There follows a list of the occupations in alphabetical order, beginning with accountants, acid makers, etc., with the rating and symbol set opposite each one.[8]

Premiums for the double indemnity benefit have usually been based on the amount of money previously paid out on life insurance losses due to accidental death. Due allowance must be made, however, for the fact that if the benefit is included, an accident is more likely to be alleged as the cause of death, and also that expenses of litigation are more likely to be incurred.

[8] See Paper by Arthur Hunter and Dr. Oscar H. Rogers, in T.A.S.A., XXI, 22, et seq.

It is customary to limit the double indemnity benefit to a stated maximum, not more than $25,000. The travel-accident benefit is sometimes not incorporated if the applicant is over 55 years of age.

Objections to the Disability Annuity and Double Indemnity Clause.—A number of objections have appeared from time to time to the granting of these benefits in life insurance contracts, among the more important of which the following may be mentioned:

1. The coverage is incomplete. The disability hazard is not covered in many cases, as indicated by the fact that about one person in 5 suffers loss from disability each year, whereas but one person in 1781 at age 30, and one in 1201 at age 40 will annually become totally and permanently disabled, according to Hunter's Disability Table. The vast number between these ratios are unprotected by the disability clause, and in order to obtain complete coverage must supplement life insurance with accident and health insurance as granted by the casualty companies.

This argument, however, while pointing out the advisability of accident and health insurance, fails to state the advantages of complete coverage. It might as well be argued that the disability annuity of life contracts should supplement one's accident and health insurance, which may be used to provide temporary benefits.

2. The clauses may be easily misrepresented by agents, and misunderstood by the insured who is led to believe that his coverage is complete.

This, however, is an argument against dishonest or uninformed agents, and furnishes a reason why the insuring public should take the trouble to inform itself on the subject of life and accident and health insurance.

3. There is no need for the disability clause, since its coverage can now be obtained from casualty companies.

Experience alone can demonstrate whether life or casualty companies can best serve the public in this regard. At the present time, many thousands of persons would not have any protection against total and permanent disability if provision for it were not incorporated in their life contracts, and there is ample room in this field for both types of companies.

4. Since double indemnity is accident insurance, and disability coverage is accident and health insurance, the incorporation of these benefits in life contracts means that life companies have become multiple-line companies.

For many years this was thought to be contrary to good life

insurance practices, but, again, experience alone can be depended on to solve this question.

5. Disability and double indemnity insurance is speculative and dangerous.

Some of our ablest and most experienced casualty insurance men have viewed the non-cancellable accident and health contract as a source of peril. The recent increase in the premiums charged for these contracts would tend to indicate that the fears of these men were not without foundation. But the extra premiums for the benefits in life contracts may also be increased if necessary to safety, hence there is little danger of serious trouble from this source.

6. In most instances these benefits are not participating and not mutual. When a purely mutual life insurance company loses money on its disability and double indemnity contracts, the deficiency must be met by drawing upon the surplus contributed for the most part by older policyholders who have nothing but life protection.

The same may be said of annuities and income policies, however.

7. Uncertainty and controversy arise because of the difficulty of defining, and interpreting the definition of, total and permanent disability, and of determining whether a death in question is the result of accidental causes. The agency force may at times suffer because the company does not disregard business principles in dealing with losses. Suppose a man is disabled and receives benefits for a time, and then partially recovers, so that the agent must take away the annuity benefit and again put him on a premium-paying basis. No agent relishes such a task, nor does he expect any improvement in good will as a result of its performance. Furthermore, an agent may lose a prospect because his company declines to include the disability or double indemnity clause.

The difficulties of definition and interpretation are not insurmountable, however, and the inclusion of a liberal disability clause meets with no opposition from the agency force.

8. It has been maintained that such clauses have no proper place in the life insurance contract. They only complicate the contract, protect against a relatively small hazard, are narrow in scope, provide benefits of little value except in large policies, and are unnecessary since the insured can make use of extended insurance or paid-up non-forfeiture values, or make a policy loan each year to cover the premium in the event of total and permanent disability.

As opposed to this series of objections, it may be stated that the clauses can be drawn with a fair degree of clearness, and can be made

quite wide in scope. The average policy is for about $3000; $30 per month would not be unwelcome. Non-forfeiture values are not available for some time after the policy is in force; extended insurance and loan values may not be sufficient to meet the needs of a long illness, and the clauses are not extended to non-forfeiture benefits.

9. The incorporation of the clauses in life contracts constitutes a movement whose ultimate destination no one can predict. Benefits of 2 per cent of the face per month and triple indemnity are now offered by a few companies. Seldom a year passes but that some company goes beyond the previous limit. Keen competition may result disastrously to some companies, or at least cause losses to old policyholders, in the opinion of some insurance men.

Concerning this objection, it may be urged that under our careful legal supervision no company is likely to get into serious financial difficulty on account of these clauses.

The provision in the double indemnity clause excluding accidents resulting from the violation of any law now constitutes a very broad exemption, and one which few persons rightly appreciate. For instance, a person who loses his life as the result of an accident due to walking on a railroad, crossing the tracks at other than regular crossings, driving an automobile at a speed in excess of the legal limit, or having illicit liquor in his possession, may lose the benefit.

One prominent company which grants disability insurance does not incorporate the extra accidental death benefit, but recommends a casualty company's accident policy to a prospective purchaser desiring extra protection against accidental death.

It may be said that from an economic point of view, disability insurance should be encouraged, either in connection with life policies or with one of the better casualty companies. The case for the double indemnity benefit, however, is not so clear. Few persons would consider insurance under a separate contract providing for indemnity only in case of accidental death. As for travel accidents, the dependents can often obtain damages from the carrier. Double indemnity might better fit the needs of survivors if paid in cases of long and expensive illness.[9] Such cases, however, are covered by the disability benefit. Deaths due to the automobile are becoming increasingly prevalent; the public is willing to pay for the double indemnity benefit, and companies are justified in offering it. If offered at cost and with no misunderstanding on the part of the policyholder as to the amount of protection he is purchasing, there would seem

[9] Papps, P. C. H., in T.A.S.A., XIX, 336.

to be no serious objection to it. From the purchaser's viewpoint, the cost and benefits under double indemnity should be compared with the costs and benefits under the accident contracts of casualty companies. Whether the cost will eventually be lower than with casualty companies, experience will demonstrate.

CHAPTER XV

Government Supervision and the Management of New Companies

General Purposes.—A majority of the contracts granted by life insurance companies run for a considerable period of years, and a number continue for over half a century. Most of them insure against hazards which increase with the passing of time. The most practical way of providing against increasing hazards is to charge level premiums sufficient to meet them; i.e., to overcharge the insured person in the early years and preserve such overcharges to meet the increased hazards of the later years when the current level premium will be insufficient. The two fundamental problems in government supervision therefore consist in (1) compelling the company to charge rates high enough to enable it to meet the deficiencies of the later years by the excess collected in the early ones, and (2) providing for the absolute safety of this excess. The problem of rate regulation as applied to life insurance is to compel the companies to charge rates high enough to insure their own solvency. Well managed and conservative companies do not need the compulsion of legislation to this end. In the absence of legal restrictions, however, there might be formed companies on an inadequate rate basis only to be abandoned when the mortality increases beyond the limits for which provision has been made in the premium rate. The problem concerning the preservation of funds to assure solvency resolves itself into supervision and regulation of the investments which a life company must make.

There are other problems in government supervision. Once adequate rates are secured, surplus funds may arise, the disposition of which may require supervision. For instance, there is the danger that the company may spend an unwarranted amount for new business. In a mutual company this would be a matter of concern to the policyholders, and those insured in such represent a large group. Then there are a host of other matters as well, such as the licensing of companies and agents, the requirement of deposits with the state, the formation of new companies, the establishment of standard

policy provisions, the limitation of surplus, the prohibition of rebating and twisting, etc., which are subject to supervision.

Though life companies are private corporations, the public is vitally interested in their well-being. This is provided for by supervision which keeps out incompetently managed companies, inspires confidence in the business, and benefits well managed and conservative ones. It reduces the number of failures among life companies, and protects the public against the more serious consequence of such misfortunes.

Jurisdiction.—Since most of the larger companies operate in several states, it is thought by many that Federal, rather than state, supervision is desirable in this country. Centralized control would protect the business against sectional and retaliatory legislation, eliminate the duplication of reports and examinations, result in uniform legislation and taxation—such uniformity cannot be attained with nearly fifty separate jurisdictions supervising the companies—and lower the expense of supervision. It is quite safe to predict, however, that centralized control can never be attained in this country as long as we have effectual state jurisdiction. At the present time, neither inter-state nor intra-state insurance comes within the authority of our Federal Government to regulate interstate commerce. Since the United States Supreme Court decision in the case of Paul *vs.* Virginia in 1868, there has not been much doubt that an amendment to our Federal Constitution is necessary before the Federal Government can regulate inter-state insurance. Federal control of inter-state insurance alone would mean this saving—that a company engaging in inter-state business would report to only two departments, the insurance department of its home state, and the Federal insurance department, instead of reporting to each state in which it does business. The work of each state insurance department would then be limited to the supervision of companies whose home offices are located within its boundaries. The situation is, and is likely to be, that insurance companies are subjected to state control in this country, with all its attendant disadvantages and lack of uniformity.

It must not be inferred from the above remarks that there is no uniformity in legislation, and that there are no advantages in state control. The insurance commissioners of the various states sit in convention from time to time, appoint committees, make investigations, agree to recommend legislation tending towards uniformity, and the welfare of sound insurance. In many particulars, the insurance codes of the various states resemble closely the laws of Massachusetts and New York. Also, a deterrent to corruption or undue

influence, is the fact that a company operating in several states would be obliged to deceive or corrupt a number of departments under our present system instead of merely one or two. Again, it is quite possible that out of officials of a number of states, one may discover some matter which needs attention and which remains unnoticed by those of the other states.

Machinery of Supervision.—In most of the states that are important from an insurance point of view, supervision is accomplished by means of a department of the state government. This is called the state insurance department. It is charged with the execution of the laws of the state in relation to insurance. Its head is a state officer called the insurance commissioner, or superintendent of insurance. He is appointed by the governor with the advice and consent of the state senate, and is required to take the customary oath, and give bond for the faithful performance of his duties. He may appoint a deputy commissioner, an actuary, and such clerks and other employees as may be provided by law. It is not uncommon to provide a special deputy on liquidation; an actuarial staff consisting of one actuary, and several actuary's clerks; a compensation staff consisting of a compensation rating expert, a compensation claim statistician, and assistants; an examining staff of one chief life examiner, one chief fire and marine examiner, one chief examiner of casualty companies and fraternal societies; a complaint and investigating staff; a clerical staff consisting of a chief clerk, an examiner of statements, an examiner of policy forms, a license clerk, bookkeeper, compiler of annual statements, other clerks, stenographers, messengers; and, with the approval of the governor, additional examiners, special deputies, or clerks for special or temporary service.

Powers and Duties of State Insurance Officials.—It is the duty of the state insurance commissioner to see that all laws governing insurance companies, agents and brokers are fully executed. In order to accomplish the execution of the laws, the commissioner is given authority to make an examination of a company whenever he sees fit. The laws of most states provide that he shall do so at least as often as every three years. He has authority to examine in person, or by means of deputies or examiners, all companies licensed to do business in his state, whether domiciled in it or not. He or his deputies have free access to all books and papers of any company or its agents, he is obliged to require a domestic company (a domestic company is one domiciled in his own state rather than some other state or foreign country) to keep its books, records, accounts and

vouchers in such a manner that he or his authorized representatives may readily verify its annual statements, and ascertain whether the company has complied with all the provisions of law. Some states require the commissioner to allow the public access to the results of his examination of any company, and others go so far as to provide for publication of the result of his examination, "whenever he deems it for the interest of the policyholders so to do."

Specific Objects of Legislation and Supervision.—*Solvency.*—To determine the financial standing of an insurance company, it is necessary to compute the total reserve liability. Minimum valuation requirements vary, some states permitting different modifications of the preliminary term method, as explained in the chapter on valuation and expenses. Policies issued prior to about 1900 are ordinarily valued on the older standards.

Once the legal minimum reserve liability has been determined, the insurance commissioner must see to it that the company holds assets sufficient to balance it. The company must have such securities as it is permitted by law to own, or funds in secure investments to an amount equal to the reserve liability, over and above all other liabilities; that is, after all other debts and claims against it have been provided for, there must remain enough assets to meet the reserve liability. In the case of a company with a guarantee capital stock, it may be provided that impairments of as much as 50 per cent of the capital are permissible before the insurance commissioner must take drastic action. When such an impairment exists in a domestic company, or when the reserves of a domestic mutual company are impaired, the commissioner must notify the company in question and its agents not to issue any new policies until its funds become equal to its liabilities. If he sees fit, he may leave the control of the company in the hands of its officers for one year, and then renew the company's authority to do business if its funds again equal its liabilities. If he feels that the company is not likely to retrieve its affairs, he may institute proceedings to determine what shall be done. Companies incorporated in other states or in foreign countries must secure a certificate of authority from the insurance officer of the state in which it wishes to transact business, and must comply with the laws of that state or forfeit the certificate of authority.

There are reasons for instituting proceedings against insurance companies other than deficiency in the capital of a stock company or the reserves of a mutual. Such are, refusal to submit books and papers, the reinsurance of substantially its entire business without

written approval of the insurance commissioner, the violation of its charter, the refusal of an officer to be examined under oath concerning its affairs, or the discovery upon examination that further transaction of its business would be hazardous to its policyholders, creditors, or the public. The course of action usually followed is for the insurance commissioner, through the state's attorney general, to apply to the court of common pleas of the county in which the state capital is located or of the county in which the principal office of the company is located, for an order directing the company to show cause why the insurance commissioner should not take possession of its property and conduct its business. The court may then enjoin the company from disposing of its property before the hearing. After the hearing, the court may direct the commissioner to take over the property. Later it may order the business restored to the company, or the liquidation of the business and the dissolution of the corporation.

Regulation of Investments and Valuation of Assets.—Obviously, it would be useless to prescribe standards of solvency if some provision were not made for the security of funds held by life insurance companies. Also, unless current market values are taken, it is necessary to set forth some method of valuation of the securities held by them. Life companies often invest in long-term bonds, and opinions might differ as to the value of these bonds. Without some standard by which to judge values, one person might believe a company's funds to be inadequate to meet its liabilities and another think the company in possession of a substantial surplus.

With respect to the limitation of investments, care must be exercised not to hamper the companies unduly, since much depends on the ability and foresight of its investment managers. It is customary to limit the investment of the capital and reserves to real estate (limited as stated below), ground rents and loans on real estate, loans on policies, and bonds. Surplus funds over and above the capital and reserves may be invested in shares of stock, or in collateral loans, with certain restrictions. It is thus seen that considerable latitude is permitted in the investment of the surplus, but not so much in that of the capital and reserves.

Confining attention first to the investment of capital and funds to meet the reserve liability, the investment of such funds in real estate is generally limited to such as is necessary for office accommodations; and such as may be mortgaged or conveyed to the company by way of security for loans or satisfaction of debts, or purchased

by it at sales upon judgments, etc., in an effort to save the company from loss from debts owing to it or to persons against whom it may have financial claims. Investments in ground rents and loans upon unincumbered real estate are permitted, but no loan shall exceed a fraction (in some states 60 per cent) of the fair market value of the property at the time the loan is made. A company may also make loans on its own policies, provided that no loan shall exceed the cash value of the policy on the security of which it is granted. The fourth form in which the investment of capital and reserves is permitted, consists in bonds. Of these, a wide latitude of discretion may be exercised by the company. Federal, state, municipal, county, township, school district, and federal farm loan bonds, certain improvement bonds such as water, sewer, drainage, and road bonds, as well as first encumbrances upon steam and street or interurban railways, or their rolling stock, and bonds of public utility companies, constitute the principal issues in which investments are permitted by the law. It is customary to permit other investments under certain circumstances. Bonds and notes other than first encumbrances on steam or street railway properties may be purchased by the insurance companies, provided no default in interest exists at the time of purchase. With the consent of the insurance commissioner, sufficient investments in the securities of foreign governments may be made to enable the company to comply with the laws of the foreign government regulating the transaction of the insurance business within its jurisdiction. Companies may also acquire and hold securities or properties taken in reorganization or foreclosure proceedings, or in satisfaction of debt previously contracted.

In the investment of surplus money, most states allow insurance companies further discretion in that they may purchase standard and listed shares of stock of solvent, dividend-paying corporations, or lend money on pledge of the same, provided the current market value of the stock pledged is at least 20 per cent more than the sum loaned upon it. Investments in stocks or evidences of indebtedness of any unincorporated business, or in assessable stocks, are prohibited. Not more than one-fifth of a company's capital may be invested in any one mortgage, no loans may be made on personal security, not more than 10 per cent of the capital stock of any corporation may be acquired; nor may more than 10 per cent of the insurance company's capital and surplus be invested in the stock of any one corporation. It is further provided that if any investment or loan is made in a manner not authorized by the act, the officers and directors making

or authorizing such loan or investment shall be personally liable for any loss occasioned by it.

Valuation of Assets.—As mentioned above, some method of determining the value of securities held by insurance companies is necessary. While many states allow the insurance commissioner to exercise discretion as to the valuation of most securities held by companies and reported as assets, there is a tendency towards uniformity in the standards established and the methods pursued in valuation. Concerning real estate, a life company's annual report to the state insurance office usually includes specific information as to property held, dates of acquisition, names of vendors, the actual cost, the value at which it is carried on the company's books, the market value, amounts expended during the year for repairs and improvements, the gross and net income from each parcel, the rental value of real estate occupied by the company, and all purchases and sales since the last annual report, together with particulars as to dates, vendors, vendees, and the consideration. Loans upon the security of real estate, and collateral loans, with particulars in each case, as well as all other property, including bonds, owned by the company, with dates of acquisition, actual cost, value at which the property is carried on the books, market value, etc., must be accurately reported. Assets or securities lawfully held or acquired are allowed as assets at "their just value in the judgment of the superintendent," to quote Section 86 of the New York insurance law. It is thus seen that the company's gross assets may be different from those admitted by the insurance commissioner as assets in arriving at the financial standing of the company.

Mortgages are generally admitted at the book value, which in turn is usually the par value. Stocks, perpetual bonds, and bonds in default are admitted as assets up to the market value only. In former times, the market value of these securities on December 31 was taken in determining the amount at which they should be admitted. More recently, the method has been to choose five fairly representative dates, taking the average of the prices on those dates as the value of such securities.

The Amortized Basis.—As an outgrowth of the Armstrong investigation, laws have been passed to the effect that bonds having a definite maturity date and not in default, may be reported to the state insurance department on the amortized basis; that is, if purchased at par, the par value may be reported; if purchased above or below par, the value reported may be the purchase price, "adjusted

so as to bring the value to par at maturity and so as to yield meantime the effective rate of interest at which the purchase was made.'' The purchase price cannot be taken at a higher figure than the actual market price at the time of purchase.

The state insurance officer has full discretionary powers as to the precise method of calculating amortized values. The simplest method is to take the value found in a bond table, as described above in the chapter on interest and probabilities. Thus, suppose a bond having 20 years to run with interest at 5 per cent payable semi-annually, were purchased 10 years ago at a figure which would yield 4 per cent if held to maturity. What is the amortized value of the bond now? Turning to the page of a book of bond values on which semi-annual values and yields of bonds having 10 years to run are found, glancing down the yield column to 4 per cent, and then to the right to the column headed 5 per cent (the nominal rate as stated in the bond), the figure in the column headed 5 per cent is the amortized value. The amortized value is the present value of all interest payments to become due in the future, plus the present value of the principal, discount in each case being at the effective rate of interest at which the bond was purchased.

Some states provide that life companies may report on the basis of either the amortized or the market value, but that the aggregate value of bonds subject to the amortized basis shall not be greater than the aggregate of the amortized values. Thus at a recent date, one company reported gross assets considerably in excess of admitted assets, and among the assets not admitted was an item entitled, ''Bock over amortized value of bonds and over market value of stocks and bonds not amortized—$————.''

Numerous items of assets not admitted appear from time to time in official reports. Some of them are: supplies, furniture and fixtures, advances to agents or officers, accounts collectible, agents' debit balances, premium notes, policy loans and other policy assets in excess of net value and other policy liabilities on individual policies, and over-due and accrued interest on bonds in default.

Advantages of the Amortized Basis.—It has been argued that the liabilities of life companies for the most part are demand, or short-time liabilities, since insured persons may demand their surrender values on very short notice. The experience in the fall of 1907 lends some weight to the argument, but it is by no means conclusive. The vast preponderance of evidence leads to the opposite conclusion. With our present Federal Reserve banking system, there is reason to believe

the future will show that in the main, the obligations of life companies are long-time obligations. The experience during the recent depression tends to support this belief. Trade cycles do not materially affect the financial demands upon life companies, since the mortality rate does not fluctuate in the same manner as does business prosperity. So long as the life company is able to collect the interest and principal of bonds when due, it will be able to meet its obligations. Temporary fluctuations in the market price of its bonds, therefore, do not affect the financial standing of the life company to any appreciable degree. To summarize, the advantages of the amortized basis for the valuation of bonds held by it, having a definite maturity date and not in default, may be enumerated as follows:

1. It gives stability to values upon which the company's condition from year to year is determined.

2. It prevents sharp fluctuations in surplus. Hence it eliminates the temptation to declare large dividends on participating policies, or on stock, when bond prices are high due to low prevailing interest rates. It also obviates the necessity of declaring a company insolvent because the market value of its bonds has declined, when in fact the company will be able to meet all of its obligations as they fall due.

3. It avoids all questions as to the determination of market values, what dates to choose for the quotations, etc.

4. It substitutes mathematics for judgment, in large measure, and is accurate enough to insure solvency.

5. It avoids penalizing a life company for buying a good listed bond instead of a mortgage which may be inferior as an investment. If the amortized method were not available, companies might hesitate to invest in long-time securities, preferring the extra expense of frequent reinvestments of the same sum in order to reduce the chances of becoming embarrassed by a decline in bond prices.

By setting up standards of solvency and providing necessary regulations and the machinery to carry them into effect, states have made it very unlikely that a life company will fail with serious loss to the policyholders.

Organization and Admission of Companies.—In addition to making adequate provisions for the solvency of companies operating within its borders, a state provides for the organization of new companies and for the admission of additional companies incorporated in other states. While the statutes governing the incorporation of life companies follow the same lines in many respects as those laid down for forming all corporations, there are usually special acts

pertaining only to life companies. Thus it is customary to require proposed mutual companies to have applications for a minimum of insurance on a minimum number of lives before issuing any policies; stock companies must have a minimum guarantee capital. The requirement in one state is that a stock life company must have a capital of at least $200,000, and net surplus of $100,000 in addition to capital amounting to $200,000. If accident and health risks are also to be written, the capital must be at least $300,000. Life companies organized on the mutual plan in that state must have applications for insurance to the amount of $1,000,000, by not less than 400 persons. Some states require life companies to deposit with the state insurance officer approved securities to the value of $100,000, to insure the payment of its obligations. Most states having a deposit requirement provide that an approved deposit of the designated amount, but made with the proper officer of some other state, may be accepted as meeting this requirement. This is to avoid compelling a company to make a deposit in each state in which it wishes to do business. The minimum insurance required of mutuals, together with the deposit requirement, renders it somewhat difficult to organize a mutual company.

The usual procedure is to organize and commence business as a stock company. Later, if desired, mutualization may be accomplished by having the policyholders purchase the stock with surplus funds. The stock may then be retired. The state ordinarily regulates the amount which stockholders may receive for stock retired in this manner.

Some states require mutual companies to have a guarantee capital before commencing business. One leading state requires of mutual life companies a minimum capital of $200,000, one-half of which must be paid in cash, before commencing business. When the surplus equals the amount of the guarantee capital subscribed, the directors may retire all or a portion of the capital, but no sum paid out in cash may exceed that actually paid in, with interest due and unpaid. Legislation as to conditions under which foreign companies, including those chartered in other states, may be admitted to transact business, usually relate to taxation, certificate of authority, compliance with standards of solvency, licensing of agents, the annual report, and the valuation of policies and assets. As regards the latter, it is customary to provide that the insurance commissioner may accept the valuation of insurance departments of other states. The state insurance officer is usually designated as attorney for service of legal processes in any action or legal proceedings against a company from another state or

foreign country. Service upon the commissioner as attorney is deemed legal service upon the company.

Publicity.—There has long been a difference of opinion as to whether direct supervision of the company or adequate publicity concerning its condition and practices is the better method of protecting the public. In England, the practice for many years was to depend mainly on publicity. In this country, from the days of Elizur Wright, we have leaned heavily towards regulation and supervision, but our states have not been unmindful of the benefits of publicity. To accomplish publicity, state statutes provide that the insurance department must publish a report setting forth certain detailed information concerning each company doing business within its confines. A number of state departments do so annually, others every two years. The results of any examination by the state insurance department may be published whenever in the judgment of the insurance commissioner it is desirable to do so. The results of examinations are ordinarily thus available to the public, that representatives of the press and of competing companies may make use of them.

While the reports of state insurance officials may not be published each year, it is necessary for every life company to render an annual report of its condition to the state insurance department. Such reports are made out on blanks prepared by the state insurance department and contain information which the companies are compelled by law to submit. For many years the insurance commissioners of the various states have been working towards a uniform blank. The reports ordinarily contain a complete statement of the company's assets and liabilities, income and disbursements, the number and type of policies issued and terminated in various ways during the year, and the amount and kind of insurance in force. Concerning each of these parts of the report, specific and detailed information must be given. The reports are due early in the calendar year, ordinarily during the first quarter, and give information as to the condition of the company at the close of business on December 31. The information is abbreviated by the state insurance department and set forth in its report. Part of it, also, must be published by the company in one or more newspapers. The state insurance department must make periodical examinations of domestic companies (ordinarily every three years) and it may make special examinations in addition to these. In making periodical or special examinations, the representatives of the department may examine all books, papers and records, and all officers and agents, under oath, in order to appraise the company's assets and

determine its liabilities, as well as to learn whether its practices conform to the law and are in accordance with the best interests of the policyholders and of the public. The insurance officer may withhold a report of periodical or special examination from public inspection for such time as he deems proper, or he may publish it in one or more newspapers. In all instances, however, the company concerned must be granted a hearing by the insurance officer of the state, prior to his filing such a report, if the company so desires.

Standard Policy Provisions.—Although there are no standard life contracts laid down by law in any state, practically all states have established so-called standard provisions which must be contained in any life contract offered for sale within their respective jurisdictions. These are what might be termed "minimum provisions," in respect to liberality; that is, companies may use clauses granting provisions more liberal than those required by law, but not less so. Also, certain provisions are ordinarily prohibited by law. While state insurance laws differ as regards required and prohibited provisions, it is not uncommon to provide that no policy of life or endowment insurance shall be issued in the state before a copy of the form has been filed with the insurance commissioner, nor at all if that officer notifies the company that in his opinion, supported by reasons, the form of the policy does not conform to the requirements of law. The commissioner's action, however, is subject to review by any court having jurisdiction. One law further specifies that no life or endowment policy, except industrial policies where the premiums are payable monthly or oftener, shall be issued unless it contains in substance the following provisions:

1. Premiums shall be payable in advance.

2. A grace of thirty days or one month shall be allowed in the payment of premiums after the first.

3. The policy shall be incontestable after not more than 2 years from the date of issue, except for non-payment of premiums or for engaging in military or naval service in time of war without the consent of the company.

4. The policy (and application, if the company desires and so states in the contract, and attaches to the latter a copy of the application) shall constitute the entire contract between the parties.

5. If the age of the insured has been misstated, the amount payable shall be such as the premium would have purchased at the correct age.

6. Annual-premium participating policies shall participate an-

nually in the surplus beginning not later than the end of the third policy-year, except on certain forms (some states still permit deferred dividends).

7. The policyholder may elect to receive the dividends in cash, to use them to pay a part of the premium, or to purchase with them paid up additions to the policy.

8. The policyholder shall have certain options in case of default in a premium payment after 3 full years' premiums have been paid.

9. Loans shall be granted after 3 full years' premiums, equal to the reserve at the end of the current policy-year plus dividend additions, less a sum of not more than 2½ per cent of the total amount insured, including dividend additions to the insurance.

10. In case of default in premium payments after the latter have been made for 3 years, the policyholder shall be entitled to a stipulated form of insurance whose net value is equal to the reserve at the date of default plus dividend additions, less a sum not greater than 2½ per cent of the amount insured plus dividend additions to the insurance, and less any existing indebtedness to the company. In other words, a surrender charge of 2½ per cent of the amount of insurance is permitted. In the provisions for both loan and surrender values the company may insert in the policy a clause reserving the right to withhold the loan or cash value for a period not exceeding six months after application for a loan or surrender value has been made. Surrender values are not required in term policies of 20 years or less.

11. A table of loan values during at least the first 20 years of the policy, and a provision for reinstatement at any time within 3 years from date of default upon payment of back premiums with interest and evidence of insurability satisfactory to the company, shall be included in the contract.

12. Settlement of a claim shall be made by the company upon receipt of due proof of death.

Usually there is a proviso legalizing any policy provisions of foreign companies which may be required by the laws of the state or government under which the company is organized. Also, domestic companies may incorporate in policies issued in other states, provisions required by law in those states.

It is usually stipulated that the policy shall not contain provisions, (1) for forfeiture because of failure to repay any loan while the total indebtedness is less than the cash value; (2) for limiting the time within which an action at law or equity may be commenced to less

than 2 years after the cause of action shall accrue; (3) for ante-dating the policy more than six months before the original application for insurance was made; (4) for a mode of settlement at maturity of less value than the amount insured on the face of the policy, plus any dividend additions, less any indebtedness to the company on the policy and less any premiums that may be deducted according to the terms of the policy.

Practically all companies incorporate more liberal provisions than the ones enumerated above; most of them more liberal than those prescribed by states having the most rigid requirements. Among provisions commonly found in state statutes, but not enumerated above, is one to the effect that statements made by the insured in the application shall be construed as representations and not warranties. It is thus seen that while laws requiring certain provisions and prohibiting others may protect the policyholders against some of the evils in policy writing and construction which might exist if there were no such legal requirements, in practice they amount to little more than mere minimum standards.

Regulation of Surplus.—As has been shown above, one of the good results of state regulation and supervision in this country is the establishment of minimum standards of valuation which in turn require rates adequate to meet the hazards insured against and pay the expenses of management. Following the Armstrong Investigation of 1905, there was a strong demand for legislation with a view to limiting expenses and the retention of surplus by mutual or mixed companies which give the policyholder a share in the latter. It was felt that too large a surplus, accumulated from the payments of policyholders because of the company's declaring dividends smaller than was necessary, might be used for the benefit of others than the policyholders. For this reason it was deemed expedient to limit the contingency reserve to 20 per cent of the reserve liability, or $10,000 whichever is the greater, in the case of small companies, and to decrease the ratio to 7½ or 10 per cent if the reserve equals or exceeds $50,000,000. Pennsylvania permits a surplus or safety fund not exceeding 10 per cent of the reserve, or $100,000, whichever is the greater, and the excess of the market value of securities over their book value. In both instances, the state insurance officer may permit a company to hold more than these amounts for a period not exceeding one year under any one permission, by filing in his office a decision stating his reasons and causing it to be published in his next annual report.

There is no doubt that life companies should maintain a substantial surplus or safety fund to meet contingencies and prevent violent fluctuations in dividend schedules in case of unusual investment or mortality gains or losses. Since life companies, however, are not as a rule subject to violent fluctuations in losses, it is unnecessary for them to hold heavy contingency reserves. Too large a surplus can be piled up at the expense of policyholders, and, once accumulated, it may lead to the misuse of funds or extravagance in the conduct of business. None of the above discussion of surplus funds applies to companies granting only non-participating business.

Limitation of Expenses.—Limitation of the amount of surplus to be held would not suffice to protect the policyholders' interests in all instances, since companies might by mere extravagance in conducting business keep the surplus funds low and the costs to the policyholders unnecessarily high. Hence there has been a tendency toward regulation of expenses as well as of surplus. The famous Section 97 of the New York insurance law, and amendments thereto, providing for the limitation of expenses, has caused much discussion in insurance literature. In brief, this section originally limited initial expenses of large companies to first-year loadings, plus the present value of assumed mortality gains under select and ultimate valuation, plus the full gross premiums on policies issued and terminated during the year less the tabular net cost of insurance while in force, according to the select and ultimate basis. Renewal expenses were to remain within renewal loadings. Smaller companies were allowed greater freedom in the matter of expenses. Agents' commissions were also prescribed within certain limits. Foreign companies, including those of other states, were obliged to comply with these provisions of the act, in order to do business within the state. This (without taking up various amendments of the original section) furnishes an instance of state limitation of expenses.

Limitation of New Business.—New York also supplies an example in Section 96, in which companies were limited, as regards new business in any year, to certain percentages of the amount in force at the beginning of the year. The large companies were limited to smaller percentages than were the smaller companies. Section 96-a provides that the superintendent of insurance may suspend the limitation of new business, under certain circumstances, when he believes it to be justified.

Concerning limitations of the surplus, expenses and new business, it may be said that the tendency toward such rigid regulation grew

out of the Armstrong investigation in New York. It has not been manifest to so great an extent in many of the other states. Of late years, the tendency in New York has been rather decidedly away from the more rigid requirements. So far as the surplus is concerned, the results of the epidemic of influenza and of the World War with attendant circumstances demonstrated the wisdom of setting up a substantial contingency reserve or safety fund. As to the limitation of new business, great care must be exercised not to hamper the development of the institution of life insurance and the extension of its benefits to greater numbers. So long as the expenses for new business are kept within reasonable bounds and surplus funds are not depleted, there would seem to be little ground for interference. The wisdom of attempting to limit expenses is open to question. The company that pays high for business, but gets and keeps it, even though the cost to the policyholders is greater, may render a greater service to the community as a whole than some cheaper company. It is generally agreed that more insurance at a greater expense per unit than the minimum is desirable. The question resolves itself into what the limit should be, and whether the state should fix it. So far as companies granting only non-participating insurance are concerned, it would seem to be certain that the state should not attempt to limit their expenses. As long as there is keen competition, the cost to the policyholder will be kept down, and the stockholders may be relied upon to keep the surplus and expenses within reasonable limits. With mutual companies also the cost to policyholders must be kept low or old policyholders who are in health will withdraw and new ones will be hard to obtain. Some sort of effective supervision is desirable, however, to prevent those in control of companies from using surplus funds to further their own interests rather than that of the policyholders.

Election of Directors.—The election of the directors of stock companies granting only non-participating business is regulated in much the same manner as for ordinary corporations. For mutual or mixed companies in which policyholders may vote for all or a part of the board of directors or trustees, some states are inclined to regulate elections with great care. The most detailed regulation is that setting forth the voting powers of policyholders, supervising the election and counting of ballots, and generally insuring a fair and honest election. To give others than the administration an opportunity to canvass the electorate, the state insurance officer may, upon request of a specified minimum number of policyholders, require the company to file in his

office a copy of its list or card catalogue of the names and addresses of all policyholders entitled to vote. This may be used to canvass policyholders for votes. The board of directors must nominate candidates for vacancies, long in advance of the election. This nomination is known as the "administration ticket." If not satisfied with administration candidates, a group of policyholders may also nominate candidates of their own and distinguish their ticket from that of the administration. These nominations must also be made sufficiently in advance of the election to prevent taking the main body of policyholders by surprise and suddenly pushing candidates into office who might not be able to secure enough votes if their names had long been before the electorate and the wishes of a majority of the voters had been carried out.

Supervision of Agents.—Most states have passed laws defining an "agent," and regulating his appointment and conduct. An insurance agent is commonly defined as an individual, co-partnership or corporation, authorized in writing by a company to solicit risks and collect premiums in its behalf; and sometimes also to issue or countersign policies in its behalf. An agent is thus defined as being the agent of the company rather than of the insured person. An insurance broker is most frequently defined as, "a person, not an officer or agent of the company interested, who for compensation, acts or aids in any manner in obtaining insurance for a person other than himself." An insurance broker is defined as the agent of the insured person. The exact relation of a broker to an insured person, or an applicant for insurance, has been the subject of many court decisions. An agent must procure a license from the state insurance officer showing that the company represented by him has authority to transact business within the state, and that he has been duly appointed by the company named. Various qualifications may be prescribed with respect to age, business reputation and experience, knowledge of insurance law, etc. In a few instances, an examination must be passed before a person can obtain an agent's license. Licenses are usually issued for one year only, at the end of which time they may be renewed, and during which they may be terminated upon severance of business relations between the company and the agent or upon revocation by the state insurance commissioner for cause. An agent whose license has been revoked may appeal to a court having jurisdiction. All companies authorized to transact business in the state must certify to the insurance commissioner the names of all agents appointed to solicit insurance. No corporation can secure an

agent's license without first being authorized to engage in the business of insurance or real estate.

Agents are also required to exhibit their books and papers, to submit to examination under oath whenever the state insurance officer shall deem it necessary, and to comply with the insurance laws generally. Practically all states prohibit, under penalty, embezzlement, fraudulent conversion or wrongful use of premiums collected. It is also customary to prohibit, under penalty, the rebating of any part of a premium, or offering any inducement, favor or advantage not specified in the policy in order to persuade an individual to take the latter. In many instances a penalty for rebating applies to the person accepting the same, as well as to the agent or company granting it. Agents are also prohibited from making any offer or promise concerning the purchase or sale of stock, bonds, or anything of value, directly or indirectly, as an inducement to insurance. The making of any false statement, or misrepresentation on the part of a prospective purchaser with a view to securing a policy, or on the part of one trying to collect its proceeds, is likewise prohibited. Most states also have an anti-twisting statute; this is a statute designed to prohibit an agent, or other person, from prevailing upon an insured person to surrender a policy in one company and take one in another. The provisions of one state law in this particular are that no one, "shall make any misrepresentation or incomplete comparison of policies, oral, written, or otherwise, to any person insured in any company, for the purpose of inducing or tending to induce a policyholder in any company to lapse, forfeit, or surrender his insurance therein, and to take out a policy of insurance in another company insuring against similar risks."

While apparently applicable to forms of insurance other than life, it is against twisting in life insurance that the law is directed. This provision, however, as well as anti-rebating legislation, is difficult to enforce, and few cases have appeared, even in life insurance.

Some states prescribe many additional regulations, such as pertain to information contained in any advertisements of the agency, advancing of money or canvassing votes for elections of directors or trustees, compensation, loans or advances to agents by the company, bonuses, prizes, etc. Concerning rebating, some states make it illegal for any employer to accept a commission or part of a premium for an insurance on the life of an employee, or for an employee or attorney to accept remuneration because of insurance on an employer or fellow-employee.

Taxes and Fees.—A great variety of fees are collected from insurance companies or their agents in the various states. For instance, one state collects fees for the valuation of policies, for filing a copy of its charter, for filing annual or other statements, also for a license to a company, or to an individual as an insurance broker, or to a firm as brokers; for each copy of any paper filed in the state insurance department, for any other certificate required, and for service of process. Taxes may be based on all or a part of the company's assets, on net receipts, or on gross premiums. By far the greater number tax gross premiums, although dividends to policyholders may be deducted in some instances. With some companies, Federal taxes constitute a very heavy item of expense. The tax laws of the various states are by no means uniform, and many discriminate in favor of domestic companies. Such discrimination has led to much retaliatory legislation.

The extent to which life companies should be taxed has long been a subject of controversy in this country. Most authorities agree that they should be taxed enough to pay the cost of proper supervision. Further taxation is difficult to justify in view of the inherently beneficent nature of the business. Life insurance men say the business should not be taxed. Legislators, on the other hand, see an opportunity to tap the large financial resources of the companies without stirring up serious opposition among the voters. Companies have never been able to induce their policyholders, upon whom the burden of taxation falls, to take an active interest in tax reduction. An effort of one state to retaliate in the case of discrimination against one of its companies by the legislation of another state and the mistaken belief that foreign companies take millions out of the state never to be returned; as well as the failure to understand the true nature of life insurance, account for many of the taxes imposed. Perhaps companies might accomplish something by enclosing a brief statement to the policyholder in each annual premium bill or dividend notice setting forth that his dividend would be about $—— greater were it not for taxes, etc., together with a few reasons why life insurance should not be taxed and an exhortation to him to communicate with his legislators concerning his views and demand action in the matter. The average policyholder, however, would probably reason that the tax-gatherer would reach him in some manner anyway, and that taxes on life insurance are no more unjust than are many others. At a recent date, the taxes of several large companies operating in many states amounted to approximately $1 per $1000 of insurance in force

per year. Total taxes and fees by life companies were recently estimated at about $35,000,000 per year.[1] The total insurance in force was $45,983,000,000,[2] making the taxes approximately 76 cents per $1000 of insurance. In practice, the tax situation seems to have little effect upon the growth of the business, although there is no more reason for taxing life insurance than for taxing building-and-loan associations and savings banks. It has been said that a tax on life insurance is a "tax on death," or a tax on thrift, or both, the principal burden of which is borne by widows and orphans; these could have had greater protection had there been no tax.

Formation, Development, and Management of New Companies

To form a new life insurance company, a minimum number of natural persons, such as ten or more, must draw up articles of agreement setting forth the name, the class of insurance, the plan for conducting business, the location, capital (if any), duration, powers, and general purposes of the proposed company. Upon complying with many technicalities of the law governing the formation of corporations in general, or of insurance companies in particular, a charter may be secured and business commenced.

In recent years there have been many new companies formed, nearly all of them on the stock plan, for reasons previously discussed. To provide for immediate working capital, the stock is nearly always sold at a premium. Thus each $100 share may be sold for $150; the $50 creating a surplus from which the expenses of selling stock and of organization may be paid. After meeting the expenses of formation and retaining intact sufficient capital to meet the guarantee requirements of the law, the company is in a position to do business. Since a new company has very limited resources with which to pay salaries, the problem of securing an adequately trained staff is a most serious one. It is essential to have one officer responsible for home office work (he may do much of it in person), and another to write business and to secure an agency force. Next in importance come the medical director and the actuary, who are sometimes engaged to devote only a part of their time to the company. The home office manager, who is usually the president, and the actuary can take care of the routine work until the company has a considerable amount of insurance in force. After that, a clerical force not exceeding one clerk to each 1½ millions of business in force may be added as the company grows.

[1] Huebner, S. S., Life Insurance, Revised Edition, p. 378.
[2] *Ibid.*, p. 5.

The medical director may examine all applicants personally, and thus save examiner's fees, until the business has grown to considerable proportions or has expanded to localities other than those in the immediate vicinity of the home office.

In the beginning, the problem of developing an agency organization and securing business is a most important one, and the one most difficult to solve. The field executive is in much the same position as a man trying to build up a general agency in a new territory. He must exercise judgment as to the amount of time to spend on personal soliciting and on securing other agents. If he can secure the services of other agents who have sufficient experience and capital, and are willing to undertake the development of agencies in various towns for reasonable commissions, the company will be fortunate indeed. Of course, business cannot be put on the books too rapidly, even if it should be possible to write it, because of the heavy initial expenses, as explained above in the chapter on the loading. Generally the most severe test of the agency organization comes after the first year when the initial enthusiasm and interest has been spent.

Valuation Basis.—Unless a new company has an unusually large capital and surplus contributed by the stockholders, it will not operate on a higher valuation standard than the American Experience table with interest at 3½ per cent, and even with this standard it cannot commence business on the full net level premium valuation basis. It is customary for a young company to evaluate on some sort of modified preliminary term basis, such as the Illinois or other standard. Too high a basis will prohibit the company from spending enough to obtain a fair amount of new business, and too low a one is likely to lead to extravagance in management. The cost of new business must not be permitted to become greater than its worth to the company. The select and ultimate basis is not ordinarily satisfactory for a new company. A company might do well to operate on the Illinois standard for the first ten years, then change all business to the modified preliminary term (ordinary life) basis, and later on adopt full level-premium valuation, if such is the ultimate aim. The Illinois standard is safe enough, however, and might be continued with satisfactory results for many years. To adopt it as the basis for the first few years is a most expedient measure.

Mortality.—The mortality likely to be experienced by a new company depends on a number of elements that are not present to any appreciable extent in the case of an older and larger company. In the first place, there is likely to be a very marked selection against

the company, unless the officers guard against this very closely. In the desire for new business, too, there is a great temptation not to be too particular in the selection of risks. Once it becomes known that a company is not rigid in the matter of selection, agents will be prone to shift applicants of doubtful insurability to it. In the next place, the normal fluctuation in mortality is likely to be greater here than in a large company; as the company grows this will tend to decrease. It has been learned that an average company of small or moderate size will be fairly well protected against normal mortality fluctuations if it provides for maximum losses of 75 per cent of those expected by the American Experience table.[3] Excessive fluctuations, such as those due to epidemics of disease or other grave disturbances, occur about once in a generation, and these must be provided for by setting aside a mortality fluctuation fund which will take care of losses up to at least 100 per cent of those expected according to the American Experience table. If the mortality fluctuation fund cannot be set aside at once, it may be built up gradually over a period of, say, 20 or 30 years. It is held by some that provision for normal mortality of 50 per cent of the expected, according to the American Experience table, and a fund for fluctuations built up by setting aside 10 per cent of the expected, is sufficient for the first 10 years.

If care in selection is exercised, a newly formed company may expect considerable mortality salvages during the early years, even after adequate mortality fluctuation funds have been set aside. These may be used to meet part of the heavy expenses incident to getting new business. As the company grows older, there may be a greater proportion of old business on the books, in which case the mortality gains will be of less consequence, hence a new company should strive for a large volume of business as quickly as possible, if this involves only a partial depletion of surplus funds.

Interest.—In the early years, a company has little to invest, but it may secure a high net rate of interest on its investments. As it grows older, interest earnings in excess of the assumed rate become a more important factor in the success of the company, and tend to neutralize an increase in the ratio of actual to expected losses. Whether the bulk of the company's funds should be invested in bonds or in mortgages depends on the conditions of the times. If real estate prices are inflated, it is unwise to invest in mortgages. On the other

[3] Halliday, W. R., in a paper, "On Some of the Problems of the Smaller Life Insurance Companies," T.A.S.A., XXII, 13 et seq., states the minimum provision as a maximum mortality of 75 per cent of the expected by the American Experience table, rather than maximum losses.

hand, conditions may be such as to indicate uncertainty concerning the stability of bonds. Good judgment and common sense are required in managing investments. The temptation to invest in less stable securities, peculiarly powerful at times when it becomes apparent that high interest earnings cannot be maintained, should by all means be avoided. It is wise to set aside a safety margin to provide for fluctuations in the value of assets, and for losses on investments. Amply secured mortgages do not fluctuate much, but some losses must be expected each year after the assets have attained considerable proportions. Those conditions which produce an unusually high mortality rate are likely to result in losses in investments. If losses on investments occur, they are likely to be more serious with a small company than with a large one; hence, not only must greater care be exercised in making investments, but also a larger proportion of surplus funds should be set aside to meet such contingencies.

Maximum Amount.—The maximum amount of insurance which should be written and retained on one life by any company depends on a number of considerations which are discussed in the chapter on reinsurance. A newly formed company should keep the maximum insurance on a single life low, possibly as low as $2500 at first, although $5000 is considered more practicable by some authorities. Mistakes are made despite careful selection, and the company must be particularly careful to avoid large risks, in view of what has been said above concerning adverse selection and mortality fluctuations as experienced by small companies. Later the company might raise its limit to $5000 or $10,000, and then increase it by $5000 at a time, waiting several years between increases in each instance. Since the effect of mortality fluctuations may result in more serious fluctuations in losses when larger amounts are at risk on single lives, care must be exercised not to make the maximum amount high enough to cause an undue strain on the surplus if a number of the larger claims should suddenly arise. Reinsurance may, of course, be secured if it is necessary to accept large risks in order to satisfy the agency force, although the young company may not make any money on the reinsured portions.

Local Conditions.—Newly formed companies are in nearly all instances obliged to confine their underwriting activities to the immediate locality in which they are founded. It is only after they have been in existence for several years that they are able to extend their efforts to broader fields. Hence the peculiar conditions in the locality of the home office are of vital importance to the company, and a

knowledge of them essential to its success. The cost of securing business varies with different communities, according as the people are better educated, and able to appreciate and pay for the benefits of life insurance. There are differences, as well, in health conditions and in mortality rates at similar ages. Even a community's success in securing of an adequate supply of good water may influence the mortality experience of the company. These factors must be properly considered by those who would be successful developers of a new life company. An additional consideration is the modern military practice of placing all the men of a community in the same regiment, so far as feasible. The persons insured in a new company are likely to be of military age, and physically fit for service. A single battle might thus prove disastrous to a small company, even if it charged extra rates for military service. It is possible to protect the company against such a contingency by inserting a war clause in the policy requiring the consent of the company for entrance into military service in time of war. Then instead of giving consent upon the payment of extra premiums, the liability of the company may be reduced by placing a military service lien against the policies of those entering the service, or by other means if liens are prohibited by law.

Expenses.—A conservative policy governing home office expenses is desirable, but economy should be observed in the number of employees rather than in the size of salaries. An able administrative officer, proper technical and professional advice, good business sense, honesty, and continual striving are essential. Agency expenses will, of course, be very high, since it is usually difficult to sell the policies of a new company, and the loading cannot be too great or the premiums will be so high that competition cannot be met successfully. Some authorities hold that 80 per cent of the first premium may be allowed for first-year agency expenses,[4] to which must be added about 10 per cent to meet the medical examination and other expenses incident to getting new business. Then at least 5 per cent should be allowed for renewal commissions. Others hold that such an arrangement is likely to attract "high pressure" salesmen, or "floaters," thus necessitating much care in selecting agents and in eliminating those of inferior ability or questionable character, and resulting in heavy lapses at the end of the first year. They suggest that not over 75 per cent of the first premium should be allowed for initial expenses, including medical examination and selection, and that higher renewals should be allowed, not to exceed, however, 10 per cent for the first

[4] Halliday, W. R., in T.A.S.A., XXII., 14. See also, Graham, Geo., in *Ibid.*, 30.

9 renewal years. An initial commission of 65 per cent, graded, and 9 renewal commissions of 10 per cent should be sufficient. The percentages may, of course, differ with the premium rates. If the company should decide to issue participating policies, the gross premium may be higher, and the commission rate lower. Advances to agents, allowances for rent, traveling expenses, or salary should be avoided, and the agents' compensation put on a straight commission basis, or care must be taken to eliminate such fixed charges as soon as production falls below the amount on which they are based.

In an effort to secure new business the capital and surplus will ordinarily be diminished during the first 4 or 5 years of the company's history, but thereafter there should be a gradual increase. As the company develops, new business expenses should decrease and dividends to stockholders may be paid. It is not considered wise to pay much more than legal interest as a dividend to stockholders before the end of 10 years. Stockholders should be informed of this at the time of subscription.

Home office expenses should be provided for at the rate of $5 or $6 per $1000 of insurance in force. Some authorities think this is too high, and hold that $3 or $4 is enough.

After determining the amounts to be set aside for safety funds, home office, and other expenses, including some provision for dividends to stockholders, and arranging to set up the reserve liability according to the Illinois standard, the managers of the company may encourage the agency force to write all the business it can. $500,000 of insurance the first year, $800,000 the second, $1,500,000 the third, and thereafter an increase in volume of 20 per cent or 25 per cent, is a worthy and not impracticable aim of a new company.

Lapses.—There is normally a higher rate of termination by lapse in a new company than in one of substantial growth. The compensation of the agent and the latitude allowed him in settling with the company have a bearing on lapses. Excessive commissions and settlements at a longer period than the customary 60 days are likely to result in the agent's extending too long credits for first premiums to persons of uncertain financial responsibility. A higher termination rate than that anticipated will, of course, defer the time when the surplus will start to increase.[5]

Among the more serious obstacles to be overcome in starting a new company are: (1) lack of knowledge of insurance practices on the part of its personnel, (2) hostility of competitors and prejudice

[5] Graham, Geo., in T.A.S.A., XXII., 30.

of the public, (3) frequent acceptance of poor risks due to lack of access to the impairment records of the larger companies and to intense desire for business, (4) difficulty of securing first-class agents, (5) difficulty and expense of securing new business.[6]

Mutualization.—As the amount of insurance increases and the company extends its activities to wider areas, the agency and home office organizations develop in size and efficiency and surplus funds accumulate. It is now possible, if a majority of the stockholders so desire, to take steps toward mutualization. The expenses of organization and the state deposit requirements have long since been met. The company is now a going concern, and its capital stock insignificant in proportion to its total liabilites. It is no longer necessary as a guarantee of safety or to meet expenses, and may be more of a hindrance than a help from the policyholder's point of view. If it was the original intention to form a mutual company, but to provide a guarantee capital during the early years, it may be provided that the policyholders shall have the right to purchase the stock and retire it according to certain stimplated terms whenever the surplus is sufficient to do so. If there has been no such original intention and stipulation, it is necessary to effect an agreement between the policyholders and the stockholders whereby the policyholders may purchase the stock with a portion of the company's surplus funds. In a pure stock company, the surplus belongs to the stockholders, whereas in a mutual company it theoretically belongs to the policyholders, although, of course, the company is under no legal obligations to a policyholder other than to carry out the terms of his contract. In purchasing the stock, the policyholders authorize the transfer of a part of the surplus to the stockholders.

By this time the Illinois or other standard should have been discarded for a higher one, if the aim is full level-premium valuation before mutualization takes place. In the absence of guarantee capital, full premium valuation has been considered the better basis. It tends to stimulate efficiency and keep acquisition expenses down, and results in showing a lower surplus. The latter is advantageous in that it reduces the temptation to pay unduly high dividends and allows maximum safety under surplus limitations by law.

Stockholders will ordinarily be content with less than all of the surplus as the price they receive when the company is mutualized, since they would receive only so much of it even if the company were not mutualized. In the more important states, so far as life insurance

[6] Watt, Arthur, in T.A.S.A., XXII., 29.

is concerned, mutualization may take place only with the approval and under the supervision of the state insurance officer. If a minority of the stockholders refuse to sell their stock, the majority may be purchased by the policyholders and placed in the hands of trustees who hold it for the policyholders' benefit, vote according to their directions, and pay dividends into the general funds of the company, except the dividends to which the minority stockholders are entitled. Companies controlled in this manner are mutual for practically all purposes, but remain technically stock companies until all of the stock has been purchased and retired by the policyholders.

The management of larger companies is ably discussed in a number of works, particularly in S. S. Huebner's Life Insurance, Chapter XXV.

CHAPTER XVI

Reinsurance in Life Insurance

Nature and Uses.—In life insurance, the need for reinsurance arises from two sets of circumstances. In the first place, a company may wish to retire, or it may become legally insolvent, in both of which cases its business is usually reinsured in bulk in another company. In the second place, a company may issue a larger amount of insurance to an individual than it cares to retain at risk on one life, hence may desire to reinsure a part of the risk in some other company, or companies.

Until recently, a person desiring a large amount of insurance was obliged to take a number of policies in as many separate companies, each with premium payments and policy provisions differing from the others. The agent, in negotiating many policies on the same life, was obliged to obtain through agents of other companies, all contracts but that of his own company. He, therefore, seldom received the full agent's commission except on the policy placed with his own company. Only in the event that other agents should reciprocate with him when they might have opportunities to write large insurances, would he receive approximately the same commission as would have been the case if his own company had issued a policy for the entire amount of insurance desired by his client.

With reinsurance facilities, however, an individual seeking insurance is enabled in many instances to secure as much protection as he desires in the company with whose standing and practice he is most familiar and most satisfied. The direct writing company is able to satisfy the needs of its customers, spread its large risks in such a way as to take advantage of the operation of averages, and build up and accommodate its agency force by taking care of large policies, thus avoiding for its agents the necessity of seeking partial connections with other companies for placing excess risks. (It is still the practice in many instances, however, when the amount of insurance desired by an individual is very large, to take policies in a number of companies.) The reinsuring company renders a distinct service

in furnishing such facilities, and may derive a reasonable profit from business so obtained. With the continued increase in the number of large policies, reinsurance of excess liability is a virtual necessity, especially for the small and moderate-sized companies. Concerning bulk reinsurance, in most instances it is better for all concerned to reinsure a failing company's business in an experienced and carefully managed one, than to continue such business in the hands of a receiver.

Reinsurance in Bulk.—The importance of reinsurance in bulk may be realized from the fact that within the past decade over 80 American life insurance companies have retired by reinsuring their business in other life companies. In some instances companies voluntarily reinsure their entire business. In others, where insolvency has occurred and companies are placed in the hands of a receiver, or of the state insurance department acting as receiver, the business is usually reinsured. In a few instances the receiver continues to accept premiums and administer the business of old policyholders, no new business being written. The latter arrangement may be resorted to when no favorable reinsurance agreement can be made.

Reinsurance in bulk is usually accomplished in one of two ways. According to the first method the total liabilities which the reinsuring company must assume are determined, and from the sum of them the purchase price which the reinsuring company agrees to pay is deducted. The balance represents the sum which the retiring company must transfer to the reinsuring company in the form of cash or selected assets. The second method consists in the reinsuring company's purchasing the capital stock of the retiring company, thus assuming all of its liabilities and all of its assets.

Whatever the method by which reinsurance is effected, the legal liabilities assumed by the reinsuring company are usually in excess of the real value of the assets received from the retiring company. Such excess is the price paid for the business, and it is frequently stated as so much per each $1000 of insurance transferred. The price paid depends on a number of considerations. So far as the retiring company is concerned, the price it may command is usually determined by competition among other companies for its business, since the retiring company is nearly always in poor financial circumstances. It is virtually compelled to cease activities, having spent, usually, too much money in the acquisition of business.[1] Of course, there is always a minimum price below which an insolvent company would not be reinsured, the receiver or state insurance department preferring

[1] Blehl, E. M., T.A.S.A., XXII., 115.

to manage the business for old policyholders rather than reinsure under unfavorable terms. The amount which a prospective reinsuring company may offer depends on one or more of a number of circumstances which may influence the minds of its managers. A company may offer to take over the business of a retiring one merely to serve the latter's policyholders, when it is reasonably certain that no loss will result. It may merely wish to uphold the record of life insurance as a safe financial institution. Again, it may be influenced primarily by the desire of a state insurance department. The prestige too of the reinsuring company may in this manner be enhanced, and the volume of its business greatly enlarged. Another consideration is the possibility of acquiring the agency staff of the retiring company, although it is to be expected that quite a number of agents will refuse to join forces with the new company. While one or more of these factors may influence the prospective reinsurer to make an offer for the business of a retiring company, in most instances it is probable that the expectation of a profit on the acquired business is the final and determining reason for making an offer. In estimating this, the type of business, the ratio of actual to expected mortality, the length of time the bulk of the business has been in force, must be considered.

Sources of Profit.—When the price paid is such that the legal liabilities assumed by the reinsurer are greater than the assets transferred, it would seem at first thought that the reinsurer could not expect to make a profit. Legal liabilities on outstanding contracts, however, are determined by net valuation. While not all of the difference between legal and gross valuation can be saved, since expenses must be met, nevertheless there is a margin worthy of consideration. When non-participating business is acquired, the reinsurer may retain, (1) all of the net excess interest earned, (2) the mortality salvages, (3) the loading salvages, (4) the surrender charges on discontinuances. Of course, expenses including taxes and commissions, if the loading should prove insufficient to meet them, must be paid before a profit can be realized on any of these items. The loading, however, is usually more than sufficient to meet these expenses. The plan of insurance, the gross premiums charged, the scale of commissions paid, the age at entry and the duration of policies in force at the time of reinsurance, must be known if an accurate estimate of profits is desired. To reduce the amount of labor involved in this estimation, it has been suggested that plans and ages be grouped, those entering under age 21 being placed in one group, those entering between ages 21 and 25, 26 and 30, etc., in separate groups. The mean age at

entry of each group may then be taken and the amount of ordinary life, 20-payment life, 20-year endowment, and term insurance, set opposite each mean age. The amount under each plan as well as an approximation of amounts at different ages of entry will then appear.[2] Average net premiums, net values, net expected mortality according to the American Experience table and an estimate of the probable actual mortality in the future may then be made for different policy-years and for the different plans. It is then possible to estimate the future profits from excess interest, mortality and loading salvages.

In estimating future mortality salvages, the past mortality experience, the geographical distribution of the business to be taken over, the proportion of rural and urban policyholders, the standards by which risks were selected, as well as the personnel of the agency force and medical staff of the retiring company, should be studied with care. Also, due allowance must be made for increased lapsation which nearly always follows a reinsurance transaction and results in an unfavorable mortality experience for at least one or two years after the transfer. There are a number of reasons for the unfavorable mortality experience following reinsurance, the principal one being the displeasure of the policyholders at the failure of the retiring company. The healthy ones are apt to drop their insurance and those of doubtful health go over to the reinsurer. Young companies, also, which are inexperienced in the selection of risks often show a relatively high mortality rate, and business reinsured is almost invariably that of a young company, usually not more than 10 years old. Loading salvages will ordinarily be less on reinsured business, since renewal commissions may be higher, and the home office expenses will be greater than for the reinsurer's regular business. Records of reinsured business must always be kept separate if for no other reason than that policy numbers will be the same as some of the numbers of the reinsurer's policies. Reinsurance also requires much attention during the first two years to counteract excessive lapsation. Profits from lapses and surrenders are negligible on ordinary and 20-payment life policies after only two premiums have been paid. The greater profits on 20-year endowment policies arise when they are lapsed after one or two premiums have been paid, since the surrender charge is generally very small thereafter. Term insurance ordinarily shows a greater profit from this source if lapsed after a considerable number of premiums have been collected. The practice of the retiring

[2] Rydgren, Adolph A., Value of Business Reinsured in Bulk, in Transactions, Actuarial Society of America, Vol. XXII, p. 84.

company concerning the surrender charge will be the determining factor in profits from this source in each case under consideration. It is probable that in practice an estimate of profits from surrenders and lapses is seldom made by a company about to bid for bulk reinsurance.

Opinion differs as to the number of years' profit to be discounted in arriving at the present value of profits expected in the future. Some authorities maintain that the whole term of the insurance should be discounted. The more conservative opinion is that the profit of a period of from 5 to 10 years' duration is all that should be considered in making the estimate. The purchase price usually comes from the surplus of the reinsuring company, being considered as a reduction of it in that liabilities in excess of assets are assumed or that stock is purchased, or the purchase money is borrowed from other financial institutions. It is desirable to replenish the surplus, or repay the loan, within a few years. Again, any additional profit which the reinsurer ultimately expects to make should also be realized within a reasonable length of time.

Profits on Participating Business.—It would seem that no profit could be made by reinsuring participating business since any saving should be returned to the policyholders. One authority [3] states that when a prosperous company reinsures participating business, if care is not exercised it will either discriminate (or appear to discriminate) against the reinsured business or else pay more to it than it is earning. Dividends should not be reduced nor kept low merely to enable a reinsurer to make a profit, yet there is at least one source from which legitimate gains may be realized on reinsured participating business. As explained in a previous chapter, it is customary to prepare a schedule of gradually increasing dividends. The initial expense charge is often spread over the entire premium-paying period as a uniform addition to each premium. As a result, the actual expense incurred because of reinsured participating policies will be less than that provided for in the loading. It may be less by as much as $2 or $3 per $1000 of insurance each year.[4] It was not the original intention of the retiring company to use this to increase dividends, and there seems to be no reason why the reinsurer should not use it to replenish its surplus, which probably suffered when the purchase was made, or treat it as an addition to its general surplus just as it would any other profit. In fact, if this gain could not be retained

[3] Mr. Robert Henderson, in T.A.S.A., XXII., 117.
[4] Rydgren, A. A., in *Ibid.*, p. 105.

by the reinsuring company, the principal reason for acquiring participating business by way of reinsurance in bulk would appear to be that of gaining prestige and good will. Deferred dividend policies may or may not yield a profit to the reinsurer and are more often subject to litigation by a dissatisfied policyholder than are annual dividend policies. Companies may wisely hesitate to offer a price for such business.

It is pointed out by Mr. J. B. Maclean [5] that the complications involved in determining the value of surrenders and lapses are probably too great to be overcome in practice, and that a form of gross valuation may be used to approximate the price which a reinsurer may offer. Mr. Maclean states: "If a valuation is made on a 'true' basis throughout, i.e., as to mortality, interest and premium actually receivable, the difference between the liability so brought out and the actual reserve that must be set up by the reinsuring company (not necessarily the same amount as that held by the retiring company) increased by the present value on the 'true' basis of the difference between the 'true' and valuation premiums will represent the 'price' paid for the expected profits from interest, mortality and net loading, or, in other words, will be the present value of such future profits." [6]

Protection of the Reinsuring Company.—The reinsuring company must be on guard against a number of contingencies which may arise. The value of business reinsured in bulk will depend largely on the percentage that remains in force with the reinsurer. The mortality rate will always be somewhat uncertain, although quite a number of companies have not had an unfavorable experience in this regard. An instance of reinsurance in bulk, which attracted much attention at the time of its occurrence, is the reinsurance by the Metropolitan, of the defunct Pittsburgh Life in 1917. It was left optional with any policyholder whether the Metropolitan should assume his policy, but 77 per cent of insurance and 89 per cent of the reserve went over to the books of the reinsurer.[7] When a company merely assumes the policy obligations of another, and takes over funds to meet the reserve liabilities on them, the matter is not so highly complicated. When one company takes over another through purchase of a controlling interest in its stock, however, the matter is somewhat speculative in character and close scrutiny is necessary to avoid difficulties. For instance, since reinsurance is usually considered only after a company

[5] T.A.S.A., XXII., 112.

[6] Mr. Maclean then cites Mr. Lidstone's paper in the Journal of the Institute of Actuaries, Vol. XXXII.

[7] Craig, J. D., T.A.S.A., XXII., 108.

has encountered financial troubles, a close examination is likely to reveal hidden obligations. Furthermore, policyholders, claimants or agents may be threatening suits which would be hastened by a move to reinsure. The objection of a single stockholder may cause much difficulty unless the retiring company is insolvent and the receiver has been ordered by the court to reinsure the business. Unscrupulous persons may have gained control of the retiring company for exploitation and liquidation, assets may have been overvalued, payments on mortgages may be in default and foreclosure would result in a loss, real estate taken on foreclosure may appear on the books at a price higher than the market value, bonds held may have depreciated, deposits may have been made in banks which are now insolvent. There have been legal decisions against reinsurance in bulk without the consent of the policyholders, some of which have returned to withdrawing policyholders all premiums previously paid.[8] Mr. Blehl suggests that some of these uncertain features can be eliminated if at first merely the insurance, and preferred assets to cover the reserve liability on account of it, be transferred to the reinsurer, the retiring company staying in business for a short time to liquidate remaining assets and to meet liabilities other than policy obligations. This method, however, is not likely to prove as satisfactory to the retiring company as either of the two methods commonly employed.[9]

Reinsurance of Excess Liability

Development.—Excess reinsurance, whereby the original or direct underwriter may transfer that part of any risk assumed by it which exceeds the normal amount customarily retained, has been extensively practiced in fire and marine insurance for many years. Though as long as a half century ago there was need occasionally in life insurance for reinsurance of a part of an individual risk, and as early as 1849 the Associated Scottish Life Offices entered into an agreement governing reinsurance,[10] it is only within recent years that the practice of excess reinsurance in life insurance attained any appreciable proportions in the United States. In earlier times, companies did not have the same confidence in one another as they now have, and instances in which a large amount of insurance was desired on a single life were comparatively rare. The growth of business

[8] Blehl, E. M., in Transactions, Actuarial Society of America, Vol. XXII, p. 114.

[9] Rydgren, A. A., T.A.S.A., XXII., 118.

[10] Moir, Henry, Reassurance of Surplus Risks, T.A.S.A., VII., 308.

enterprises, the increased use of life insurance for business purposes, greater experience on the part of life underwriters in the matter of selection, and greater confidence in themselves and in each other, has made possible the development of reinsurance facilities so that at the present time large policies may be issued with safety and reinsurance of standard risks may be accomplished with ease.

While it is probable that toward the close of the past century our larger companies had some sort of facilities for reinsuring excess lines in foreign companies, the first real advance in our life reinsurance came in 1903 when certain German companies entered this country for the specific purpose of accepting reinsurance. They had no agency force and made no attempt at direct underwriting, but they were able to accept individual reinsurance of large amounts because of their connections with other companies abroad. The principal innovation introduced by them was that they made it possible for companies to cede reinsurance in large amounts without obligation to accept any reinsurance as a return favor. Until this time, also, many American companies hesitated to assume responsibility for a part of the risk of another from fear of adverse selection.

The German companies were obliged to cease doing business here after the United States entered the war in 1917. But the great growth of life insurance following the war, the general prosperity of the country, the increased use of life insurance to protect corporations against loss of officers and other valuable employees, and the use of life policies to meet inheritance taxes, led to an unprecedented demand for insurance of large amounts. As a result, well established American companies have supplied the deficiency occasioned by the withdrawal of the German reinsurers by effecting agreements among themselves to assume each others' excess risks. They also assume excess insurance written by smaller companies without expecting the smaller ones to reciprocate. Some American companies, however, do not reinsure, preferring to limit the amount which they will write on a single risk. At the time of the withdrawal of the German companies in 1917, it was difficult for an American company with a $50,000 retention limit to obtain satisfactory reinsurance for another $50,000 in order that one of its agents might write $100,000 of insurance on a single life.[11] Within five years from that date a good company with a maximum limit of from $5000 to $100,000 could readily obtain reinsurance for at least as much as it retained, and on much more

[11] Bagley, W. N., and Laird, J. M., Life Reinsurance, in Transactions, Actuarial Society of America, Vol. XXIII., p. 65.

favorable terms than was formerly the case. Many companies have been able to increase the amount accepted on one life. Reinsurance pools involving a number of companies have been formed to exchange reinsurance among themselves, and a movement is now under consideration in some of the larger pools to simplify the work by operating through a central receiving and distributing bureau.[12] With the increased demand for large policies which it is reasonable to expect in the future, reinsurance should attain even greater significance as a means whereby life companies can render still further service to the insuring public.

Purpose.—The principal purpose of a company in reinsuring a portion of its large amounts at risk is to avoid the encroachments on surplus funds which might result from accidental fluctuations in the mortality rate among those heavily insured.

Elements to be Considered.—A successful plan of reinsurance requires that each company shall have implicit faith in the integrity of the others involved. Mr. Henry Moir points out [13] that the slightest evidence that one of the companies is trying to take advantage of another is likely to result in the cancellation of the reinsurance contract. Experience demonstrates that the most satisfactory results are obtained when the original company retains a substantial part of each risk, and when the reinsurer follows the policy conditions and rules of the original underwriter.[14] The same applies to reinsurance in marine and fire insurance. When reinsurance agreements are so drawn as to be profitable both to the original company and the reinsurer, and when each makes every effort to protect the other, both interpreting the contract in a liberal spirit, there is little likelihood of any difficulty.

Mortality.—While some companies have had an experience to the contrary, it is quite definitely established that the mortality on policies for large amounts is greater than on those of moderate size. Since reinsurance is applied to large policies a mortality rate higher than the average may be expected on reinsured business, and on the parts of reinsured policies retained by the original company. The question, then, as to what constitutes a "large policy" immediately presents itself. A large company with ample surplus funds might view a \$50,000 policy as a normal risk, whereas a smaller institution might look upon a \$25,000 policy as relatively large. It is impossible to

12 *Ibid.*, 66.
13 See T.A.S.A., XXIII., 81.
14 See Bagley and Laird, *Ibid.*, 29.

define a large risk as one in excess of any specified sum. For purposes of reinsurance, Messrs. Bagley and Laird define a large risk as one in excess of the company's limit of retention. Mr. M. D. Torrey states,[15] that the amount which a company can retain depends on the amount of insurance in force, the average size of its policies, and its surplus. As pointed out in a previous chapter, the element of judgment must enter into the final determination of the maximum to be placed at risk on one life. Hence a conservative company might establish a limit of $10,000, whereas another of approximately the same size and strength, but a little more venturesome, might retain as much as $25,000. According to the definition a $20,000 policy would be considered a large policy in the one but not in the other. Nevertheless, the retention limit shows what the company considers a large policy. It is the company's estimate of the maximum that can be retained without undue fluctuation in surplus because of one or more "limit" losses and without diminishing the profit to the company because of the extra mortality likely to result from retaining an undue share of large risks. It is thus an important factor in estimating the probable mortality on reinsured business. For instance, suppose a company will retain $100,000 on the ordinary life or endowment plan at age 30, but only $50,000 on these plans at age 60, and, say, $25,000 on term policies at any age. Obviously, unfavorable mortality on larger policies is thrown on the reinsurer. If the original company finds its mortality is more favorable than that expected on term, and less so on life at age 30, it may vary the retention limits, subjecting the reinsurer to unfavorable mortality due to this adverse selection. If the original company retained but $5000 and reinsured $195,000 of a $200,000 policy, the burden of extra mortality will fall heavier on the reinsurer than if the original company retained, say, $100,000 of the risk. Also, if the original company retains a large part of each risk, it will be stimulated to exercise more care in selection.

In the Specialized Mortality Investigation, mortality on policies of $20,000 or more was shown to be 108 per cent of the standard. The American Men Experience shows a mortality of 117 per cent on policies of $50,000 or more, and 131 per cent on those of $100,000 and upwards. Various reasons have been advanced to account for such experience. The more valid ones probably are that large policies are frequently written on the lives of prominent executives and business men liable to labor under an intense nervous strain, and that large insurances are particularly subject to adverse selection. The

[15] T.A.S.A., XXIII., 67-68.

latter is due to the applicant's knowledge of some physical condition, or other circumstance, which the company cannot ascertain.

For the various reasons enumerated, a comparatively unfavorable mortality rate may be expected on reinsured business, the highest being on very large policies reinsured in several companies. Thus far the experience of most companies on reinsured business has tended to bear out this expectation, although it is believed that a better class of risks are now taking large policies because of the decline in the purchasing power of the dollar, and for other purposes, as mentioned previously.

Residence will also have to be taken into consideration the same as in direct writing, since some sections of the country show a higher mortality rate than others.

Methods of Reinsurance.—*Facultative.*—Reinsurance may be either "Facultative," or "Automatic." Under the facultative plan, there is no obligation on the part of the direct insurer to submit its excess insurance to any particular company, and the reinsurer to which it may be submitted is free to reject or accept each offer presented. The direct insurer will usually submit all papers pertaining to any particular risk to the company or companies with which it wishes to place its excess. Each reinsuring company passes upon risks submitted to it, accepting or rejecting the same according as each comes within its own underwriting standards. The facultative method is usually applied reciprocally among a number of companies. When the home offices of these are not far apart, and when their underwriting rules are similar, the plan seems to be fairly satisfactory. For the direct underwriter it has the advantage that reinsurance may be sought wherever desired. The direct underwriter also secures the additional benefit of the opinion of another, often a larger and more experienced underwriter, concerning the desirability of the risk. The latter advantage may sometimes prove of value, especially in borderline risks. If the direct insurer is a small company, however, it is likely to look upon advice to cut down the insurance applied for, as a mere attempt to curtail its business. In some instances this is perhaps unfortunate, for when a large company rejects an application, agents will be likely to try to place it with smaller companies which may be more anxious for business. The smaller company in turn is likely to have a medical staff less experienced in life insurance matters. Mr. M. W. Torrey states,[16] "I have run across a number of medical directors who are very prone to look at a risk from their

[16] T.A.S.A., XXIII., 77.

individual medical practice point of view and say: 'Oh, I had a man like that in my practice, and I think he is a good risk. He lived so many years.' Well, that one man may have, but 10,000 men with the same impairment he had would not live the same number of years as 10,000 standard risks.'' An honest opinion from a more experienced medical department should not go unheeded. The reinsuring company has the advantage of applying its own rules and determining whether it will accept or reject the risk. To both, the method commends itself in that any profit made on reinsurance by those in the reciprocal agreement will be shared among themselves since each receives reinsurance from the others, but if the reinsurance should prove unprofitable the companies involved must suffer the losses.

There are several objections to the facultative plan. In the first place, it is uncertain as to how much reinsurance can be secured by the direct underwriter, hence its agency force may be somewhat uncertain as to how much can be written on a single life. In practice, however, there seems to have been little complaint from this source. The more serious objection is the delay in securing the necessary reinsurance.[17] If the companies are far apart, it may be several days before the direct underwriter can secure enough reinsurance to warrant delivery of the policy. For many years, however, the reciprocal facultative method has been followed, and is still in use, sometimes involving as many as 5 or 10 companies.[18]

Automatic Reinsurance.—An automatic reinsurance agreement, or treaty, as it is called, requires the direct insurer to cede its excess insurance to the reinsuring company, and the reinsuring company to accept the reinsurance ceded to it, regardless of whether the latter desires to do so or not. The excess ceded is the amount above the direct underwriter's retention, but the excess which the reinsurer must accept is usually, but not always, limited to an amount not greater than the direct insurer's retention. It is also provided that the excess which the reinsurer is required to take shall not be greater than a stated sum.

When a company desires to make a practice of insuring more than its retention, it may be able to secure protection through an automatic reinsurance treaty with a larger company. Usually, however, such excess is placed with more than one reinsuring company, the excess going to one until the limit of that one is reached, and the balance of it to the others. Again, the insurer's excess may be divided evenly

[17] Bagley and Laird, T.A.S.A., XXIII., 32.
[18] *Ibid.*, 33.

among the reinsurers, no one of them being obligated to take more than the insurer's retention. Some treaties stipulate that the original company cannot bind the reinsurer if the latter already has its limit of retention at risk on the life on which reinsurance is sought. Such information can be transmitted by telegram and the direct insurer can then seek reinsurance elsewhere on the facultative plan. When a risk is within the provisions of the treaty the judgment of the originating company regarding its desirability must be accepted by the reinsurer without question. For this reason it is even more necessary for the reinsured company to act in good faith toward the reinsurer under the automatic method, than it is under the facultative.

Reinsurance of border-line risks is frequently submitted on the facultative basis, even though the original company is empowered to bind the reinsurer under the terms of the treaty. Though on the automatic basis the reinsurer must accept from the direct writing company an undesirable risk, the former will usually notify the latter of the objectionable features of it. The companies joined in an automatic agreement will thus have the advantage of at least a partial exchange of information and views governing the acceptance of risks.

The principal advantage of the automatic arrangement to the direct underwriter is, that it may immediately bind the reinsurers and thus lose no time in issuing the policy. Also, it may issue policies of three or four times the amount of its own retention and immediately secure protection by transferring its excess to several reinsurers. The greatest facility is offered by the arrangement of a treaty among a group of companies. Since its introduction in 1903, the automatic plan has met with increasing approval. A single company has been known to arrange 10 treaties with other companies to handle its surplus lines, and more recently the automatic plan has been operated reciprocally among a group of companies. The principal requirements for the successful operation of automatic reinsurance agreements are that the underwriting views of the companies involved must be somewhat similar, and that there must be no attempt on the part of the original company to unload its liability merely because of its knowledge that the risk is undesirable. When it possesses such knowledge it should acquaint the reinsurers with all the facts, and voluntarily submit the risk in question on a facultative basis.

Plans for Fixing the Reinsurer's Liability.—Whether reinsurance is effected on the facultative or automatic basis, the conditions governing the reinsurer's assumption of liability may vary considerably.

In the first place, it may be agreed that the reinsurer shall apply its own policy provisions, charge its own premium rates, credit its customary commissions to the original underwriter, and allow the latter its usual non-forfeiture values, and dividends if the business is participating. In other words, the original company is named as the beneficiary and treated in the same manner as any other applicant, except that an allowance for commissions is made, and under an automatic treaty the reinsurer must accept the risk. The same type of policy is issued by the reinsurer as is granted by the direct underwriter to the individual who originally applied for protection. The objection to this plan is that the reinsurance does not cover exactly the same contingencies as the direct underwriter's policy, due to differences in the two contracts as regards such matters as dividends, surrender and loan values, the application of the suicide and incontestable clauses, etc. The second arrangement consists in using the policy form of the direct underwriter, but applying the commissions, premiums, and dividends of the reinsurer. This modification of the first plan overcomes some of the difficulties, but still leaves room for dissatisfaction because of differences in the cost of insurance. The plan may be further modified by reinsuring on a non-participating basis, but this is also open to obvious objections. According to a third plan, the reinsurance may be effected on a special 1, 5, or 10-year renewable term contract for a level amount of protection. This plan, however, is not satisfactory, since the amount at risk under the original policy decreases as the reserve increases, and quite rapidly so if it is a 20-payment life or short-term endowment insurance. Troublesome adjustments are thus necessary if each is to bear a just share of the risk. The objections to all three of these plans have caused them to be abandoned, except in a few instances, in favor of (1) yearly-renewable term insurance for the excess amount at risk, often called the "amount at risk" plan, or of (2) coinsurance.

Coinsurance.—The coinsurance plan requires the reinsurer to duplicate exactly the coverage provided in the contract of the original company. The reinsurer receives a pro-rata share of the premiums collected by the direct underwriter, pays first year and renewal commissions and other expenses including medical and inspection fees, and taxes, at the same rates as those of the original company. It also allows the same non-forfeiture values, disability and accidental death benefits, and dividends if participating, and applies to the reinsurance all other provisions of the direct underwriter's policy. The exact arrangement of the details governing the reinsurer's share

of expenses vary somewhat under different treaties. When the reinsurer is licensed in the state where the insurance originates, the legal reserve liability on the excess reinsured may be set up by it, thus relieving the direct insurer of a considerable proportion of the strain on surplus funds which results when a company writes a large amount of new business and sets up the full level premium reserve liability on December 31 of the year of issue. The reinsurer sets up a reserve liability at least as high as that established by the original company.

The chief advantage of this plan to the direct insurer is that it remains in virtually the same financial position as if it had issued only its retention, and at the same time is enabled to take care of customers desiring large amounts of insurance. The principal objection on the part of the original company is that it pays high premiums and thus accumulates considerable funds in the hands of the reinsurer to meet the reserve liability of the latter on the reinsured business. The reinsurer may make a profit if it earns a higher rate than that allowed by the original company in computing dividends. The expense arrangement may be such that the reinsurer receives insurance at a relatively low cost. For instance, advertising expenses and indirect agency expenses are practically nil. From the viewpoint of the reinsurer, the objectionable features are that reinsurance is concentrated on large policies and for the most part at the higher ages where business is likely to be unprofitable.

Yearly-Renewable Term for Excess at Risk.—As the name implies, the yearly-renewable term or excess amount at risk plan is one whereby the direct underwriter reinsures its excess on the yearly-renewable term basis. Since on life and endowment plans the original company accumulates reserve funds, the amount at risk steadily decreases with the passing of time, and the amount of reinsurance necessary decreases each year. It is customary under this plan to reduce the amount of reinsurance each year, by an amount equal to the policy-value increases. The direct writing company pays the reinsurer a non-participating yearly-renewable term premium as of the age attained and for the amount then at risk on the reinsurance; that is, for the original amount reinsured, less the policy-value or reserve. Thus while the premium rate increases each year the reinsurance is in force, the amount on which it is paid steadily decreases. The essential purpose of reinsurance, that is, the protection of the original company against excessive net loss on a single life, is accomplished by this method, since the portion reinsured plus the reserve on it, which is held by the original company, will meet any claim in excess

of the original company's retention. As a rule the terminal reserve is taken, but either the initial or the mean value may be used. The rates for the reinsurance may be calculated on the basis of the American Experience table and 3½ per cent, with a small loading of from 5 per cent to 10 per cent, or the new American Men basis with larger loading. No renewal commissions are paid by the reinsurer, but it is customary for the direct underwriter to pay only one-half of the first year's reinsurance premium to the reinsurer, the other half being the latter's contribution toward the first year's commission which is paid by the original company.

Sometimes it is arranged that the reinsurance shall be reduced by the amount of the reserve liability on the entire original policy instead of merely that on the reinsurance. To illustrate, if the original policy were for $100,000, and $50,000 were reinsured, the reinsurance may be reduced each year by the reserve liability on the entire $100,000 instead of merely on the $50,000 reinsured. The direct underwriter's amount at risk remains at $50,000 under this plan, until the reserve liability equals $50,000, after which the reinsurance ceases and further reserve funds reduce the original company's net amount at risk. By this arrangement the reinsurance is less, and it decreases faster, than under the plan whereby the amount at risk is reduced only by the reserve accumulations on the reinsurance. Manifestly, if the original policy were on the term plan, reserves would be inconsequential and, therefore, could be ignored, the reinsurance being carried for a level amount.

X (Proper adjustments may be made to take care of income policies; mortality salvages may be divided with the original company; and liability for disability and double indemnity benefits also may be reinsured. The latter may be kept entirely separate from life reinsurance, even though all three benefits are combined in the original policy.)

The principles underlying this plan are quite simple and easy to apply. The details concerning commissions, dividends and surrender values are avoided. The original company secures adequate protection at minimum rates and retains control of the funds necessary to meet the legal reserve requirements. The plan is especially well adapted for use when several policies on different plans and issued at different times are in force on the same life, or if under a joint-life policy reinsurance is required on only one of the lives because of other policies. To the reinsurer the plan is advantageous in that premiums and reserves are independent of the form and source

of the original policy. In a reciprocal arrangement involving several companies, all of the companies can use the same reinsurance rates. On this plan, also the cost of reinsurance to the original company may be met by a share in the mortality salvages on both the retained and reinsured part, by loading salvages on the original premiums, and by interest earned in excess of the rate used in dividend calculations, or that assumed if the original policy is non-participating. Under the coinsurance plan, extra expenses on excess policies are borne by the reinsurer, thus leaving the original company in the same position as if it did not issue insurance in excess of its retention, so a number prefer it to the excess-at-risk plan.

Small companies, especially those earning a fairly high rate of interest on their investments, prefer to reinsure their excess liability on the amount at risk plan.[19] It enables them to build up their assets, and after a time to offset mortality and loading salvages shared with the reinsurer by a higher interest rate than that of the latter. Companies of moderate size sometimes prefer the coinsurance plan, since decreases in net cost are reflected in this plan. Generally speaking, the coinsurance plan is more popular in the Eastern part of this country, the amount at risk, in the Western.

X **The Treaty.**—Under the facultative plan, no reinsurance treaty is necessary since the provisions governing the reinsurance may be stated in each policy or certificate of reinsurance. A written statement of the terms and conditions under which reinsurance will be accepted, or at least a complete understanding of them, is essential, especially when reciprocal facultative reinsurance is arranged among several companies. For reinsurance on the automatic basis a written agreement called a treaty is necessary. Under an automatic treaty, the liability of the reinsurer begins simultaneously with that of the direct underwriter. The direct underwriter, however, notifies the reinsurer, or reinsurers of amounts ceded, as soon as convenient, the time usually being limited to a certain number of days, and forwards all papers pertaining to the risk. The reinsuring company then issues a certificate of reinsurance to the original company. Facultative reinsurances are not effective until accepted by the reinsurer, or until the original policy becomes effective, if that date is later than the time when reinsurance is agreed upon. Whether facultative or automatic, premiums are usually payable on the reinsurance at the time at which they are due on the original policy, except that all premiums are on an annual basis. The treaty is so drawn that liability on the

19 Torrey, M. W., Reinsurance, in T.A.S.A., XXIII., 70.

part of the reinsurer begins and, with certain exceptions, ends with that of the direct underwriter.

The specific liability of the reinsurer must be clearly stated in the treaty as regards, (1) the excess of each risk which is to be ceded to and accepted by it, (2) total disability and accidental death benefits, (3) standard or sub-standard lives if the latter are subject to reinsurance, (4) original policy forms on which reinsurance will be granted and the limits on each, (5) the method of calculating mortality savings or other profits that may be shared with the original company, and (6) the method of calculating the yearly amounts of reinsurance on which premiums are payable, if the excess amount at risk method is used. It is also essential that a complete schedule of premiums, commission and expense allowances, if any, and the rate-book and dividend schedule of the original company, if the business is participating and reinsured on the coinsurance plan, be considered a part of the treaty. Provisions must be made also for changes in the form of the original policy, such as the conversion of term policies to life or endowment contracts, or the conversion of life or endowment policies to extended term insurance under a surrender option. When the original policy goes to extended term; that is, when extended term insurance is granted following lapse, the reinsurance also goes to extended term if the coinsurance method is used. If the amount at risk plan is followed the reinsurance may continue at a level amount to the end of the term of extension, the original company paying either a single premium, or the yearly-renewable term premium. In any case, when the original insurance goes to extended term, the reinsurance certificate is returned and a new one issued by the reinsurer. The new certificate may provide for a level amount of reinsurance, as just stated, or for yearly increasing reinsurance. Since extended term insurance is granted at net single premium rates, and since the single premium reserve for term insurance decreases with the passing of time, the amount at risk increases. The increasing reinsurance, however, must not exceed the amount reinsured in the first place. Cancellations and reductions of the original policies result in cancellations and reductions of reinsurances. Reductions of reinsurance may be either the same as on the original policy, or in the same proportion as the original policy. Reductions and terminations are automatically effective on reinsurances at the same time that they become effective on the original policy, and unearned reinsurance premiums from the date of reduction or termination are credited to

the original company. Reinstatements apply to reinsurance at the same time that they become effective with the original company.

When a claim arises, prompt notice of it with copies of proof, is sent to the reinsurer, and upon receipt of them together with copies of checks of the original company, the reinsurance becomes payable to the original company. Reinsurance claims are paid in a lump sum, whatever the mode of settlement in the original policy, if the reinsurance is on the excess amount at risk plan. In the case of continuous installments, the commuted value of the installments for which the reinsurer is liable, is paid to the original company. Under the coinsurance plan, the payments made by the reinsurer to the direct underwriter correspond to those made by the latter to the beneficiary under the original policy. Though the direct underwriter may consult the reinsurer as to the advisability of contesting a claim, final decision rests with the former. The reinsurer participates in any reduction by legal defense or compromise, bears a proportionate part of the expense of such defense or compromise, and is automatically liable to the direct underwriter under the treaty if the latter is held liable for the payment of a claim by a court of law. When a claim is paid in full, under coinsurance the reinsurer pays the direct underwriter in proportion to the total amount of insurance, and under the amount at risk plan it pays the part agreed upon of the original company's net loss above the reserve.

While the original company has the privilege of increasing its retention limit on new business, upon due notice to the reinsurer, it cannot, according to some treaties, apply the new retention limits to old reinsurances and thus take back a part of the reinsured business. In other agreements such a procedure is provided for. The direct writing company is rarely permitted to cancel one reinsurance and allow another to continue if the original policy is, in both instances, still in force. Either company can cancel the treaty upon 60 or 90 days' written notice, the agreement continuing in full force during this period, and indefinitely in force as regards reinsurances previously granted. Provision may be made for the arbitration of disputes by three arbitrators, one appointed by each disputant, and a third by these two arbitrators, as is done in cases of disagreement between fire companies and insured persons concerning the value of damaged goods. There is also a reservation to the effect that the reinsurer shall have the right to examine books and documents pertaining to reinsurance effected under the agreement, at the office of the original

company. This privilege is seldom if ever made use of, though it might be useful in unusual cases, especially when there is a radical change in the management or control of the direct underwriting company.

Administration.—The work connected with the management of reinsurances may be performed by the company's regular organization, under the direction of someone who is thoroughly familiar with all of the company's reinsurance arrangements, or by a special reinsurance department. When a company either cedes or accepts a substantial amount of reinsurance, a separate department is much more satisfactory. This department handles new business, renewals, changes, and reinstatements, keeps the accounts, and renders statements and reports from which the statistical or actuarial department may obtain data and derive valuable results. All correspondence concerning reinsurance is conducted by this department,—an important advantage.[20]

It is customary for the original company to render a statement early in each month showing first year and renewal premiums payable during the preceding month, together with adjustments, and to forward the amount due the reinsurer. If balances are due the reinsured company, the reinsurer may pay them at once, or credit the original company with them in the next statement. As a matter of convenience, reinsurance premiums are usually paid on the annual basis, even if the original premiums are paid semi-annually or quarterly. A lapse is not reported to the reinsurer until the original company is reasonably certain that the policy will not be reinstated. Adjustments and readjustments in the case of lapses and subsequent reinstatements are thus largely avoided. The monthly statement is made out in duplicate and sent to the reinsurer which signs and returns one copy as the insurer's voucher for payment of premiums, thereby avoiding the necessity of issuing individual premium notices and receipts.

When many policies of large amounts are taken from time to time by an individual, it may happen that a company will receive by way of reinsurance a greater liability on that life than its own limit of retention. It may then cede a part of its risk to still another reinsurer, if necessary availing itself of the facilities offered by European reinsurers.

Legal Features.—The person insured under the original policy looks only to the direct writing company for the fulfillment of all

[20] For a suggestion concerning the manner in which records may be kept, see Bagley and Laird, T.A.S.A., XXIII., 61 et seq.

of the policy provisions. He is in no way concerned with any reinsurance his company may cede. He need therefore satisfy himself only with the stability of his company, his policy contract, and the reasonableness of his premium payments.

Quite a number of states have attempted to place legal safeguards about reinsurance. North Carolina forbids a domestic company to reinsure in a company not authorized to transact business in that state. New York allows reinsurance in companies not licensed there, but does not allow the original company to reduce its reserve liability because of reinsurance placed with outside companies; [21] i.e., its laws favor the amount at risk plan when reinsurance is placed with nonlicensed companies. Several other states have similar regulations. Most states allow the reduction of liability if the reinsurance is placed in licensed companies. Massachusetts, North Carolina, Mississippi, Minnesota, Maine and Indiana limit the amount of reinsurance to one-half of the total insurance on a single risk. Oklahoma permits two-thirds of the individual risk to be reinsured, while New York permits all of it to be reinsured upon consent of the state superintendent of insurance. Connecticut specifies that no more reinsurance shall be placed in any one company than the amount retained by the original company. Most states allow premiums paid for reinsurance in licensed companies to be deducted from the premium tax return. Some states tax the entire original premium and not the reinsurance premium income; a few of them tax both the entire original premium and the reinsurance premium. In general, state insurance laws do not apply to reinsurance agreements between companies.

21 New York Insurance Law, Sec. 22.

CHAPTER XVII

Group Insurance: Group Life Insurance

Nature and Development.—The law of the State of New York defines group life insurance as, "that form of life insurance covering not less than fifty employees with or without medical examination, written under a policy issued to the employer, the premium on which is to be paid by the employer or by the employer and employee jointly, and insuring all of his employees, or all of any class or classes thereof determined by conditions pertaining to the employment, for amounts of insurance based upon some plan which will preclude individual selection, for the benefit of persons other than the employer, provided, however, that when the premium is to be paid by the employer and employee jointly and the benefits of the policy are offered to all eligible employees not less than 75 per cent of such employees may be so insured."

By this law group life insurance is limited to employees of one employer only. It is ordinarily written without medical examination and at a lower cost than would be the case if an ordinary policy were issued to each of the members in the usual manner. The fundamental purpose of this plan is to supply a substantial amount of protection at low cost. It may be applied to all forms of protection, such as life insurance, accident and health insurance, and insurance against superannuation.

It is essential that the group be sufficiently large, so that the inclusion of one or more impaired lives will not be a material consideration, that it include most of the employees, and that it be kept together by some motive more powerful than the desire for insurance, thus avoiding the disastrous effects of adverse selection. While the premiums for the individuals first included will increase, the average age and consequently the average premium per 1000 of insurance will remain about the same for most employed groups, because of the addition of new and young employees and the occasional withdrawal of an older one. Also, since the employer pays a substantial part of the premiums, there will not be the tendency for the healthy members

of the group to drop the insurance. Aside from legal restrictions, the plan cannot be applied to fraternal societies or other voluntary associations because adverse selection, as indicated above, would probably continue to cause losses on the fraternal groups as it has done in the past. Fraternal societies and voluntary associations cannot be insured on the group plan unless a sufficiently high premium is charged to enable setting up the regular legal reserves. This would largely defeat the purpose of group insurance so far as low cost is concerned. Moreover, if the association were a voluntary one, the impaired risks would join it, in the absence of the medical examination, and the healthy lives might not. Thus in the early days of group life insurance before legislation restricted it to employees of one employer, business done with associations showed a heavy selection against the insuring company.[1] It has been found, however, that the desire of an individual to hold his position is a sufficiently powerful motive to induce him to remain in the group, and if he retires from his position, another, and often a younger individual takes his place. Hence, not only is the group maintained without effort so far as soliciting insurance is concerned, but its average age is kept low. It follows, therefore, that group insurance is properly restricted to the employees of one employer where the employer pays the entire premium, though assessing his employees if he so desires for all or a part of the premium.

The employer's responsibility for premium payments on account of an employee ceases when the latter leaves his service, but the employee may continue the insurance under an ordinary individual policy without medical examination, provided he makes application within 31 days and pays the premium for his attained age and the type of policy applied for. This individual conversion option does not ordinarily entitle him to a term insurance policy. The amounts of insurance, also, cannot be large, yet they may be substantial. Thus, at a recent date, eleven of the largest companies covered a total of 1,711,833 lives with $1,615,779,646 of group insurance, or an average of $944 per person.

Group insurance also serves as an efficient aid to the modern welfare movement. It has long been recognized by those familiar with social welfare problems that insurance is one of the most effective means of solving many of them. Yet in most instances ordinary

[1] Cammack, E. E., and Morris, E. B., Joint Mortality Experience of the Aetna Life and Travelers Insurance Companies on Group Policies. Transactions, Actuarial Society of America, Vol. XIX., pp. 34 et seq.

employees are either unable to provide, or cannot be induced to provide, adequate protection for their dependents. Industrial insurance has done much, but it is expensive and necessarily limited to small amounts. Thus the average industrial policy provides less than $150 of insurance. In a majority of instances the recipients of the benefits of group policies have little or no other protection. Group insurance should not, however, be urged as a substitute for other insurance, but rather as a supplement to it.

The origin of group insurance is very recent, the New York Insurance Department having issued its first approval of a group policy in February, 1911. Even as late as April, 1914, only six companies were writing the business. On December 31, 1918, a little over $600,000,000 of it was in force. During 1919 this amount almost doubled, the amount in force at the end of that year being over $1,100,000,000; while, on December 31, 1920, nearly $1,700,000,000 was operative on over 1,700,000 lives. During 1921, there was a considerable slackening in the increase in new business, and also a considerable amount was canceled by industrial concerns whose business was curtailed by the depression. This was a temporary condition, however, which speedily disappeared when business became more normal. Some insurance companies provide for immediate re-instatement of employees upon the reopening of the plant, and one company states that its practice in case of shut-downs is to permit the continuance of the insurance for a period of from two to three months, provided it is continued by the employer for all employees laid off. This company agrees to "give consideration" to the request of an employer applying in writing for a further continuance of the insurance. As a rule, cancellations of group policies rarely occur, except in times of industrial depression, as employers do not wish to create trouble for themselves by changing conditions for their employees. For the same reason, employers hesitate to change insurance companies, once the policy has been accepted, and furthermore, companies generally refuse to accept groups previously covered by another one. The companies which insure the groups in the first instance, therefore, find that their business thus automatically increases as the business of the insured employers and their forces of employees increase.

Premium Computations.—Level premiums based on the ordinary life or endowment plans could be used in group insurance, and in a few instances they have been so used, with a considerable saving in expenses such as those of medical examiners' fees and agents' commissions. The calculation of these premiums for each individual,

and the determination of the proper policy-values of such contracts, involves no new principle; the methods being explained in detail above. In practically all instances, however, the premiums charged are at present based on the natural premium rate, and the policies of life insurance are of the yearly renewable term type with a provision that the policy may be automatically renewed annually by the payment of the premium as adjusted for each year to meet the ages attained at that time by the employees covered. This means that the premium rate per $1000 of insurance will increase each year if the group of employees remains the same, or if the younger employees are discharged and the older ones retained, as sometimes happens in times of business depression. If a considerable number of younger persons, however, are added to the force, as happens in many instances during periods of prosperity, the average age of the group will be reduced, with the result that the premium rate will be lowered. In practice the premium rate paid by employers has remained practically the same or has been reduced, hence the premium is to all practical intents and purposes a level premium, although it is based on the yearly renewable term plan. This type of policy protects employees during the period of employment, and does not compel the employer to pay the higher premium required on legel reserve types of policies, which would be highly disadvantageous when the employment is temporary. The policy also does not involve the return of a surrender value upon discontinuance, a complicated process when the employer and employees have contributed jointly to the payment of the premium.

The process of arriving at the actual rate which the employer pays is as follows: the company adopts a table of basic gross or office premium rates, which may or may not be participating, depending on the type of company and its practices. These basic rates are applied to groups of employees that produce the lowest mortality, such as the clerical forces of banks, insurance companies, etc., and those who work in mail order houses, retail stores, and industries in which there is no occupational or other extra hazard. The amount of insurance at each age is then multiplied by the premium rate for that age and the results totaled. This total divided by the volume of insurance gives the average rate per unit of insurance, which is the flat rate that the employer pays. The rate is subject to recalculation at intervals, according to the terms of the policy.

To illustrate, suppose each of the employees of a standard group are to receive $1000 of insurance, and that their ages are as follows: 50 persons aged 20, 25 aged 40, and 12 aged 60. From the standard

rate schedule of one participating company, it is ascertained that the rates per month for $1000 of insurance at these ages are respectively, $0.54; $0.72; and $2.69.

$$
\begin{array}{rrl}
\text{Then,} & 50 \times \ \ .54 = & \$27.00 \\
 & 25 \times \ \ .72 = & \ \ 18.00 \\
 & 12 \times 2.69 = & \ \ 32.28 \\
\hline
\text{(total persons)} & 87 \qquad & \$77.28 \text{ (total premiums)}
\end{array}
$$

77.28 ÷ 87 = $0.8883, the rate per $1000 of insurance per month quoted to the employer. If later on the number of employees should increase to 100 persons, and if there has been no recalculation of the premium, the amount, according to the usual plan, which the employer should pay would be: 100 × 0.8883 = $88.83, per month.

The normal age distribution in large standard groups is such as to produce an average minimum rate on a participating basis of about 90 cents per month per $1000 of insurance. The average minimum rate on a non-participating basis is about 85 cents. In small groups the average age distribution varies considerably from the normal, but 90 cents per $1000 of insurance per month may be safely used to furnish a rough estimate of the gross cost (without deducting dividends) in *large standard groups.* For non-standard groups, namely, groups of employees engaged in hazardous industries, extra premiums are charged, the amount of premium depending on the circumstances. Thus one company would add $0.09 to the rate of $0.8883 calculated above, as a minimum extra premium for groups engaged in brick, tile and terra cotta works; electric light, power and gas companies; iron and steel foundries; potteries; steam railroading; shipbuilding; and industries involving similar hazards. This same company adds $0.17 to the rate for groups employed in transportation by water, subway and elevated railroads, quarries, surface metal mines, ice companies, manufacture of acids, coal delivery, electric and street railroads, and poisonous gas manufacture; and $0.26 for groups of city firemen, glass blowers, policemen, employees on tugs and barges, and harbor and dock employees. When more than two-thirds of the employees covered are underground metal miners, $0.34 is added, and $0.42 is added for structural iron and steel workers employed in constructing buildings and bridges.

In the determination of premiums and the classification of groups, a careful inspection is made to ascertain conditions as regards hygiene, sanitation, and any unusual accident or health risk. Allowance is

made for catastrophe hazards such as fire or explosion. Individual adverse selection is guarded against by insuring the group on a basis which does not give an individual the option to decide whether to enter the group, or whether to leave it, except in so far as discontinuance is brought about by ceasing his employment.

Fallacy of Average Age.—From the above it is seen that the *average premium* is the basic rate used, and *not the average age* of the employees. The following example will illustrate the difference:

Age	Amount of Insurance	Monthly Premium
20	$1000	$0 54
40	1000	0 72
60	1000	2 69
Total, 120	$3000	$3.95
Average, 40	1000	1.32

The average age of the three persons is 40 years, yet the average premium is $1.32, or 60 cents per month greater than the premium at age 40. The premium of $1.32 is nearly that for an individual aged 51, according to the rates of the company whose figures are used throughout this illustration. It is usually found that the *average premium* corresponds to an age from 10 to 12 years *higher than the average age.* A rough estimate of the cost can sometimes be made for an employer on this basis if he knows the average age of his employees. The expression "average age" may be correctly used if by it is meant the age corresponding to the average premium. Thus the "average age" for premiums in the above example is 50⅔ years.

The rates given in the foregoing illustrations are those used by one of the largest group insurers. This company operates on the participating plan, so that employers may expect a considerable reduction in cost in the form of a dividend at the end of the year, if the mortality experience is at all favorable. If the employer prefers to pay the premium annually in advance, a reduction of 4 per cent is allowed from the aggregate monthly premiums. Most of the non-participating companies quote annual rates only, although monthly rates may usually be arranged, and in any event the employer pays only for those employees covered during each separate month. Thus when an additional employee becomes eligible, or when some other one becomes entitled to increased insurance, the employer notifies the

insurance company and is charged pro-rata premiums from the middle of the policy month in which the additional or increased insurance takes effect. When insurance on an employee is terminated, the employer is credited with the pro-rata unearned premium from the middle of the policy month in which the employment ceases if he notifies the company within a specified time, such as 14 days. If the employer fails to notify the company within the time limit, he is credited with the unearned premium from the middle of the policy month in which the notice is received.

Legal Reserves on Group Policies.—In those rare instances in which group insurance was written on the level premium basis, the determination of policy-values offered no problems other than those explained in Chapters IX and X. When group insurance is written on the basis of the net natural premium rate according to the standard prescribed by law, there is no reserve problem because the law does not require a reserve on yearly renewable term business. When the gross premium rate charged is actually less than the net natural rate, the question of an adequate reserve liability to insure solvency immediately arises. It so happens that group mortality at the younger ages, like the mortality on individual policies, has been considerably lower than that indicated by the American Experience table. In anticipation of the mortality salvages, therefore, the companies have quoted rates on standard groups lower than the net natural rate. The following table comparing basic gross rates per $1000 of insurance on standard groups with the net natural rate will serve to illustrate:

Company	Age 20	Age 40	Age 60
Net natural premium per annum, American Experience and 3½%.	$7 54	$9.46	$25.79
Equitable (participating, includes total and permanent disability benefits, aggregate monthly rates discounted 4% for annual).	6.23	8 31	31.04
Metropolitan (participating, includes total and permanent disability benefits).	6.18	8.27	30.94
Aetna (non-participating, includes total and permanent disability benefits).	5 87	7.85	29.39
Travelers (non-participating includes total and permanent disability benefits).	5 87	7.85	29 39

In groups engaged in hazardous occupations where a high extra premium is added, the legal requirement of minimum values according to the American Experience table and interest at 3½ per cent, might cause no difficulty. When the policy is participating, also, the initial premium matters but little. Non-participating companies, however, if held strictly to the letter of the requirement, would be at a great disadvantage when competing for groups at very young ages. Various states have recognized this and have adopted measures to remedy the situation. Thus Colorado passed a law in 1919, making the Medico-Actuarial table of mortality and 3½ per cent the basis. New York, Massachusetts and other states allow the deficiency to be charged as a separate liability. In other words, companies which charge less than net rates are charged with a deficiency reserve liability equal to the present value of the difference between gross premiums and the net premiums calculated upon the state valuation basis for the term of years for which the rates are guaranteed.

Distribution Among Employees.—The face value of the protection which an employee receives under a group policy is set forth in his certificate. The amount may be an arbitrary sum for each member of the group. This is called by one company the *"Flat Sum Plan."* It is not popular with employers and is not urged by insurance companies. The main criticism directed against it is that it places all employees on the same level, thus enabling those in the lower ranks to receive the same benefits as those in the higher positions, regardless of length of service or salary.

"The Salary Plan," whereby the amount of insurance is based on the salary earned by the individual, is more satisfactory. The usual basis is one year's salary, with specific minimum and maximum amounts. A minimum of $500, a maximum of $5000, and a probationary period of six months' employment before becoming eligible, is the customary arrangement. Piecework may be included for an amount equal to what the employee would be likely to earn during the year. Payment of the claim may be in installments, the checks being drawn by the insurance company in favor of the beneficiary and turned over to her at the office of the employer on "pay-day." Thus the family of the deceased is taken care of for one year as it was while the insured was living, and other employees are strongly impressed with the benefits of the insurance and the interest of the employer in their welfare.

"The Service Plan," is perhaps more popular with employers than any other. It bases the amount of insurance in each case solely on

the length of time the employee has been in the service of the employer and provides for increases in the amounts of insurance based upon increases in length of service. It may, and usually does, give credit to employees for service already rendered when the group policy is taken out.

Combinations of plans are sometimes demanded by employers in order to take care of the higher salaried employees, such as officers, foremen, and superintendents. When plans are combined, certain additional regulations are required by the companies. Thus one insurance company requires that the character of employment of each member of the higher-salaried organization be set forth as "president," "treasurer," "chief accountant," "foreman," etc. It also limits the amount of insurance for any individual to 2½ times the average amount per individual in the main body of employees.

To any of these plans a funeral benefit may be added, limited in most instances to $150. To all of them it is customary to fix a minimum of $500 of insurance on any one life, and a maximum of $5000, the maximum depending on the total insurance on the entire group. Thus one company furnishes the following schedule to its agents:

Amount of Insurance in the Group	Maximum on Any One Life
Under $50,000	$1500
50,000 to $75,000	2000
75,000 to 100,000	2500
100,000 to 150,000	3000
150,000 to 200,000	3500
200,000 to 300,000	4000
300,000 to 400,000	4500
400,000 or over	5000

The maximum may be $10,000

Practically all group policies now provide total and permanent disability benefits, the usual stipulation being that in case the employee becomes totally and permanently disabled before age 60, the entire amount of his insurance becomes payable six months after such disability. It may be paid in a lump sum or in installments as the employer may elect. It is not used in extra-hazard groups. Some companies also give the services of their welfare departments, including a free nursing service, a health literature service, a "sales" service to explain the benefits to employees, and an "industrial relations" service to make suggestions and assist in settling employment problems.

A method of providing for a pension, or of creating a retirement fund, is often adopted by the employer in connection with group life insurance. The funds may either be left with an insurance company to purchase a deferred annuity or other benefit, or deposited in bank, or left with the employer to invest. Thus the Westinghouse Company requires employees to save at the rate of 2 per cent of their wages before they receive insurance in addition to the $500 minimum, and the Curtis Publishing Company stipulates membership in its benefit association which in turn necessitates partial payment towards the cost of health and accident insurance.

Policy Conditions in Group Life Insurance.—There are two application blanks, to be filled in preliminary to the issuance of the policy. One is signed by the employer, whereby he simply makes a request for insurance on those lives whose names shall appear upon the schedule of employees already attached or which may subsequently be added thereto. This provides for the payment of the premium as stipulated by the contract, subject to subsequent adjustments on account of termination of employment and the addition of new employes in accordance with the schedule of rates provided in the contract. The other application blank is signed by the employee and contains certain specific information as to age, nationality, wages or salary, period of service with the company, amount of insurance, exact duties performed, height, weight, present health, name of beneficiary, relationship of beneficiary, etc. The policy is issued on the basis of these applications, consideration being given also to the condition of the plant. Though the policy is issued to the employer, each individual in the group receives a certificate of insurance made out in the name of the individual and containing the name of the beneficiary. This certificate constitutes the evidence of obligation upon the insurance company for the payment of the claim to the stated beneficiary. The certificate prescribes the conditions under which it continues in force; viz., the continuance of the group policy through the payment of premiums by the employer and the continuance of service with the employer on the part of the holder of the certificate. The group policy issued to the employer is usually accompanied by a card index or register which gives the names of the persons insured, the amount of insurance in each case, the names of the beneficiaries and the amount of premium attaching to each individual.

Among the specific conditions of the policy, the more important may be mentioned as follows:

(1) The term is usually for one year.

(2) The agreement is made in consideration of the application for the policy, which application is made a part of the contract.

(3) The policy upon its expiration may be renewed and continued indefinitely, or for a stated number of successive terms of one year each without medical examination. The company also promises that the rates for individuals at different ages as they are stated in the contract will be continued for successive terms of the policy for a definite or indefinite period of years as the contract may provide.

(4) The policy is incontestable after one year from its date of issue except for non-payment of premium or engaging in military or naval service in time of war; but if the age of any insured has been mis-stated, an equitable adjustment is made. The practices of companies differ as regards the exact method of adjustment.

(5) The policy is issued and accepted with the express understanding that, in addition to the persons originally insured thereunder, the company will at any time during the continuance of the policy upon application therefor by the employer insure other employees of the employer who have become eligible under the terms of the contract. The company also agrees under the same conditions to increase the insurance of any employee of the employer to a total not exceeding a stated sum.

(6) It is agreed that upon the return of the card representing any individual insurance under the group policy, if properly signed by the employer, the insurance on such individual employee shall immediately cease, and the unearned premium paid on account of such insurance shall be returned by the company.

(7) The insurance continues only so long as the insured remains with the employer, but, "Upon written application therefor by any individual insured who is not above the insuring age of —— years, within 31 days after the insurance upon the life of such insured is terminated by the individual's leaving the employ of the employer, the company will issue upon such life, without a medical examination, at its then regular rate for the attained insuring age of the insured a level premium whole life or endowment policy for an amount not exceeding the amount of insurance terminated upon the same life under this policy."

(8) The employer may elect to have the insurance on any employee's life paid in a fixed number of installments, not oftener than monthly nor for more than one year. If default occurs in any pre-

mium payment the policy may be reinstated upon evidence of insurability satisfactory to the company.

(9) In case of total, permanent disability the policy is payable after six months, either in a lump sum or in installments as the employer may elect.

Services Rendered by Group Insurance.—While some of these have already been referred to, the more important ones may be summarized as follows:

(1) This form of insurance reaches the masses, and gives protection to those whose need for it is greatest, and who cannot afford ordinary insurance because of low wages. Since lives are accepted in groups, the stronger lives support the weaker in preserving the insurability of the whole. Many persons are therefore enabled to have the benefit of a limited amount of insurance, who would be unable to meet the medical standard employed in individual insurance. (2) It tends to keep the employee with the employer and thus proves valuable to the latter. Constant changes in the working personnel are expensive in the following respects: loss of time of the instructors, loss from idleness of machines, from waste of materials, and from the smaller productiveness of new hands. Efficiency requires a low rate of labor turn-over. Group insurance helps to create this by forming a bond of sympathy between worker and employer. (3) In the case of group insurance, agents for the most part solicit the employer and not the employees. Large economies in sales time are thus brought about. Commissions and renewals for group insurance, it is stated, are approximately only one-third of the usual rate. Employer and employees, as a result, profit materially. Moreover, only one policy is issued instead of hundreds of them. The wholesale plan of insurance also leads to economy in that the accounts are handled in the aggregate. The cost of medical examinations is also obviated and this, averaging between $3 and $5 per person, is an important consideration. There are also administrative economies, such as saving in postage, in premium notices, etc. Moreover, because business is held together by the group there is a diminution in loss from lapses. The latter is an expensive feature in individual insurance. (4) Group insurance has been instrumental in bringing about an allied plan for group health and accident insurance. Compulsory workmen's compensation acts protect workers only during the hours of employment. This plan grants protection outside of actual working hours. (5) Group insurance is regarded as a part

of the modern welfare movement. Group insurance should not be considered a substitute for individual insurance, but as an important supplement thereto. It is essentially pay-check insurance. (6) Many are insured under group insurance who would otherwise be indifferent. (7) It relieves employees from worry and does away with the necessity of "passing the hat" upon the death of one of their numbers—an advantage which may be stressed upon explaining the subject to employees. (8) It is a most practical method of rewarding faithful service.

Obstacles to be Overcome in Writing Group Insurance.—Unless the insurance company, upon writing a group policy, explains to employees the advantages of such coverage, the workers may not be favorably impressed with it. They are likely to fear that group insurance will result in keeping wages lower than would otherwise be the case. Should this be the result, there would be no way of measuring the cost of protection to the employees, since there is no way of ascertaining the extent to which wages may be kept down because of it. This fear is strengthened by the common practice of terminating the employee's protection under a group policy if he participates in a strike. Furthermore, group policies are usually written so that an employee of long standing cannot shift to a new employer without sacrificing the greater part of his protection. For instance, suppose that a person who has remained in one employment until his protection under a group policy has reached, say, $4000, now shifts to another employer. Even if the new employer should hold a group policy, there probably would be a waiting period of six months during which this employee would receive no protection, and after which he would begin once more to build up his protection,—the sum of $4000 being reached only after years of service with his new employer. Labor leaders fear, therefore, that group insurance, as at present conducted, may seriously lessen the mobility of labor.

Employers do not relish the task of convincing their working personnel of the benefits of group coverage. This is a task the insurance company should perform, with the employers' coöperation, if the latter is to reap the advantage of the loyalty and efficiency which it is the purpose of group insurance to engender. To do this, and to overcome the hindrance to the mobility of labor, as well as to overcome the laborer's objection that group insurance interferes with the efficacy of the strike,—these constitute the principal obstacles in the way of the expansion of this particular branch of the insurance business.

Group Health and Accident Insurance

Origin and Nature.—Workmen's compensation laws compel employers to insure against accidental deaths and disablements of employes arising out of, and in the course of, employment. In some states, compensation laws also cover industrial disease. Compensation laws do not compel insurance against deaths or disabilities arising outside of employment. Many employers, however, have taken group life insurance policies for their employees and have fostered the welfare of their workers in other respects. It is realized that although group life insurance protects against death, and workmen's compensation insurance protects against accidents arising in the course of employment, still further protection is necessary if employees are to be protected against ill health and accidents not incident to employmet. Group health and accident insurance supplies this need. It is a part of the modern welfare movement, one of whose relief measures is the insuring of employees as a class—a measure which appears to bring about increased production and a more stable and contented working personnel.

The contingencies insured against are those which are not covered by the workmen's compensation acts, the intention being to supplement compensation insurance. In states which have no compensation laws, occupational accidents as well as non-occupational accidents and full health coverage may be granted. Full coverage may also be granted when an employer wishes to augment the benefits provided by compensation insurance, since occupational accidents may also be included in group insurance. When occupational accidents and diseases are covered by compensation insurance, the group health and accident policy may be limited to non-occupational accidents and diseases. Full health coverage, however, that is, coverage of both occupational and non-occupational diseases, and insurance against non-occupational accidents, constitutes the benefit most commonly sought in this type of protection.

To be eligible for protection, under a group accident and health policy at the time it goes into effect, employees must be actively at work for full time and for full pay. In other words, such a policy does not retroactively cover those who may be sick on the date when the contract becomes effective. If an employee, however, becomes incapacitated on the last day the policy is in force, the benefits are payable subsequently, even though the policy expires.

Coverage and Period of Indemnity.—The benefit consists in a weekly indemnity for a stipulated period of time. The maximum coverage is ordinarily not greater than 66⅔ per cent of the average weekly earnings, and not to exceed $40 per week. Twenty-six weeks is the most usual period during which the benefit is payable, with no benefit payable during the first 7 days of disability. This is for non-occupational accidents, as well as for both occupational and non-occupational diseases. Other plans are available, however, such as provide a waiting period of 3 or 5 days, no payment for the first 3 days unless the period of disability exceeds 15 days, no payment for the first 7 days unless the period of disability exceeds 30 days. The period may be limited to 13, 26, or 52 weeks. Periods longer than 52 weeks might be arranged, but after this period the insurance gradually changes to total and permanent disability insurance, so that the demand for a longer period is negligible, and few companies extend the benefit beyond one year. The percentage of wages may be less than the maximum, or the policy may provide for a flat sum payment of so much per week for the limited period. Thus one company quotes separate rates for the various plans, such as 3, 5, or 7, etc., days waiting period, with limits of 13, 26, and 52 weeks, and also for $10 weekly indemnity to each employee, or per $100 monthly payroll for a benefit of 50 per cent of wages. Rates for 66⅔ per cent of wages are found by increasing the quoted rates by ⅓.

The standard coverage provides weekly indemnity for temporary total disability resulting either from sickness or from accidental injury suffered while not engaged in the duties of the regular occupation. House confinement may or may not be required, the stipulation often being that full indemnity will be paid to the employee as long as he "is wholly and continuously unable to perform the duties of his occupation," subject to the contract limits. On the other hand, however, the insurance does not ordinarily cover sickness or injuries which are not of sufficient severity to require treatment by a physician. While accident coverage may be made effective from date of employment, the contract often contains a clause stipulating that no indemnity shall be payable for sickness commencing during the first 15 days of employment; but, subject to this stipulation, the insurance on individual employees may begin either at the time of employment or at the end of a specified probationary period. Such insurance will continue as long as the employee remains on the payroll of the employer, or until he is retired on pension, or has reached the maximum age.

Policy Provisions.—The insuring clause sets forth that subject to the provisions of the formula the company, upon notice and proof of total disability caused by non-occupational accident or illness contracted during the term of the policy, will pay to an incapacitated employee the stipulated weekly indemnity. The waiting period, the requirement of attendance by a licensed physician, and such other restrictions and exceptions as may be made by the company, are also stated. The clause governing premium payments provides that the policy is issued in consideration of the application of the employer and the payment of premiums at the rate of so much per each $1.00 of weekly indemnity (or so much per $100.00 of monthly payroll, benefit — per cent of wages with maximum of $— per week) as determined by the application of the formula to the schedule of employees filed with the company, subject to certain provisions for premium adjustment, treated below. The "formula" commonly states that the insurance shall cover employees actually working on the date when the policy becomes effective, employees then absent after they return to work in good health, new employees after they have completed "an aggregate period of service of — months on full time and for full pay, and then employed on such basis." The amount of insurance as to each employee covered, and the period for which indemnity will be paid, is then stipulated. Concerning the eligibility of employees, the employer must furnish the company with the names of all employees as they become eligible, with necessary data as to each, in order that the company may determine the amount of the premium. New or additional insurance must be paid for at half the monthly premium rate for the month of entry, and the regular rate thereafter. Insurance on an employee ceases when the latter discontinues working for the employer, regardless of the manner in which the employment is terminated. On a stipulated day of each month the employer must furnish the company with the names of all employees whose insurance is reduced, the amount and date of the insurance discontinued, the names of all persons ceasing to be in his employment during the preceding month, upon whom insurance is discontinued, with the date of termination and the amount of insurance discontinued in each instance. An unearned premium is then returnable on account of discontinuances equal to half of the monthly premium payable for such insurance for the particular month during which the insurance ceased. The company is then in position to furnish the employer with a monthly statement of premium adjustment, taking into account the

changes in coverage due to additions, increases, reductions and discontinuances. The employer is credited with unearned premiums returnable, and debited with premiums for continuances, additions and increases to arrive at the amount he must pay for each month's protection. The company reserves the right to inspect the payrolls or other records of the employer in order to verify or determine the amount of insurance and the premium payable.

The contract does not apply to injuries or sickness suffered outside the continental limits of the United States or Canada north of the 60th degree, nor to those suffered as a result of war or participation in aeronautics. The policy may be renewed "from year to year for a further term of one year by and with the consent of the Company at such premium rates as may be determined by the Company." It is thus observed that the company may increase the rate, or refuse to renew the policy at the end of any policy year, if it so desires. The contract may be either participating or non-participating. An individual certificate is issued to each employee covered, as in group life insurance.

Selection.—Selection on the part of the company begins by eliminating certain groups of employees, such as those engaged in aeronautics. from the list of possible prospects. Other groups may then be classified with a view to adjusting premiums to the hazards involved. There is no legal definition of group accident and health insurance. In writing insurance of this character, the companies follow in general the definition of group life insurance. In other words, group accident and health contracts must include all the employees, or all of any class or classes thereof grouped according to conditions pertaining to employment, and upon a basis which will preclude individual selection. It is customary to follow the practice in group life insurance of limiting the policy to employees of one employer. To encourage membership in an organization including only a part of a group of employees, an employer may take a policy covering only members of the association. In such cases it is stipulated that at least 75 per cent of the total eligible employees shall be members of the association, in order to prevent adverse individual selection. The requirement that the employer shall pay a substantial proportion of the premium is designed to prevent malingering by making the employee's return to work after illness a matter of pecuniary interest to the employer; i.e., any considerable malingering on the part of employees would soon influence the employer's premium rate. Moreover, the employer is made responsible for carrying out the

terms of the contract including the payment of premiums. To require him to contribute a considerable part of the premiums will make him feel his responsibilities more distinctly and exert himself more diligently in collecting the employees' contributions.[2]

To eliminate individual selection, then, the contract as indicated in part above, is very specific as to the employees covered, as to what percentage of the contributory group must accept the insurance, as to the date when and the length of time for which they are covered and as to the method of determining the weekly benefit so that it will apply automatically to each employee covered. It is impracticable to allow an individual employee to determine the amount of his own benefit, since this would encourage adverse selection in that those in poor health would be inclined to select high coverage. The floating laborer may be eliminated by requiring three to six months' employment before becoming eligible for the insurance benefits. All of those who are employed at the inception of the contract, however, are generally covered, regardless of the waiting period applied in the case of subsequent additions to the force of employees. Were it not for the fact that the contract excludes those who are ill when it becomes effective until such time as they shall return to work in good health, considerable difficulty might result from the willingness of employers to accept a policy at times when a large number of employees are on the sick list.

It is not impossible to grant protection of this kind to part-time workers who may have more than one employer, although the contract usually excludes them. If it were desired to grant such protection, the amount of weekly benefit apparently should be a percentage of weekly wages and the premium computed as so-much per $100 of payroll. The risk of adverse selection through an individual employee's carrying heavy personal accident and health insurance at his own expense in addition to the benefits under a group policy, is something the companies cannot well avoid. The remedy for abnormal loss is to raise the premium rate, or if the policy is participating, to reduce the dividend. General over-insurance of all employees must be avoided.

Premiums.—Premium computations for this type of insurance involve many of the same principles as do other premium determinations. The loss ratio, corresponding to the mortality rate in life insurance, must first be estimated with a fair degree of accuracy and

[2] Craig, James D., Group Health Insurance, in Proceedings, Casualty Actuarial and Statistical Society of America, Vol. VII, Part I, p. 80.

on a conservative basis from the company's point of view. A proper allowance for expenses and contingencies may then be made, and the gross rate arrived at in the usual manner. The Manchester Unity Tables, considered the most satisfactory basis for health insurance rates in this country, have been generally adopted. They are also used in computing rates for non-occupational accident and full health coverage, the usual insurance provided by group health and accident policies, as explained above. The Manchester Unity Tables show the rate of sickness for each age and as regards duration, but the need for simplicity has resulted in the general adoption of an average rate. Mr. E. E. Cammack of the Ætna Life Insurance Company, states that if all employees insured under group policies were considered to be age 40, the resulting aggregate group health premium would be approximately the same as if individual ages were taken. Hence, calculations have generally been based on that age. On commercial risks, Mr. Cammack has estimated that 13 per cent of the total amount of disability arises from occupational accidents. The probability of occupational accident varies with the nature of the industry, since some occupations are more hazardous than others. Mr. James D. Craig, in a paper published in the Proceedings of the Casualty Actuarial and Statistical Society of America, Vol. VII, Part I, describes Mr. Cammack's method as follows:

"He (Mr. Cammack) then subdivided the rate of sickness of the Manchester Unity A H J group into periods of sickness according to the formulae on pages 591-593 of the Report for 1912-1913 on the Administration of the National Insurance Act, Part I, and prepared premium rates for a full health and foreign to occupational accident group contract. The net premium at age 40 according to the A H J table for a benefit of $10 a week from the second to the 27th week is $7.28 and deducting 13 per cent for occupational accidents, the net premium for foreign to occupational accidents and full health benefit is $6.34. The experience of the Metropolitan Life on a group policy during the years 1915-1918 indicates that from the 2d to the 14th weeks the claims actually incurred were 71½ per cent of the expected under the Manchester Unity A H J Table. For the 15th to the 27th week the Metropolitan experienced 158.3 per cent, or a total percentage for the 2d to the 27th week of 86.6 per cent as compared with Mr. Cammack's 87 per cent. The Metropolitan's experience was on male lives exclusively, whereas the experience of the Manchester Unity includes some females. It may be that the Metropolitan's rate is higher than would be experienced under commercial policies generally,

but if it is assumed that the Metropolitan's rate represents the normal for an average grade of men in a non-hazardous occupation insured under a health policy, the net premium of Mr. Cammack would apply only to such groups. If, however, a group is assumed to consist of 90 per cent males and 10 per cent females and that the female sickness rate is 200 per cent of the male, then the net premium would be raised from $6.34 to $6.97 and a gross premium of $10 a year would be required to provide for a loss ratio of 70 per cent.

"For risks covering more than 10 per cent females the rate would have to be increased accordingly, and while a reliable experience on females has not yet been published the experience of the Metropolitan indicates that substantial increases will be necessary. The general scale is

"When the group consists of from 11 per cent to 20 per cent females increase of 15 per cent.

"When the group consists of from 21 per cent to 30 per cent females increase of 25 per cent.

"When the group consists of from 31 per cent to 40 per cent females increase of 35 per cent.

"When the group consists of from 41 per cent to 50 per cent females increase of 45 per cent.

"When the group consists of from 51 per cent to 60 per cent females increase of 55 per cent.

"When the group consists of from 61 per cent to 70 per cent females increase of 65 per cent.

"When the group consists of from 71 per cent to 80 per cent females increase of 75 per cent.

"When the group consists of from 81 per cent to 90 per cent females increase of 85 per cent.

"When the group consists of from 91 per cent to 100 per cent females increase of 95 per cent.

"With a basic rate of $10 for a benefit of $10 a week from the 2d to the 27th week inclusive so derived Mr. Cammack proceeded to prepare rates for other periods. To the premium so computed for the last few days of the first week an additional 20 per cent was added, as there are certain circumstances of a practical nature which enter into a rate covering first week's sickness. It is a grave question whether or not such full coverage should be granted, as malingering is encouraged and the cost of administration of this sickness will

probably be increased out of proportion to the increase that could be made in the premium. The final rates so prepared by Mr. Cammack to cover benefits from 3 days to 54 weeks are presented in the table below."

Period for which Indemnity is Payable, Counting from Commencement of Disability	Annual Premium for Indemnity of $10 a Week	Period for which Indemnity is Payable, Counting from Commencement of Disability	Annual Premium for Indemnity of $10 a Week
3 days	$1.35	24 weeks	$12.74
5 "	2 20	25 "	12.85
6 "	2.63	26 "	12.94
7 "	3 05	27 "	13.04
8 "	3 45	28 "	13.11
9 "	3 81	29 "	13 20
10 "	4 15	30 "	13 28
11 "	4 46	31 "	13 35
12 "	4.75	32 "	13 43
13 "	5 01	33 "	13 50
2 weeks	5 24	34 "	13 56
3 "	6.91	35 "	13 63
4 "	7 85	36 "	13 69
5 "	8 40	37 "	13.75
6 "	8.85	38 "	13 81
7 "	9 25	39 "	13 86
8 "	9 60	40 "	13 91
9 "	9 92	41 "	13 98
10 "	10.20	42 "	14 03
11 "	10 48	43 "	14 08
12 "	10 73	44 "	14 11
13 "	10 98	45 "	14 16
14 "	11.20	46 "	14 20
15 "	11 41	47 "	14 25
16 "	11 61	48 "	14 29
17 "	11 80	49 "	14 33
18 "	11.96	50 "	14 36
19 "	12.11	51 "	14 40
20 "	12.26	52 "	14 44
21 "	12.39	53 "	14 48
22 "	12.51	54 "	14 51
23 "	12.63		

When occupational accident coverage is desired as well as non-occupational and full health, an additional premium must be charged

depending on the degree of occupational hazard. To avoid over-insurance, one company stipulates that the weekly indemnity thus provided, together with the Workmen's Compensation benefits, shall not exceed $40.00 nor 66⅔ per cent of the employee's weekly wage.

Rates can be quoted only after inspection of working conditions, and after full information has been received. The following schedule, however, gives the minimum participating rates quoted by one large company for five different plans referred to above, these rates being applicable to the better classes of risks where all conditions are favorable and there are no special occupational hazards.

MINIMUM MONTHLY PAYMENTS

Plan	For Each $10.00 of Weekly Indemnity (Benefit, Flat Sum for Each Employee)			Per $100.00 of Monthly Payroll (Benefit, 50% of Wages)		
	Limit			Limit		
	13 Weeks	26 Weeks	52 Weeks	13 Weeks	26 Weeks	52 Weeks
(1) No payment for first 3 days (3 day waiting period).	88	1.06	1.19	1.02	1.22	1 37
(2) No payment for first 5 days (5 day waiting period).	81	.98	1.12	.93	1 13	1 29
(3) No payment for first 7 days (7 day waiting period).......	.74	.91	1.04	.85	1 05	1 20
(4) No payment for first 3 days unless period of disability exceeds 15 days (3–15 exception).........	.93	1.11	1.25	1.07	1.28	1.44
(5) No payment for first 7 days unless period of disability exceeds 30 days (7–30 exception)........	.77	.95	1 09	.89	1.10	1 26

Rates for 66⅔% of wages are found by increasing the above rates by ⅓.

Modification of Premium Rates.—These rates are increased when:

(a) It is found that there is an unusually large proportion of employees at the older ages.

(b) The percentage of women employees is in excess of 10 per cent; the extra rating being determined as a percentage of

the minimum rate for the plans selected, determined according to the following rules:

Per Cent of Women	Extra Rating as Per Cent of Minimum Rate
11 to 20	15
21 to 30	25
31 to 40	35
41 to 50	45
51 to 60	55
61 to 70	65
71 to 80	75
81 to 90	85
91 to 100	95

(c) The nature of occupation or conditions of employment are such as to require an extra premium.

This company urges its agents not to solicit risks involving an additional disability hazard without securing permission and quotations from the home office. As examples of hazardous risks, the following are quoted: Lead workers, pottery employees, felt workers, furriers, tannery employees and workers in dusty trades or in the manufacture of chemicals or in processes involving the use of arsenic, mercury, wood alcohol, benzol, carbon bisulphide, aniline dyes, fuming acids or poisonous gases, or exposure to extreme heat. Considerations that govern the acceptance of the risk and the determination of the premium, as well as the advantages and disadvantages of this form of protection, are similar in most respects to those enumerated under group life insurance.

Commissions.—Group health and accident insurance partakes of many of the characteristics of compensation insurance, although it differs in one important particular; namely, it is not compulsory. Its existence, therefore, depends on the favorable attitude of employers, and this can only be brought about to a satisfactory degree by the efforts of agents. In other words, this type of business is similar to life insurance in that a well trained and well paid agency force is necessary to place any considerable amount of it on the books of the companies. England's experience under the National Health Insurance Act indicates that such insurance should be carried by private underwriters, that the loss ratio should constitute 70 per cent of gross premiums, that the companies should limit their margins for

expenses of management, and their commissions and profits to 30 per cent of premium income, and that commissions should be limited to 5 per cent of premium income.[3] When an agent persuades an employer to take voluntary group health and accident insurance, however, his remuneration probably should be more than 5 per cent of the first premium in many instances, since it is often as difficult to sell a small policy as a large one, and since his greatest task is accomplished when he first persuades the employer to purchase a policy. Hence, a graded scale of commissions is desirable, under which larger commissions are payable the first year, and a higher percentage of premiums is allowed for groups paying small premiums than for those paying large ones. It would seem that a scale of commissions might be determined from experience, with a view to producing the volume and kind of business most satisfactory to the company. In other words, if an increase in commissions is followed by an increase in desirable business, the company will not lose, and a greater proportion of the population will be covered. The cost per unit in force is not likely to be thus excessively increased, owing to the competition and the difficulty of selling high-priced insurance.

The practice of companies as regards first-year commissions varies quite widely, some paying as high as 17½ per cent when the premium is $5000 or less. Renewal commissions also vary, one company paying 5 per cent for 5 years when the premium exceeds $10,000, with greater percentages for smaller amounts. In general, first-year commissions are greater and renewals less than 5 per cent, the average being slightly in excess of this percentage, and total administrative expenses are generally kept within 30 per cent of the gross premiums.

[3] Craig, James D., referring to conclusions of the Committee appointed to consider proposed changes in the British Workmen's Compensation Law, in Proceedings, Casualty Actuarial and Statistical Society of America, Vol. VII, Part I, pp. 90 and 91.

CHAPTER XVIII

Industrial Insurance

Industrial life insurance is especially designed for the wage-earning or industrial classes who would have difficulty in meeting the annual, semi-annual or quarterly premium payments of ordinary insurance. It consists in issuing a small amount of insurance to an individual on the level premium plan, the premium being payable weekly,[1] and usually being some multiple of 5 cents. Hence it is often called "weekly premium" insurance, and sometimes "nickel," "dime," or "quarter" insurance. The amount of insurance per unit of premium varies with the age of the risk. The premiums are usually collected each week by house to house visitation. The principal purpose is to insure each member of the family for an amount sufficient to meet last illness and burial expenses.

Burial associations and friendly societies have provided a sort of burial insurance for many centuries. The first industrial insurance company in England, however, was not established until 1849. It was known as the Industrial and General Insurance Company. In 1854 the Prudential Mutual Assurance Investment and Loan Association began writing this form of insurance. This company later changed to the Prudential Assurance Company, under which name it has become famous. Young children were not accepted at first by this company, but later on business was accepted at any age from 3 months to 60 years. The British Government Post Office Department commenced writing industrial insurance in 1865,[2] but the post office plan has met with but little success.[3]

The success of the Prudential of England began to attract serious attention in this country in 1875. In that year the Prudential Friendly Society, which afterwards became the Prudential Insurance Company of America, issued its first industrial policy. The John Hancock

[1] Monthly premiums, payable every four weeks, are frequently used by British companies, especially for endowments.

[2] Buchanan, J. D., in Transactions of the Actuarial Society of America, Vol. XXII, pp. 36-38.

[3] Gore, John K., *ibid.*, pp. 72-73.

Mutual Life, and the Metropolitan Life Insurance Company followed in 1879. Many difficulties were encountered in the early years, expenses were heavy, mortality experience was not particularly favorable, and because of the failures in the early 70's and the activities of fraternal insurance enthusiasts, old-line companies were not at that time held in high esteem. Following the initial period, the business grew quite steadily until at the close of 1922 there was $8,650,000,000 of industrial insurance on the books of the companies writing it. The Metropolitan, the Prudential, and the John Hancock, now write the bulk of the business. In addition to mere growth, the outstanding features of the progress of industrial insurance are the development of the peculiar agency organization necessary to its successful conduct, the increasing liberality of the policy-contract, the systematic prosecution of health and welfare work, and a gradually decreasing expense rate, which has made possible a return to policyholders in the form of dividends, although the nature of this business is such as to make its cost high, compared with ordinary insurance.

Policy Contracts.—In this country, early industrial policies provided for a weekly indemnity during sickness, an annuity in old age, and a burial fund in case of death. The sickness insurance feature and the annuity provisions were speedily abandoned, however, inasmuch as they resulted in a high claim rate for sickness and were beyond the means of the working man. In addition to a disability benefit, discussed below, infantile and adult burial funds constitute the principal benefits granted in recent years, although non-cancellable sickness benefits have been successfully issued in connection with industrial life insurance abroad since 1909. The amount of insurance on infantile risks is limited by statute, a table being printed in children's policies showing the amount of the benefit each year. Since the probability of death decreases from birth to about age 10, the amount of insurance per unit of premium is greater at older ages of entry. Also, the level rates being lower per unit of insurance at older ages up to age 10, if a policy is taken at age 2, say, the amount of insurance payable may be increased as the policy grows older. Thus for a weekly premium of 5 cents it may be provided that in the case of a child insured at age 2 next birthday, $12.50 will be paid if death occurs when the insurance has been in force less than 6 months, $25.00 when the duration is more than 6 months but less than one year, $34.00 when the duration is one year, and sums greater for each succeeding year's duration, up to 8 or 10 years. It is customary in both infantile and adult policies to pay but half of

the benefit if death from any cause other than accident occurs during the first 6 months. This compensates in part for the absence of a medical examination.

During the decade from 1890 to 1900, companies began issuing so-called intermediate policies consisting in most instances of contracts for $500 of insurance with level premiums payable quarterly. Such policies have been adapted to industrial business between ages 14 and 50, the premium varying with one company from 19 cents per week at age 14 next birthday, to 71 cents at age 50, for $500 of insurance. These intermediate policies are similar to ordinary policies in that the amount of insurance remains the same at all ages, but the premiums are payable weekly and are greater at the older ages. Adult industrial policies are for relatively high amounts at the younger ages, the amount of insurance per unit of premium decreasing as the age advances.

Industrial policies may be written on the whole-life, limited-payment life, endowment, or other plans. It is desirable that a company confine itself to a relatively small number of contracts, and that the provisions of each should be drawn up in such a form as to be easily understood by the agent and the policyholder. Routine work in carrying out the policy's provisions should be kept to a minimum. Many of the provisions of industrial policies in this country are similar to those of ordinary policies. A few of the more interesting features, and some of those not found in ordinary policies, are herewith set forth:

1. The weekly premiums may be paid to any authorized agent or at the home office of the company. The latter provision is necessary because circumstances may arise which make it necessary for the policyholder to remit directly to the home office in order to make the payment within the grace period, which is usually four weeks. Some industrial companies grant postponement of premiums in case of strikes or lockouts. An allowance of 10 per cent of premiums is sometimes made if premium-payments are made directly to the head office or district office. Most industrial policies permit the policyholder to surrender within two weeks, if dissatisfied, and receive a refund of the premiums paid.

2. The company retains the right to pay the proceeds of a policy to any relative by blood or marriage, or to any other person, who, in the opinion of the company, is entitled to the proceeds on account of having incurred expenses because of the insured or of his burial.

3. The policy is void if at the time of its issue the insured has

another industrial policy with the same company, unless it is endorsed so as to permit the policies to run concurrently. This is to enable the company to keep insurance within the limits it will take, without trying to check up every policy at the home office by means of a name index.

4. Some companies do not permit the assignment of industrial policies.

5. The total and permanent disability benefit usually becomes payable only in case of loss of eyesight, of both hands, of both feet, or of one hand and one foot, the benefit consisting of one-half of the insurance in cash and one-half as a paid-up policy.

6. Industrial policies are written both on the non-participating and the participating plan. Dividends on participating industrial contracts may begin at the end of the second year, but usually they do not commence until the end of the fifth year. After that, they are distributed annually. To avoid administrative work, policyholders are not given many options concerning the use of their dividends. Some policies stipulate that dividends will be granted only in the form of paid-up additional insurance, while others provide for distribution in cash or for the elimination of a certain number of premium payments.

7. Non-forfeitable surrender values are granted, usually after 3 years if all premiums have been paid, although some policies have no value until after 5 years. The values are granted in cash, in extended term, or in paid-up fractional insurance, but most policies do not provide for a cash payment until after 10 years from date of issue. A surrender charge is usually made if the policy is surrendered before the end of 20 years. It is customary for the company to make the extended term or the paid-up fractional insurance privilege automatic. Extended term insurance is likely to prove unsatisfactory to the policyholder, but paid-up fractional insurance is more expensive to the company, the care and final settlement of lapsed policies involving considerable difficulty. In fact the company is likely to lose all trace of the policyholder who in many instances does not know that his policy has a value under an automatic non-forfeiture privilege. Automatic premium loans are not granted. Such loans might assist in keeping insurance in force during periods of unemployment, but they would involve administrative expenses and are subject to the other objections to automatic non-forfeiture privileges.

Another reason for not granting surrender values in cash is that 80 or 90 per cent of industrial business is done through women. It

is the woman who insures the whole family and keeps the insurance in force. It is asserted that it would be unfair to allow a husband to surrender his policy for cash only in order to satisfy some transient need, thus depriving his dependents of protection for which they have provided out of their weekly allowance.

Premium Calculations.—The distinctive features of industrial insurance are that premiums are payable weekly, are collected by house to house visitation, and are of even amount—the amount of insurance being the variant. Some authors include a fourth feature; namely, that industrial insurance affords facilities for insuring the entire family.[4] As in other premium calculations, the principal factors in determining the amount of insurance corresponding to a unit of premium are interest, mortality, and expenses. In estimating the interest rate which may be safely used in the computations, practically the same considerations apply as in the case of ordinary business. Concerning mortality, most companies of reasonable size might base rates on mortality tables constructed from their own experience. Both population tables and experience under ordinary business have been used in the past, but of late years the Standard Industrial table has been extensively employed. It is probably our most reliable table for industrial business. Loading for expenses constitutes quite a problem since most of the elements involved in loading ordinary premiums must be considered, and in addition the high expense of house to house calls, together with the possible future trend of expenses, must be estimated. The expenses of this business have always been high despite the efforts of the companies to keep them down. Considerable progress has been made towards reduction by better organization, the larger industrial companies being very efficiently managed, but it is doubtful if any further great economy can be brought about in the future if the business is to continue to increase as rapidly as it has in recent years.

It is customary to calculate the amount of insurance for a unit of premium. Then for premiums higher than the unit premium, the amount of insurance is usually varied in direct proportion to the amount corresponding to the unit premium; that is, for twice the unit premium, twice the amount of insurance may be granted. Suppose that the unit premium is taken as 5 cents per week, and that a cost analysis shows that a charge of K per cent of the gross premium is sufficient to meet expenses and provide for contingencies. The percentage method

[4] Huebner, S. S., Life Insurance, p. 277, and Buchanan, J. D., in Transactions of the Actuarial Society of America, Vol. XXII, p. 37.

appears to be quite equitable for loading this type of business since the greater proportion of the expenses is a percentage of the gross premium, the agent's remuneration (the largest single expense) usually being a percentage of sums collected. Collection expenses, commissions and taxes are a percentage of the gross premiums. An analysis of expenses indicates that home office expenses are the only ones which do not vary roughly with premiums. Theoretically, a more correct method of loading would consist in using a relatively large percentage plus a small constant. But since the Standard Industrial table, generally taken as the basis of net premiums, overstates the mortality, the resulting concealed loading is higher at the younger ages of entry. In practice, therefore, the percentage method yields satisfactory results. It is demonstrated in Chapter VIII that when the percentage, K, is taken on the gross premium, the gross premium may be found by multiplying the net premium by $\frac{1}{1-K}$. If the gross premium were given, the net premium may be found by multiplying the gross premium by $\frac{1-K}{1}$. Thus if the gross premium were 111.111, and if $K = 10$ per cent, $111.111 \times \frac{1-.1}{1} = 111.111 \times \frac{.9}{1} = 100$, the net premium. It appears that 5 cents per week may be considered as roughly equivalent to \$2.60 per year; that is, $.05 \times 52 = 2.60$. If \$2.60 is considered the gross annual premium, the net annual premium would be, $2.60 \times \frac{1-K}{1}$, or $2.60\,(1-K)$. Now if P_x has been determined as the net annual level premium for 1 of insurance according to any mortality table and interest basis, the insurance equivalent to a net annual premium of $2.60\,(1-K)$, on the same basis may be found by proportion. Thus if S denotes the amount of insurance for a 5-cents-a-week policy, $S : 1 :: 2.60\,(1-K) : P_x$, whence $S = \frac{2.60(1-K)}{P_x}$, the amount of whole-life insurance equivalent at age x to a gross premium of 5 cents per week for an adult industrial policy.

For an n-year endowment insurance:

$$S = \frac{2.60(1-K)}{P_{x\overline{n|}}}.$$

And for an n-payment life insurance:

$$S = \frac{2.60(1-K)}{{}_tP_x}.$$

These formulas may be used to determine roughly the amount of adult industrial insurance corresponding to a unit of premium, and they serve to illustrate the principles involved in the calculation. They are based on the false assumption that premiums are paid in advance at the beginning of the policy-year and that claims will be paid at the end of the policy-year in which they arise, since P_x is calculated on that basis. It is shown in a previous chapter that P_x may be determined for insurance payable at the moment of death, and when one mth of P_x is payable m times a year. Determine P_x on the assumption that the weekly premium will be paid $52\frac{5}{28}$ times a year, adjust for payment at the moment of death, and substitute the resulting values for P_x in the formulas, if greater accuracy is desired. To provide for initial expenses, charge enough to meet them and carry the risk that year, after which let K' represent the percentage of the gross premium necessary to pay renewal expenses, and substitute it for K, and P_{x+1} for P_x, in the formulas. More detailed calculations are required for varying amounts of insurance in the early years of an infantile policy. Many elements that cannot be calculated with precision, such as the rate of lapse, what proportion of lapses will result in a loss to the company, the effect of unemployment on the operation of adverse selection, variations in administrative and clerical expenses, the advisability of meeting competition, etc., enter into the final determination of the schedule of gross premiums. The participating plan has many advantages, therefore, although if any considerable business appears to be unprofitable, whether it is participating or non-participating, it may be allowed to lapse by the exercise of a little ingenuity in managing the agency force. Since the lapse ratio is so great, it seems probable that industrial policies remain on the books but a short time, on the average, so the correction in rates might easily be made. Nevertheless, in practice it has generally been found that rates are redundant rather than too low.

Valuation.—The three principal tables used in the valuation of industrial business in this country are the Actuaries' table, the American Experience table, and the Standard Industrial table. Three and one-half per cent is the customary rate of interest, 4 per cent being used on business issued prior to 1901. The same principles apply in the valuation of industrial business as in the case of ordinary insurance, but it must be remembered that the premiums are payable weekly. In ordinary business the net annual premium is the initial value at issue, but this is diminished by mortality more than it is increased by interest earnings, so that the terminal value at the end of one year is less than the original premium. The second year's

premium is then added to the terminal value to arrive at the initial value at the beginning of the second year. This again diminishes to the terminal value at the end of the second year. The mean of the initial and terminal values is, therefore, greater than the terminal value. In industrial insurance, however, it is not desirable to require the company to hold the mean of the initial and terminal values as its legal reserve liability under the policy, since the premium is payable weekly during the year instead of annually in advance. Thus the first year, the company does not have the entire annual premium in its possession from the outset, and at the beginning of the second year the company has only the preceding terminal value plus one weekly premium. Hence the reserve liability to be reported to the state insurance department is the mean between two terminal net values, one at the end of the preceding policy-year and the other at the end of the current policy-year. The assumption is that on December 31 policies written during the closing calendar year have been in force for an average duration of six months.

This method has been criticized because the heavy lapse ratio during the first two policy-years results in a larger proportion of lapses among policies written during the first half of the calendar year than during the last half. Consequently, of the policies remaining in force on December 31, more were written during the last half of the calendar year, and the average duration has been less than six months. Also, when business is steadily increasing, more policies are written during the last six months than during the first half of the year. The method therefore results in requiring companies to report too high a reserve liability on account of industrial business, especially during the early policy-years. Of course when policies are lapsed after having been in force for some time and yet not long enough for the non-forfeiture feature to have become operative, the company is released from reserve liabilities on account of them. Nevertheless there is a considerable strain on the company when it is required to set up the full premium value of recently issued policies and also meet the high initial expenses. It is practically impossible for a young company to establish an industrial business unless some method other than full valuation is permitted. Some states permit a deduction from the full reserve liability. For instance the insurance commissioner of Massachusetts may allow a deduction from the full value of seven-twelfths for the first year's issues and one-fourth for the second year's issues of policies issued within two years of the date of valuation. Other states permit preliminary term valuation, still others have no statutory requirement

governing the valuation of industrial business, and a few require full premium values.

Office Methods of Valuation.—In preparing to make the valuation of a company's industrial business, it is convenient to calculate and tabulate the reserve liabilities per unit of premium. This may be done, just as the amount of insurance per unit of premium may be ascertained. To quote Mr. Buchanan's excellent paper, published in the Transactions of the Actuarial Society of America, Vol. XXII, pp. 43 et seq., "The actual work of preparing the data and making a valuation, considering the large number of policies on the books of an Industrial company, may be carried on with comparative simplicity by a Group Valuation method. The details of the work would vary in different companies on account of varying conditions but the following is a general description of a method of conducting the work for the main body of an Industrial business. In some companies more subdivision of the work would be required than is here shown.

"The Actuarial records are kept on cards. Cards for new business are written from the applications accepted each week. The records are adjusted weekly by picking out the cards for the policies the agent reports for lapse and for the policies becoming claims and inserting from the lapse file cards for revivals by use of the revival applications. Tabulations are made each week by policies and units. A unit may be taken as a 5 cents a week premium. The use of the unit rather than the actual amount of insurance in the intermediate stages of the work reduces the size of the figures to a minimum. The cards for issued policies are sorted into groups by plan and age and each group totalled. The cards for lapses, claims, and revivals are sorted by plan, year of issue and age. Mechanical sorting and tabulating machines may be used very advantageously for this work. The results of the tabulation are recorded each week on weekly tabulation sheets. These sheets are totalled and the totals are checked by means of the agency records. At the end of a month the weekly tabulation sheets for the month are combined into a monthly tabulation sheet which shows the issues, lapses, revivals, etc., in each valuation division for the month. After the monthly tabulation sheets have been checked the figures are transferred to a register containing a record of the policies and units in force for each valuation division. It also contains columns in which may be entered each month, according to the number of policies and units, the new issues, revivals, lapses and claims in order to make the adjustments necessary to carry forward the business existing from month to month. When the exist-

ing business has been obtained from each valuation division, to make the valuation it is simply necessary to transfer such data from the register to ruled valuation sheets headed according to plan and year of issue with columns for the age, number of policies, number of units, amount of insurance per unit, amount of reserve per unit, etc. Multiplying the number of units by the amount of insurance per unit and the reserve per unit, accumulating and totalling gives the insurance in force and the amount of the reserve liability.''

Selection and Mortality.—Since the average amount insured under an industrial policy is small, heavy medical or inspection expenses are not permissible. Companies set up a minimum of from $250 to $500 of insurance, below which no medical examination is required. The company may provide for an inspection by a representative other than the agent, but risks are often taken on statements concerning health made by the insured and a responsible agent. Inspection or a medical examination may be required in doubtful cases, but even when the amount of insurance applied for exceeds the company's limit for non-medical business, only the short form of examination is required, which means the examination is not so thorough as it is for ordinary business, and a lower fee is paid for it.

Inquiry is made as to the history of past illness, as to occupation, and as to present state of health, but companies do not obtain as definite and complete information as in ordinary insurance. Because of this, and because industrial business is drawn from a different stratum of population, the mortality rate experienced on it is higher than that on ordinary business. There has been a distinct improvement in the mortality of the general population in recent years, however, due to public health work, better medical service, improved housing conditions, disease and accident prevention campaigns, and the welfare work of the companies. Lower mortality rates among industrial policyholders might therefore be expected. Other factors that may tend to lower the mortality of industrial risks are better distribution of business by districts, the development of a higher degree of responsibility on the part of the field force in the selection of risks, the avoidance of classes and races of unfavorable mortality experience, and the encouragement of endowment insurance. The mortality of industrial risks, however, is higher than that of the general population. This may be due to the fact that a large proportion of the insuring public take ordinary insurance.[5]

[5] Little, J. F., in Transactions of the Actuarial Society of America, Vol. XXII, p. 60.

Surplus.—While the mortality rate is higher among industrial than ordinary risks, when care is used in selection, the actual mortality is lower than that on which premium calculation and valuation is based. Interest earnings are usually higher than assumed, and there may be a considerable saving in the matter of expenses provided for in the loading. Thus surplus funds may arise in the same manner as in ordinary business. Over 90 per cent of industrial business in this country is participating,[6] although the methods of dividend distribution are somewhat limited, as indicated above in the discussion of policy provisions.

The Organization and Management of the Field Force.—Industrial insurance is confined primarily to the cities and large industrial towns, where a considerable number of people who receive weekly or monthly wages or salaries live near each other. It is not adapted to agricultural communities where people live far apart and where incomes are received at irregular intervals, such as at the marketing of livestock, wool, or grain. In the more thickly populated areas, house-to-house calls are more practicable, and in many instances industrial insurance is better fitted to the needs of the wage-earning class than is ordinary insurance. When the circumstances are such that a person would have difficulty in saving enough to meet the ordinary premium, an industrial policy should be recommended by the agent. Then if the insured's financial position improves, ordinary insurance may be substituted.

In organizing the field work it is customary to divide the company's territory into districts. The size of a district varies with different companies. A medium-sized city, or a number of relatively small towns, may constitute one district, whereas a large city may be divided into several districts. A superintendent, assisted by a number of deputy or assistant superintendents, has charge of a district and is held responsible for the details of the business and the supervision and training of the agency force. Accounting with individual policyholders is done at the district office, the home office usually knowing nothing as to whether a policyholder is in advance or in arrears as regards his premium payments, except in case of lapse or re-instatement, although a duplicate of the register may be kept at the home office. The home office deals only with the account of the agent. Correct accounting in district offices is maintained by inspectors sent out from the home office. Regular agency meetings are held for the

[6] Carpenter, R. V., in Transactions of the Actuarial Society of America, Vol. XXII, p. 62.

instruction and encouragement of agents in their work. Each agent starts out on Monday morning with a list of policyholders from whom he is supposed to collect premiums. This is sometimes spoken of as his "debit," or "debit book," although it is commonly referred to as the "register" or "life register." It is changed each week, new issues and revived policies being added, and matured endowments, lapses, paid-up insurances, surrenders and claims, being deducted. The term "debit," as generally used, means the sum total of the weekly premiums he is required to collect. This is the sum with which he is said to be "debited." It is made up of the premiums corresponding to the business shown on the life register. Premiums paid in advance, and premiums in arrears for a period shorter than four weeks, however, do not affect the debit.

The number of policies per debit varies greatly, the average number probably being between 750 and 1500, the higher numbers obtaining in those sections of the cities where the homes of large numbers of wage earners are close together. In one large company the average debit covers at least 1500 policies, with numerous debits covering over 2000 policies. The average premium per policy is about 10 cents per week, making debits from $75 to $200, the policies averaging less than $150 each. If the policyholders of a debit are concentrated in one locality, the time consumed in making the necessary calls may be reduced to a minimum, thereby giving the agent more time to write new business. In most instances, collections should be completed by Thursday morning, so that the rest of the week may be devoted to canvassing for new business. If an agent writes a risk outside of his district, it is ordinarily transferred to another agent. As a result, all agents gain, since on the average they receive as many policies by this practice as they lose, and the consolidation of debits enables them to do more business than would be possible if a number of long-distance calls were necessary. Special attention to the consolidation of debits has enabled the companies to increase the average amount of debits, keep down the cost of operation, at the same time increasing the earnings of agents. The agent is required to record the amounts collected in his collection book and also in the policyholder's receipt book or card. An inspection by a deputy superintendent, or inspector, of the receipt books of policyholders and a comparison of them with the agents' collection book serves as a check upon the collections reported. The inspection of the debit is made at irregular intervals, so as to obviate the possibility of the agent's straightening out temporarily any irregularities. The agent must report a policy as a lapse

if the policyholder's premium payments are four weeks in arrears. The superintendent, or deputy superintendent, is then required to interview the insured with a view to keeping the policy in force, since lapses result in a loss to the insured, and are unsatisfactory to both agent and company.

For collecting the debit the agent may receive a salary, based on the size of the debit, but more often he receives a percentage of the premiums collected. He also receives commissions on new industrial business, and the usual initial and renewal commissions on any ordinary business he may write. It is customary to allow initial commissions only on increases in the debit. That is, if an agent writes new industrial business on which the weekly premiums total \$1 and lapses \$0.25, he receives initial commissions of so much times \$0.75. To illustrate further, suppose the agent receives 15 per cent on the amount of the debit collected each week, and 15 times the increase collected. If his debit were \$100, of which he collects \$99.75, his compensation will be \$14.96. If in addition to collecting the \$99.75, he writes \$1 in new premiums, his compensation will be increased by, $15 \times .75 = \$11.25$, making a total of \$26.21 for that week. If he collects all of the \$100 debit, his compensation for increase in debit will be \$15, which added to 15 per cent of the full debit collected, makes a total of \$30 for that week's work. Revivals are thus encouraged, since every revival offsets a lapse. An unfavorable record as to lapses reduces an agent's commission. It is, therefore, unprofitable for him to neglect his debit in order to devote his time primarily to new business. The superintendent and deputy superintendents receive a salary with a bonus for increases in premium income from debits under their supervision. Members of the field force who render efficient service are advanced to more responsible positions, one of the advantages being that the business is capable of indefinite expansion so that the number of responsible positions is unlimited. The remuneration is likely to increase with the cost of living during a period of inflation. Thus during the World War and the period of inflation that followed it, it was easier for an agent to write new insurance and maintain old business in force, hence the incomes of industrial agents increased without a proportional increase in the expense rate of the company.

Objections to and Services Rendered by Industrial Insurance.— In the literature pertaining to industrial insurance, three principal objections to this form of insurance appear. In the first place, it has been urged that the insurance of children might lead to their

destruction in order that the insurance might be gained. Experience has conclusively demonstrated that such fears are entirely without foundation. In the second place, it has been stated that the heavy lapse rate results in huge losses to the industrial classes in the aggregate, since so many pay premiums for which they receive nothing. As regards this matter, it should be pointed out that those who lapse have at least had protection over the period during which premiums were paid. Furthermore, the foundation for this criticism is becoming more and more undermined by the successful campaigns waged by the industrial companies against lapses. For instance, with one large company, the percentage of terminations less revivals to mean number of policies in force declined from 20.13 in 1900 to 6.67 in 1920. As regards terminations during the first policy-year in which so large a proportion of lapses occur, this company reduced its lapse ratio on idustrial policies from 61.87 per cent in 1896 to 21.11 in 1918,[7] the decline over the period being remarkably uniform. The third objection is the high cost of industrial insurance compared to that of ordinary life. The reasons for the higher cost of industrial business are the higher mortality and expense rate. Improved mortality, and improved administrative methods have resulted in a material reduction in the cost during the past two decades, in spite of the increased cost of supplies, clerk hire, etc. Thus one company increased the amount of whole-life insurance as of age 10 for a 5-cents-a-week premium from $120 in 1906 to $162 in 1920. Increases are roughly proportionate at older ages, and on infantile policies they are even greater. Nevertheless the business is still expensive, the expenses being justified by its exponents on the grounds that the services rendered are commensurate with them, and that there would be no protection for the industrial classes were it not for the weekly house-to-house system. Others state that the benefits of industrial insurance should be looked upon in a manner similar to deductible average in marine insurance; that is, it costs more to administer than the protection is worth. A fourth objection is sometimes met, but this is taken up below under services rendered.

The services rendered by industrial insurance are quite numerous and varied. In general they may be classified under three main headings: welfare work, savings, and insurance. As regards welfare work, it is urged that industrial insurance has assisted materially in the improvement of living conditions, the compilation of statistical data

[7] Carpenter, R. V., in Transactions of the Actuarial Society of America, Vol. XXII., p. 64.

regarding diseases and occupations with a view to lower mortality; and that in coöperation with public health bodies it has assisted in educating the public to correct ways of living by the publication of health literature and through medical and nursing services. Mr. R. V. Carpenter summarizes the health and welfare work of one large company as follows: "The appropriateness of this sort of work for Industrial companies would seem to be beyond question. With fire insurance companies striving after fire prevention and casualty companies spending large sums for the prevention of accidents, is it not reasonable that life insurance companies should devote a very considerable portion of their energies to the conservation of that most precious possession, human life, and is it not particularly fitting that the Industrial companies, in close touch as they are with those elements of our population who know so little of the laws of health, should devote very special efforts to this cause? The industrial company, in a peculiar way, has a double opportunity. Through its agency force, going directly into policy-holders' homes, often their friends and counselors, the company has an unparalleled opportunity of conveying to these policy-holders such messages on behalf of their health and welfare as it considers desirable to send. On the other hand, the great amount of material coming into its Home Office, through death claims and otherwise, together with the availability of the agency force for gathering statistical data, gives the company an unusual opportunity for the study of facts relating to health and the common weal.

"These opportunities have not gone by unnoticed. Probably every Industrial company of importance has a policy-holders' magazine, and probably in every case this is to a certain extent a health magazine. The larger companies have also published additional health literature, conducted Home Office studies, and in other ways aided in health work.

"The company with which I am associated has entered into health and welfare work to an unprecedented degree. For many years it published its health magazine and other health literature. In 1909, however, the company determined to enter more intensively into health and welfare work, and since that time it has devoted a great amount of energy to work of this nature.

"One of the most important branches of the work is the Nursing Service, inaugurated in 1909. This service has been established in the principal cities and towns, about 2700 in number, and is available to any Industrial policy-holder in need of the service, under estab-

lished rules. Last year over 1,600,000 visits were made. Another important activity is the widespread distributions of pamphlets and leaflets at the rate of 15,000,000 or 20,000,000 yearly, dealing with the principal preventable diseases and a large variety of health topics. Welfare exhibits are frequently held at county fairs and similar occasions. The officers and field representatives are continuously coöperating with social agencies, schools, health departments, etc., in matters of public health. The members of the Field Force assist in, and sometimes inaugurate, 'Clean-up Campaigns' and 'Baby Weeks,' and frequently assist in securing enactment of health legislation. Last year an active campaign was carried on in California and Oregon which played a part in defeating proposed anti-vaccination amendments in those states. Numerous studies of mortality and morbidity have been conducted, and many sickness and other field surveys have been made, resulting in the accumulation of valuable material and sometimes in concrete recommendations to remedy unfavorable conditions disclosed. I will not weary you by enumerating the investigations that have been made, except to mention one of the most important, a community tuberculosis experiment, known as the 'Framingham Experiment.' In 1916 the company suggested this experiment to the National Tuberculosis Association, offering an appropriation to carry on the work, and the offer was accepted. The experiment consists of the adoption by the city, which has extended its hearty coöperation, of certain special measures relating to medical examinations, surveys, medical and nursing care, dispensary and institutional treatment, etc., with the view of determining to what extent tuberculosis can be controlled. The experiment was started in 1916, and the degree of its success and importance can be partially measured by the fact that in 1920 the tuberculosis death rate for the city was 64 per 100,000, against an average of 121 for the decade preceding the experiment.

"The company's justification for its welfare work is not alone that it is beneficent, but that it saves insured lives, and saves a great many lives. Just how many we of course do not know. We do know that our Industrial death rate is decreasing to a marked degree, and calculations have been made showing that our death rates for certain important preventable diseases have declined more rapidly than the corresponding rates indicated by United States Census returns. We do feel that 11,000,000 or 12,000,000 of nursing visits to people many of whom know so little about personal hygiene, and the dissemination of over 200,000,000 pieces of health literature, in addition to the com-

pany's magazine, among these people, must inevitably have brought about significant results in the saving of lives and the prevention and cure of disease, not only among our own policy-holders, but to a certain extent among the general population. This story of health and welfare work is of course familiar to many of the members of the Society, but I feel that you will be indulgent, because this work is to-day so important a part of Industrial insurance that the record would be far from complete if it were not briefly described.''

Endowments, especially children's endowments, supply a means of saving and industrial agents are in a position to encourage thrift amongst industrial classes. The habits of thrift and the appreciation of insurance thus developed may often prove beneficial, but as a means of saving, industrial insurance can scarcely be urged as an improvement over other facilities, since the expenses of the former are so great.

As regards insurance, the facilities furnished by industrial companies are such as to extend protection to millions who would not now be insured at all. It is customary to point to the huge sums paid out annually to industrial policyholders. Of course such figures should be compared with sums paid in by policyholders. It is undoubtedly true that there are fewer pauper burials than would be necessary if there were no industrial insurance, and herein is to be found the chief question concerning this form of protection. Most persons would probably agree that one should provide for decent burial of himself and his dependents. The question is, what shall be the method of making such provision? If industrial insurance supplies the only means, the question seems to be answered. When a person, however, is in a position to meet such expenses should they arise, without the use of industrial insurance, the case is different.

Nevertheless, companies should be encouraged to reduce mortality and expense rates, and continue their services in this field until and unless some other method is evolved to secure these benefits to the industrial classes at a lower cost.

CHAPTER XIX

Accident and Health Insurance

Development.—*Early Health Companies.*—Disability benefits as granted by friendly societies and similar organizations, constitute one of the oldest forms of protection. In former times, the benefits were provided by assessments which had no scientific basis. This is also done to-day by some of the assessment societies operating in this country. The first attempts to write commercial health insurance in the United States were made during the decade from 1840 to 1850. The Massachusetts Health Insurance Company of Boston, the Health Insurance Company of Philadelphia, the Spring Garden Health Insurance Company of Philadelphia County, and the Eagle Life and Health Insurance Company of Jersey City, were among those founded in this period. During the next half-century numerous organizations were founded, but down to nearly the close of the century the business was characterized by a large number of failures. The companies were in charge of persons not experienced in insurance matters. Adequate contingency reserves were not set up. None of the companies had adequate financial support. They had to depend on the statistics of the English friendly societies giving the number of weeks of genuine, actual sickness in England among the middle and industrial classes at different ages. Their managers had no knowledge of the expenses of running such a business, or of the claims they would have to pay because of malingering. The few men of ability and integrity connected with health companies during this period soon withdrew and their places were usually filled by incompetent and irresponsible persons. None of the companies founded during the early period survived, nor did any of them furnish from their experience data of real value concerning the rate of sickness.[1]

Accident Companies.—The first attempt to write commercial accident insurance, as distinguished from accident benefits provided by friendly societies and similar organizations, was made in this country in 1863. On June 17 of that year the Travelers Insurance Company

[1] Messenger, H. J., The Rate of Sickness, in T.A.S.A., Vol. X., pp. 371, 372.

of Hartford, Connecticut, was chartered to write accident insurance. It began business April 1, 1864. Prior to this date, English companies had granted insurance against hazards involved in railway travel, but insurance against accidents in general may be said to have begun at this time. The Travelers was organized with a capital stock of $250,000, so that its financial stability was assured from the outset. Other companies entered the field of accident insurance until by the close of the century the amount of personal accident insurance in force had grown to considerable proportions. With the growth of the business, competition with fraternal, assessment, and benevolent societies, as well as among the stock companies themselves, has resulted in a broadening of the field of general accident insurance. It now includes liability insurance of all forms, and property damage due to accident, as well as sickness and burglary insurance. Companies granting protection against such contingencies are commonly referred to as casualty companies, and the branch of insurance as casualty insurance.

Accident and Health Benefits Combined.—The particular branches of casualty insurance with which this chapter has to do are accident and health insurance. As explained above, accident insurance was first conducted scientifically in this country by the stock accident insurance companies. They were also the first to offer health insurance with adequate financial security. During the closing years of the last century the accident companies were competing so keenly for business that each vied with the others in adding some new feature to the policy contract in order to make it more attractive. Finally one company offered what was known as a special health policy in connection with its regular accident policy. For a small additional premium the company promised weekly indemnity in case of disability from any one of fifteen diseases. Among the diseases specified were typhoid or scarlet fever, smallpox, measles, and appendicitis. All of them were such that their contraction would partake of the nature of an accident. Other companies followed until soon the practice of combining both accident and health insurance of a limited character in one contract became quite general among the stock accident companies.

The accident companies were in a much better position to carry on this business successfully than were the early health companies, since they had plenty of financial resources and were managed by men experienced in insurance as it was then conducted. These men, however, were merely casting about for salable contracts and lacked knowledge of the rate of illness, or alleged illness, according to which the

company would be obliged to pay claims. They first discovered that policies limited to a few stipulated diseases were not sufficiently attractive to the public. Hence this list was extended to include 21 diseases, and later to over 30 besides providing certain payments for surgical operations. By 1910 the issuance of policies covering all diseases, with few exceptions, as well as death or injury by accidental means, became a common practice.

Development of Non-Cancellable Policies.—The next important step in the development of this business was taken in 1915. In that year the Pacific Mutual Life Insurance Company granted the first non-cancellable accident and health policy issued by an American company. Prior to this time accident and health contracts had been written to cover a limited period of time, ordinarily one year. At the end of the year, provided the insured were not then in receipt of benefits, the company might renew the contract at the same or a different rate of premium, or it might refuse to renew the contract altogether.

From the company's point of view, the cancellable policy has many advantages, among which the more important are that it may relieve itself of liability if the risk should become undesirable in any way, and it may advance premiums if they prove to be inadequate to meet claims and expenses. Such a contract is disadvantageous to the insured in that it is likely to be cancelled if he becomes disabled or diseased, or if he presents a number of small claims of a suspicious nature. Hence he may be unable to secure additional protection when most needed. Also, at the end of any policy year he may find it necessary to pay an increased premium for the next year's protection. The principal advantage of the cancellable policy to the insured is that he may secure protection against temporary impairments, the benefits being payable from the beginning of the impairment, at lower rates than those deemed necessary when benefits under the non-cancellable policy are payable from the beginning of disability.

The progress of the non-cancellable policy has been somewhat slow. Its particular appeal is the disability benefit. It was thought that business and professional men who could withstand a waiting period of from two weeks to three months after impairment and before the benefits were payable, would be attracted by the low rates for protection against the more permanent impairments. Considerable interest has been displayed by these classes.

At first, as Mr. John M. Laird has said,[2] "In the general public

[2] See T.A.S.A., XXIII., Part Two, p. 386.

the only person who fully appreciated a non-cancellable policy was the one who had recently had a commercial policy cancelled because of impairment.'' Furthermore, the accident and health companies were handicapped, in that they could not carry on an extensive selling campaign for non-cancellable business without endangering their ordinary cancellable business. Most life companies, too, were not inclined to extol the merits of a non-cancellable accident and health policy as compared with the benefits under their own disability clauses. These difficulties together with the dearth of statistics and the fear that a business depression would result in an unusually large number of claims, led the companies to develop this type of business slowly and cautiously.

Due to experience showing that about one-half of the claims under ordinary cancellable policies arise during the first two weeks of disability, and 85 per cent during the first three months, companies began issuing the disability benefit with a waiting period of from two weeks to three months, whereby the insured person agreed to wait two weeks, a month, two months, or three months, according to the wording of the contract, after the occurrence of disability before being entitled to the benefits. Policies requiring the longer waiting period were sold at a much lower premium rate than those with shorter waiting periods. A medical examination, not necessary for cancellable policies, was required. Early non-cancellable policies were sold at a premium rate the same for all ages. Policies were not issued to persons older than age 50, and at first it was provided that they should only be effective to age 60. Some policies now extend the coverage to age 65. Following an investigation by the Bureau of Personal Accident and Health Underwriters in 1921, companies began charging rates according to the age of the insured person when the policy is issued and also began setting up reserves to provide for increasing probability of impairment as the insured becomes older.

Types of Organizations Offering Disability Protection.—Disability protection is afforded by most of the old-line or legal-reserve life insurance companies in the form of a disability clause. These clauses, however, provide benefits only in case the disability is both total and permanent. Such clauses are considered at length in the chapter on disability and double indemnity benefits. Benevolent and fraternal societies and assessment associations have for centuries furnished protection of one kind or another, often on an unscientific plan. There are many of such organizations to-day, and a large proportion of them furnish a death benefit as well as weekly indemnity for im-

pairment. Some are of undoubted financial stability, while many are of such a character as to require careful investigation on the part of the insuring public. Accident and health departments of several legal-reserve life companies offer cancellable and in some instances non-cancellable accident and health policies. These, of course, may be taken without serious question so far as the financial stability of the company is concerned. A large percentage of the commercial accident and health insurance written in this country is conducted by so-called casualty companies. For the most part these also are well established institutions which operate on a sound financial basis.

Types of Policies.—*Fraternal and Assessment Certificates.*—Certificates issued by assessment, fraternal and benevolent societies providing disability benefits are subject to the conditions of death benefit certificates discussed in the chapter on fraternal and assessment insurance.

Travel-accident Policies.—As regards the contracts issued by the commercial companies, several types prevail. The so-called travel-accident policy is designed to protect the insured against the hazards incident to traveling on public conveyances. This policy provides certain stipulated benefits for death or injury by accidental means. Usually such policies may be purchased at railway stations and other ticket-selling places. Some hotels present departing guests with policies running for a short period of time, ordinarily covering the next 24 hours. Travel-accident policies usually cover but short periods of time. They may be purchased to cover periods of 1, 7, 30, 60, or 90 days. The customary one-day policy costs 25 cents for $2500.00; that is, for 25 cents the company agrees to pay $2500.00 to the insured's estate should he die within 90 days by means of an accident occurring within one day following the date of issue. A one-day policy usually protects the insured for a greater period than 24 hours, since policies issued after noon run the next 24 hours and until the following midnight, and those issued before noon run until noon of the following day. Disability benefits consist in $1250 for loss of two members, $12.50 per week for 52 weeks for total disability, and $6.25 per week for 26 weeks for partial disability. These benefits are doubled if the disability is caused by accident while the insured is being transported in a railway passenger car, or a steamship licensed to carry passengers. The contract is hedged about with numerous restrictions to the effect that no indemnity will be paid in cases of disappearances, injuries without a visible wound or contusion, poisoning, sunstroke, freezing, injuries due to firearms, fireworks, racing,

suicide, or while entering or leaving a moving conveyance or occupying a place in the same not intended for passengers. In other words, the purpose of such policies is to protect against the hazards of accident incident to travel, and they are strictly limited to indemnity for losses resulting therefrom.

Such contracts are expensive in comparison with contracts extending over a year or more, or with the accidental death benefits granted by some life insurance companies. The latter, however, do not cover impairments. On the daily basis, policies issued just after noon or just after midnight run for nearly 36 hours. Those issued just before noon or just before midnight run for a trifle over 24 hours. The average time is probably 30 hours, since few policyholders take the trouble to time their purchases so as to get the maximum number of hours of protection. Taken at intervals of 30 hours, therefore, 292 policies would cover one year's time. At 25 cents per policy, the cost of one year's protection would be $73 for a death benefit of $5000, if the death is caused by accident while on a common carrier, otherwise $2500, and the additional coverage mentioned above. These rates would not appear to be unreasonable when one considers the expense element. Also, the company may be subject to the effects of adverse selection in that a person is more likely to seek protection if about to undertake a trip rendered unusually hazardous by some cause or other, such as stormy or foggy weather at night, or high water in a flood country.

Specific Policies.—Specific policies were discussed in the historical development of cancellable insurance sketched above. Existing policies of this character provide sickness benefits for as many as 85 specified diseases, indemnity for injuries occasioned in 16 different ways, and lump sum payments for the fracture of any of 16 bones enumerated in the contract, such injury being not otherwise covered. Hospital fees and emergency benefits such as identification, notification of relatives or friends, and financial aid not exceeding a stipulated sum, are also provided, in addition to a benefit for loss of life or for continuous total disability. These policies are not so desirable as general policies since the coverage is not so complete, but they cost less.

General Policies.—General or unlimited policies cover losses, with a few exceptions, resulting from bodily injury through accidental means, and disability by disease, according to certain schedules set forth in the contract. The usual exceptions as regards accidents are those resulting from bodily or mental infirmity, or ptomaine poisoning, bacterial infections (except pyrogenic infections occurring

through accidental cut or wound), the hazards of war, and aeronautics. Disease contracted in the tropics, in Alaska or in the British Possessions in America north of the sixtieth degree, or while engaged in military or naval service, is not covered. Renewal of a cancellable policy is generally not permitted beyond age 50 or 51. To prevent over-lapping claims, it is provided that the insured person shall not be entitled to the health benefit if he has made claim or may become entitled to indemnity under the accident provisions of the contract, and the payment of an accident benefit will preclude collection under the health provisions.

Benefits Payable Under General Policies.—The customary practice in providing accident benefits, is to pay a principal sum under certain circumstances such as death through accidental means, loss of both hands, both feet, the sight of both eyes, one hand and one foot, either hand or foot and the sight of one eye. Provision is then made, according to a schedule, for amounts payable in case of loss through less serious accidents, such as one-half the principal sum for the loss of either hand or foot, one-third for loss of the sight of one eye, etc. If more than one of these losses results from the same accident, the company will pay the highest benefit for any one of the losses sustained, but it is not liable for the others. Loss of hand or foot is ordinarily defined as dismemberment by severance at or above the wrist or ankle joint. As above mentioned, the policy is ended upon the payment of any one of these benefits. Weekly indemnity is provided for total and for partial disability according to another schedule contained in the policy. It is quite common to provide double indemnity, and sometimes triple indemnity for these injuries if the insured sustains them by accidental means while a passenger on a common carrier or passenger elevator (excluding mine elevators). Sometimes double indemnity is payable if the injuries are caused by the burning of a building (if the insured is in it at the time the fire begins), the collapse of the outer walls of a building while the insured person is in it, a stroke of lightning, the explosion of a steam boiler, or by a cyclone or tornado.

In lieu of the weekly indemnity, the insured is entitled to certain elective benefits in one lump sum. The amount of the lump sum payable depends on the nature of the injury. Again it is necessary to set forth a schedule of injuries for which elective benefits may be secured, and the compensation for each one of them. Thus it may be provided that for a complete dislocation of the shoulder, $4 will be paid in a lump sum for every $1 of weekly indemnity, or $100

in a lump sum for every $25 of weekly indemnity, if the insured so elects in writing within 20 days following the accident. The reason for requiring the insured to exercise his option within so short a period of time is to prevent adverse selection in that those recovering quickly would be likely to choose a lump sum settlement, whereas those requiring a longer period than the average for recovery would probably choose weekly indemnity. The lump sum payable is equivalent to so many weeks' indemnity, the number of weeks being the average period required to recover from the particular impairment.

Concerning the health benefits, it is necessary to state the weekly indemnity payable. For temporary total disability, whereby the insured is prevented by bodily disease from performing any and every kind of duty pertaining to his occupation, a stipulated weekly indemnity will be paid. If partial disability which renders the insured incapable of performing one-half of the duties of his occupation follows the total impairment, one-half of the weekly indemnity is payable. It is customary to limit the period during which the benefits are payable to a definite number of weeks, such as 52 consecutive weeks for either or both of the above contingencies. The "permanent disability benefit" under the health insurance part of the contract provides for weekly payments in case the insured should lose "the use of" both hands, both feet, one hand and one foot, or the sight of both eyes, or become totally and permanently disabled. The company will then pay, in lieu of all other benefits except surgical or hospital indemnity, the same weekly indemnity as would be due in case of total partial disability. "Permanent" disability benefits, however, are also limited to a definite period of time, such as 100 weeks. Specific payments for surgical operations varying with the severity of the operation, hospital benefits, and expenses incident to identifying the insured and to placing him in the care of friends, are also provided; each benefit being definitely limited in amount.

Lack of Uniformity.—Though such provisions and benefits as these are the common ones in accident and health policies, emphasis should be laid upon the fact that there is by no means the same uniformity of contracts in this business as may be found in other forms of insurance. Differences in contracts and in the practices of organizations offering this form of protection are so numerous and important that the prospective purchaser must be advised to study the policy he is contemplating with great care.

In analyzing an accident and health contract the first clause that must be given attention is the insuring clause, setting forth the losses

which the policy covers. The provision as to accidental death is usually that the company insures the applicant against loss resulting, "directly and independently of all other causes," from bodily injuries occurring during the term of the contract "solely through external, violent and accidental means." As to disability by accidental means, however, quite a number of policies omit the words "directly and independently of all other causes," and the words, "external," and "violent." In other words the phrase is, "solely through accidental means." According to court decisions, the words "external" and "violent" are of little importance in determining liability.[3] The omission of the provision concerning all other causes, however, is of importance in that the company is not liable if coöperating with the injury in causing loss there should be preëxisting disease, bodily infirmity or abnormality.

Accidental Means.—Practically all companies provide that indemnity for accident shall be due only upon loss through "accidental means." The reason for this is that the definition of an accident is so broad that death or impairment from disease might be construed as "accidental," especially if the disease is infectious or contagious, and it is not the intention of the company to pay such losses under accident coverage. A distinction between the result and the means is therefore necessary. The company grants protection against loss effected by accidental means and not against accidental loss. For example, suppose a railway mail clerk, in lifting a bag of mail, should suffer hemorrhage of the lungs, resulting in death within a few minutes. If the facts fail to show that the insured slipped or stumbled, that anything fell upon or against him, or that he did something he did not intend to do, even if the *result* of his exertion is considered an accident, it is not considered as caused by *accidental means.* It was caused by exertions which he fully intended to make. In other words, loss through accidental means is covered, but unexpected results of intentional acts are not.

Sole Cause.—After it has been shown that the insured received an injury caused by accidental means it is necessary, should the policy so provide, to show that the injury was the sole cause of the loss, independently of all other means. Thus in the above illustration, if the insured were suffering from a disease which had weakened the blood vessels ruptured by his exertion, the company would not be liable even if accidental means were in evidence. An injury is not

[3] Laird, John M., Personal Accident and Health Insurance, published in T.A.S A., Vol. XXIII., Part Two, p. 369.

considered the sole cause of death if any disease, bodily infirmity, or abnormality existing at the time of the injury coöperates with the injury as one of the causes of loss. Furthermore, a trivial accident which merely accelerates the loss does not render the company liable. On the other hand when an insured person who is suffering from a fatal malady receives an injury severe enough to cause death independently of the disease, as when a person near the final stages of tuberculosis is struck by lightning, the company must pay the claim. If an injury, too, caused by accidental means brings about a disease which in turn results in death, the injury is regarded as the sole cause, or cause of causes, and the company is liable. The question is similar to that of proximate cause in fire insurance in that the sole cause is considered to be the sole original cause of an unbroken chain of events bringing about loss even though the final link may be a disorder which is ordinarily called a disease.

Suicide.—Virtually all accident and health policies stipulate that no payment shall be made if death results from suicide, sane or insane. Many courts, however, have construed this clause to mean that the company is excused from payment in all cases in which the insured takes his own life, "except where he was so devoid of mind at the time that he did not realize or contemplate the nature of his act and the physical consequences thereof. In other words, that he was so devoid of mind that he did not know what he was doing, and did not intend to commit the act which resulted in his death."[4] Though in case of doubt as to the cause of death, the burden of proof is technically on the claimant to show that death was caused by accidental means rather than suicide, nevertheless, if the company cannot establish a motive for, or present evidence showing, suicide, it will probably be obliged to pay the claim, since in the absence of evidence there is always a presumption against suicide. An autopsy is often of great value in determining the cause of death, hence most policies provide that the company may make one, except where it is forbidden by law.

Total Disability.—If the companies insist upon it, injuries resulting in total disability must also be caused by accidental means in order to entitle the insured to the benefits. Companies often pay these benefits, however, if disability results from accidental injuries whether or not the injuries are due to accidental means. The companies can afford to be more liberal in the interpretation of the accidental disability provision since disability due to disease is not likely to be

[4] See Legal Notes, T.A.S.A., XXIII., Part Two, p. 452 for citations.

construed as accidental, because total disability by disease is also covered in many instances under the same contract. The definition of total disability in specific accident and health policies may require that the insured be disabled to such an extent that he cannot pursue any occupation for wage or profit. In unlimited policies, the usual provision is that total disability shall be defined as inability to perform any and every kind of duty pertaining to the insured's own occupation. One may well take the trouble to read the definition of both total and partial disability before selecting his accident and health policy.

Death, to be considered accidental within the meaning of the policy, must occur within a limited time after the accident, such as 90 days. The double indemnity benefit in some instances may be extended to cover injuries sustained while riding in a private conveyance, such as an automobile, although this extension requires an extra premium.

Standard Provisions.—In accident and health policies there are to be found so-called standard provisions. Some of these are required by the laws of the states, while some are required in some states and are either not required or are prohibited in others. It is customary for a company to include 16 or 20 (sometimes more) of such provisions in its contract. As regards a provision required or permitted in some states and prohibited in others, the company will state that the provision shall be effective, "where it is not forbidden by law." These provisions relate to endorsements, to the application, to the agent's lack of power to waive any of the policy's provisions, to the requirement of notice to the company of injury or sickness within 20 days of such, and of accidental death immediately after its occurrence, the right of the company to cancel the policy at any time (except in non-cancellable policies), and to numerous other matters. Provisions required by law, as indicated above, are not uniform in the various states, nor are the standard provisions in the policies of different companies uniform. It is advisable for the applicant to read these carefully.

Additional Provisions.—Additional provisions ordinarily consist in exceptions; that is, statements of circumstances and conditions under which the company will not be liable for payments. They have already been mentioned briefly. While the exceptions noted are the usual ones, it is highly important for the prospect to examine them with care.

Care in Purchasing.—Most agents do not supply prospective pur-

chasers with sample policies except on request. This is true of all forms of insurance. It is probably just as well that such is the case for most prospective purchasers would not take the trouble to read them, or would not understand them anyway, and the company's printing expenses would be increased if agents distributed samples freely.

Nevertheless it is important for the prospective purchaser of cancellable accident and health contracts to study: (1) the insuring clause, (2) the definition of total disability, (3) the risks excluded, (4) the limiting ages, (5) the waiting period if any, (6) the maximum number of weeks during which indemnity will be paid, (7) the time following an accident within which death must occur in order to entitle the beneficiary to the accidental death benefit.

Non-cancellable Policies.—The principal difference between a general accident and health policy which is cancellable and one which is not so, consists in the company's inability to cancel the latter before age 60 or 65. Non-cancellable policies, however, contain the standard provision that the benefits will be reduced if the insured changes to a more hazardous occupation than that stated in the policy, just as do cancellable policies. The reduced benefits will be the same as the premium paid would have purchased at the rate for the more hazardous occupation. The insured may also notify the company of a change in occupation, surrender his policy, and receive in its place a contract containing the same provisions with benefits reduced.

Advantages.—The principal advantage which the non-cancellable feature holds for the insured is quite obvious. Once he obtains such a policy he is certain that it will not be cancelled because of ill health, nor will an impairment rider be attached to it because of his being no longer desired by the company.

Some companies provide indemnity under non-cancellable policies for loss of life as well as of business time. Others, while covering accidental death under their cancellable policies do not provide for such a benefit in the non-cancellable policy. Neither cancellable nor non-cancellable policies provide for payments when death results from disease. The reason given for the omission of the accidental death benefit in some non-cancellable policies is that if unfavorable court decisions should result in an extension of the death coverage, the company could not protect itself under the non-cancellable policy, whereas under the cancellable contract no difficulty in this matter would be encountered.

Benefits.—Since the benefits under non-cancellable policies are

confined primarily to disability, an examination of them is important to the prospective purchaser of such a contract. In general they are similar to those under cancellable contracts, but there are differences in certain respects which make it necessary to scrutinize the provisions of the particular contract under consideration. One important difference between cancellable and non-cancellable benefits is that in cancellable contracts disability benefits are usually limited to a specified number of weeks, whereas under non-cancellable policies they are usually payable as long as the insured shall live. Another is that cancellable contracts often provide indemnity beginning with disability (except disability from ill health within two weeks after the policy is first issued) whereas non-cancellable policies generally require a waiting period of from two weeks to three months following disability during which no indemnity is payable. Under the non-cancellable contracts of American companies, payments because of disability occurring before the limiting age of 60 or 65, once begun, continue until the death of the insured. Some British companies cease disability payments at the limiting age of 65, a factor to be considered if one is contemplating the purchase of such a contract from a foreign company.

In purchasing a non-cancellable policy, therefore, the prospective purchaser should satisfy himself as regards the following: (1) death benefits (if any) and the circumstances under which they are payable, (2) the waiting period, (3) the amount of the benefits payable, (4) whether the benefits, once begun, continue until the death of the insured irrespective of the limiting age.

Underwriting Practices and Problems.—Ordinary cancellable accident and health policies are usually issued without a medical examination for a term of one year only, but they are renewable for succeeding terms of one year each, provided both the company and the insured are willing. The company may cancel the contract at any time, in which event it must return the pro-rata unearned premium to the insured person. In other words, if the company cancels after the policy has run six months from the last annual premium payment, it must return one-half of the premium collected for that particular year, and one-fourth of the premium if the policy has run 9 months, etc. When accident insurance is issued alone, it is renewable at a flat rate regardless of age, except for age limits, such as 18 to 65, beyond which policies will not be issued. When health insurance is issued alone, it is renewable at a flat rate from ages 18 to 50, with a higher flat rate for ages 51 and over, whether the

insured had the policy prior to age 51 or not. When accident and health insurance are combined in one policy, the contract is usually renewable to age 50 at a flat rate, and it may be renewable for a higher flat rate at ages above 50. Since the company, however, may cancel the policy at any time or refuse to renew it at any anniversary, it may raise the premium rate as it sees fit whether the insured has reached age 50 or not.

Reasons for Cancellation.—According to Mr. John M. Laird the rate of cancellation and non-renewal is probably from 1 to 2 per cent each year on health insurance, and from ½ to 1 per cent on accident insurance. According to the same authority the principal causes for cancellation are: (1) misstatement in original application, (2) impaired physical condition, (3) material reduction in the earnings of the insured, (4) repeated claims for recurrent injuries or disease, (5) unsatisfactory moral hazard.[5]

It is the practice of companies to cancel policies in case any one of the above conditions develop. This practice tends to substantiate the claim that the policy is likely to be cancelled at a time when the insured needs it most, but this is no reason for condemning accident and health insurance. The fundamental purpose of the cancellable policy is to provide indemnity for losses due to unusual happenings, or as brought out in the account of the historical development of it, to provide indemnity against losses due to accidents and to diseases of an accidental nature. If the company discovers that an insured person has contracted a disease of the kidneys, has degenerated morally, or has experienced a sharp decline in his earnings, thus increasing the temptation to malingering, there is no reason why the company should not cancel the policy in order to protect itself. The company granting cancellable policies at currently quoted rates has no intention of affording protection against such hazards. In fact the companies must be very alert to protect themselves against the designs of many persons, who are honest enough in most transactions yet little consider the impropriety of taking an unfair advantage of an insurance company.

Lapses.—The lapse ratio among cancellable accident and health policies is very high. It is estimated that about 20 per cent of the amount in force lapses each year. Again, Mr. Laird states the causes of lapses as follows: "(1) The protection is frequently taken as a temporary sporting proposition. (2) The insured has little equity in the policy and can usually replace the insurance in another company,

[5] See T.A.S.A., XXIII., Part Two, p. 378.

without medical examination or increase in premium. (3) The salesman generally considers that he has a vested interest in the insurance and if he changes his allegiance to some other company, he transfers a large proportion of his business. (4) New policies come on the market or the insured's situation changes, and the old contract is replaced by a new one."[6] It might be added that, (5) when an insured person's accident and health policy is about to expire, the agent of a rival company may canvass him without fear that the stigma of twisting, or an attempt at twisting, will rest upon his action. Under these circumstances there is a tendency for the good risks to drop or change their insurance while the poor ones remain with the company. An example of this tendency recently came to the attention of a certain consulting insurance expert when a client asked his advice in regard to seeking a cheaper policy than he was carrying. As a matter of fact the client was then taking treatment from a noted specialist. After his attention was called to the customary question in the application blank concerning recent medical advice or treatment, the client decided not to disturb his present policy!

Expenses.—In view of the practical immunity of agents from charges of twisting in this business, and because of the fact that as much salesmanship is often required to secure a renewal as to sell the policy in the first place, it is customary to pay the same percentage of premium as a renewal commission as is paid for new business. These percentages vary from 20 per cent to 40 per cent of the premium, and combined with overhead, taxes, and settlement expenses take up from 45 per cent to 55 per cent of the premium income.

General Problems.—The principal underwriting questions which a company granting this type of protection has to answer before issuing or renewing a policy are: (1) What is the probability of accidental death? (2) Is the total amount of insurance excessive? (3) What is the probability that the applicant will become disabled? (4) If he becomes disabled, what will be the probable length of time the disability will continue? (5) What is the probability of a recurrence of disability, and for how long will it persist upon recurrence? (6) How reasonable will the applicant be when a claim arises? (7) What is the probability of a fraudulent claim and malingering? (8) Do the applicant's earnings warrant the amount of insurance applied for, in view of his other insurance? (9) What must be provided for administrative expenses incident to the benefits?

Good underwriting practice requires that the company take rea-

[6] *Ibid.*, p. 379.

sonable steps to keep the above probabilities and expenses as low as is consistent with a progressive business program. This the companies attempt to do in several ways. They urge their agents to insure only persons whom they believe will prove good risks, stimulate agents with good claim records to greater production, and discharge agents who show a consistently bad claim record. Since weekly indemnity covers loss of business time, the companies insist that the applicant shall be steadily engaged in a remunerative occupation so that the value of the business time can be definitely measured. Students at college, authors, artists, musicians, and those engaged in like pursuits are likely to be intermittently employed, hence as a rule only policies covering accidental death or dismemberment, but not weekly indemnity, are issued to them. Evidence of malingering is more likely to appear when the insured is not continuously employed. Teachers are more likely to become disabled during the vacation period, nurses during the dull season, and farmers during the winter when there is little necessary work to be done. The moral hazard must be guarded against in certain occupations such as the liquor business, or where the insured's place of business is located in the same building as his residence so that the actual period of disability could not readily be determined. The nature of some occupations renders those engaged therein especially liable to disability because of accident or disease; and certain others while showing no special hazards, yield high rates of disability and cause much difficulty in claim adjustments.

Age.—The best ages from the company's viewpoint are those between 25 and 55. At ages below 25 the insured's habits of life may not be fully formed, nor his occupation and success fully determined. Persons above age 55 seek accident and health insurance if they feel themselves slipping as regards health or earning power, whereas others do not. Hence the company must guard against adverse selection at the older employment ages. Between these ages, however, the company may limit the amount of weekly indemnity to a sum substantially less than the insured's earnings, and grant protection against loss of business time only to those regularly employed in desirable occupations, thus eliminating many of the undesirable features with which it has to contend. In fact health insurance has so often been unprofitable that companies usually refuse to write it unless the insured carries an equal amount of weekly indemnity under the accident provision. Accident insurance without health protection, however, is readily obtainable.

Family History and Physical Condition.—Family and personal

history are of the greatest importance in this business. Any record of tuberculosis, cancer, or disease of the nervous system in the family history, or of accidents or diseases in the personal history, must be given careful attention.

As regards the applicants' physical condition, particular attention is paid to the eyes, ears, teeth, varicose veins, or any serious deformity. These matters are taken care of in the application blank, since lack of accuracy in answers to questions in the latter affords the company a strong defense against a claimant. Considerations similar to those in life insurance govern under- and over-weights.

Over-Insurance.—Over-insurance must be especially guarded against in this business as it is particularly subject to heavy losses when insured persons find it profitable to become disabled. The maximum amount of weekly indemnity should not exceed 80 per cent of the insured's earnings, and some provision should be made for a reduction of the benefit if earnings decrease. Such a reduction may be accomplished by providing that indemnity greater than 80 per cent of weekly earnings will not be paid, and that on request the insured may at any time receive back a part of the pro-rata unearned premium if he desires to reduce his protection to correspond to a reduction in his earnings. A contribution clause might be used providing that the company shall not be liable for a greater proportion of the 80 per cent than its insurance bears to the total protection carried by the insured, but it is probably more practical to limit the protection in the first place when this is possible. A few cases of over-insurance are bound to arise despite the usual precautions.

Women.—Women show a higher disability rate than men. Hence they are granted less liberal contracts, are not compensated for diseases peculiar to them, are charged a higher premium rate, and are not insured at all unless engaged in a gainful occupation requiring daily absence from residence. Claim adjustments are likely to be more difficult with women, partly because they do not understand that the contract primarily covers death, dismemberment and loss of business time, partly because it is often more difficult to determine the extent of injury, or impairment due to ill health.

Considerations on Renewal.—As each policy approaches the renewal date, the company reviews its records of the risk and again refers to statements in the application blank and to the inspection report. If there is nothing to indicate that the risk is undesirable, the company offers to renew the contract. If the risk has become impaired, the company may refuse to do so. A full medical examina-

tion may be required at any renewal date. A new inspection report is usually required on large risks at each renewal. If a person appears to be a first-class risk with one exception, a policy may be issued with an "impairment rider" excluding any loss resulting wholly or partly from the specific impairment. An impairment rider may be attached at issue or on any renewal date, or the company may cancel the contract between renewal dates and offer one with an impairment rider attached. This latter procedure is quite unusual in practice.

Premium Rates.—A company cannot eliminate losses entirely by exercising the precautions indicated above, nor would it appear to be the part of wisdom to so operate as to reduce losses to an absolute minimum. Expenses are necessary to the conduct and development of the business, the company cannot always limit its business to select risks, and if it limits its policies by too many restrictive clauses, it will render them undesirable. Having determined the limits of coverage, the company must reckon the probabilities of loss and the necessary premiums.

Sickness Insurance.—The principles involved in determining rates for sickness insurance are the same as those explained before in the chapters on net rates for insurances and annuities. Thus if a group of m persons aged x experiences a total of r weeks of illness in one year, the rate of sickness may be expressed: $z_x = \frac{r}{m}$. If each person were to receive 1 for each week of his illness, z_x would be the net premium each must pay for one year of protection. If, however, premiums are payable at the beginning of the year, the present value of z_x discounted for six months will be sufficient, since some claims occur in the early part of the policy-year, and others in the latter part, so that six months approximately represents the average length of time the money will be held before it is paid out. This sum will be $v^{\frac{1}{2}}z_x$. At the beginning of the second year, $v^{\frac{1}{2}}z_{x+1}$ will be required, provided the person is alive, hence $v^{\frac{1}{2}}z_{x+1}$ becomes a pure endowment, or

$$E_x(v^{\frac{1}{2}}z_{x+1}) = \frac{D_{x+1}v^{\frac{1}{2}}z_{x+1}}{D_x}.$$

And the present value of the nth year's insurance is $\frac{D_{x+n-1}v^{\frac{1}{2}}z_{x+n-1}}{D_x}$. Hence an insurance of 1 for two years is the sum of $v^{\frac{1}{2}}z_x$ and $\frac{D_{x+1}v^{\frac{1}{2}}z_{x+1}}{D_x}$, or substituting commutation symbols and indicating the addition, the sum may be expressed, $\frac{D_xv^{\frac{1}{2}}z_x + D_{x+1}v^{\frac{1}{2}}z_{x+1}}{D_x}$. If in this expression, the values in the numerator be continued to the end of the mortality table,

the value of an insurance of 1 for the whole of life, indicated by s_x, would appear. If the values of $D_x v^{\frac{1}{2}} z_x + D_{x+1} v^{\frac{1}{2}} z_{x+1}$, etc., be summed from each age to the end of the table and set down in the usual manner to form Column K,

$$s_x = \frac{K_x}{D_x},$$

$$|_n s_x = \frac{K_x - K_{x+n}}{D_x},$$

$$_n|s_x = \frac{K_{x+n}}{D_x}.$$

Hence when the tables have been constructed, the net premium for sickness benefits may be readily ascertained for yearly renewable term, whole life, or term insurance such as that granted under non-cancellable policies.

Accident Insurance Rates.—The computation of accident insurance premiums presents no particular problem once the probability of accidental death or dismemberment is determined, especially in cancellable, yearly renewable policies.

As regards accident insurance, it is obvious that the occupation of the insured is a very important factor in determining the premium rate he should pay. It is not customary to change the premium rate if the insured changes his occupation, the common practice being to reduce the benefit, as indicated above in the discussion of the provisions of the contract. Thus if a person secures an accident policy while engaged in an occupation classed as "select and preferred" and later changes to one classed as "medium," any claim presented under his policy is adjusted on the basis of 50 per cent of the amount he would have received had he remained in an occupation classed as "select." The Bureau of Personal Accident and Health Underwriters has considered over 3000 different occupations and has divided them into 9 different classes, assigning to each class a separate rating. The classes and ratings are as follows:

1. Select and preferred	100 per cent
2. Extra preferred	120 per cent
3. Ordinary	170 per cent
4. Medium	200 per cent
5. Special	250 per cent
6. Hazardous	300 per cent
7. Extra hazardous	400 per cent
8. Perilous	500 per cent
9. Extra perilous	600 per cent

The factors which influence the bureau in placing an occupation in one class rather than another may be shown by a few illustrations. Persons in clerical pursuits, bank officers, etc., are placed in class 1; dentists, oculists, and physicians in general practice, in class 2; barbers, fish wardens, and watchmakers, in class 3; automatic sprinkler installers, automobile dealers (demonstrating and repairing but not racing), pipe fitters, in class 4; bricklayers, supervising farmers, machinists, telegraph or telephone linemen, in class 5; automobile truck drivers, carpenters, sheep-herders, in class 6; cowboys on ranch, farm laborers or farm hands, fire works exhibitors, in class 7; lumbermen in woods—hewers, loggers, choppers, sawyers, in class 8 (not insurable); animal trainers or attendants in a circus, cartridge makers, chemists manufacturing acids or explosives, in class 9 (not insurable).

As yet there is no separate classification showing one rating for the accidental death benefit and another for the accidental disability benefit. Hence a high accidental death rate or a high accidental disability rate, or both, may be the cause of a high occupational rating. If the latter is high because of a high rate of accidental disability, it may have been occasioned by numerous minor injuries of short duration, or by an abnormal number of the more severe injuries which result in prolonged disability. Thus a dentist is rated up to 120 per cent because even a slight injury to his arm, hand or fingers is likely to result in loss of business time, and a carpenter using machinery in shopwork to 300 per cent because of frequent short periods of disability due to minor injuries, whereas a telephone lineman is rated up to 250 per cent because of the high probability of serious injury or accidental death.

As regards health insurance, however, the rates are the same for all occupations in the first 3 classes; that is, from select to ordinary, inclusive. This is because, according to the best underwriting judgment based on what statistics are available, certain occupations, while showing a high rate of disability from accidents, will show a rate of disability from sickness no higher than the rate in the select class. Those in industries comprised within class 7, extra hazardous, or above, are given less liberal health policies. To obviate adverse selection, policies exclude sickness beginning within 15 days after issue. If such were not the practice, persons exposed to communicable diseases, or who for any reason suspect that they will soon become ill, might be tempted to rush to the nearest agency and secure health coverage, and in many instances the company would have great diffi-

culty in ascertaining and proving the facts. It is also customary to apply this 15-day period following the date of issue to the health insurance benefits under cancellable policies covering both accident and health losses.

Data for Non-Cancellable Rates and Reserves.—*Legal Standard.*—The matter of compiling data from which to form tables of net premiums and reserves for non-cancellable business has not progressed as far in this country as might be expected. Accident companies have generally kept an account of their experience in the form of a comparison of losses with gross premiums. When records are kept in this manner it is difficult to combine the experience of several companies with varying policy provisions, underwriting practices and premium rates, so as to arrive at a satisfactory loss ratio. Furthermore, experience with cancellable business is not considered a satisfactory basis for non-cancellable rates and reserves, since under the non-cancellable contract the company is deprived of the protection afforded it by the cancellation privilege and since the experience is mostly on benefits limited to a maximum period of 52 weeks. Hence the Manchester Unity experience is generally considered as the most satisfactory basis of net rates and reserves for non-cancellable business. On cancellable, yearly renewable business the customary legal requirement that the company retain one-half of the premiums received during the year as a reserve is considered sufficient. On non-cancellable business, a reserve against the increasing probability of loss due to disability as age advances, and a level annual premium rate sufficient to set up this reserve, is deemed essential. Methods of computation are explained in detail by Mr. E. E. Cammack in the Proceedings of the Casualty Actuarial and Statistical Society of America, Vol. VII, pp. 267, et seq. The minimum valuation standard established by Section 93 of the New York Insurance Law is, "the net premium basis, according to the British Friendly Society Tables (876, 880) and with interest at 3½ per centum per annum."

CHAPTER XX

FRATERNAL INSURANCE

Fraternal insurance is that furnished by fraternal benefit societies as contrasted with old-line or legal-reserve companies. According to the New York Conference law, a fraternal benefit society is, "Any corporation, society, order or voluntary association, without capital stock, organized and carried on solely for the mutual benefit of its members and their beneficiaries, and not for profit, and, having a lodge system with ritualistic form of work and representative form of Government, and which shall make provision for the payment of benefits in accordance with Section 5 hereof."[1] Section 5 of this law provides that benefits may be paid for death, disability, old age, etc., with certain restrictions. The provision that the society must have a lodge system with ritualistic form of work in order to be considered a fraternal benefit society is of primary importance in distinguishing a fraternal society from an assessment association.

Early Experience and the Development of Fraternal Rate-Making.—Fraternal insurance has had a very remarkable and somewhat varied career in this country. The first society providing it was founded by John J. Upchurch in 1868. Mr. Upchurch created an organization known as the Ancient Order of United Workmen primarily to provide workingmen with a union on a somewhat broader scale than that of other trades-unions of that time. Protection to the dependents was included in the plan of organization by a provision in the Constitution which stipulated that when the members numbered 1000 an insurance office should be created and a policy issued, "securing at the death of the member insured not less than $500, to be paid to his lawful heirs."[2] On October 6, 1869, an "Insurance Article" was adopted which provided that each member should contribute $1 to an insurance fund of the subordinate lodge and that upon the death of a member the funds of all the subordinate lodges

[1] Sec. 1, New York Conference Law.

[2] Basye, Walter, History and Operation of Fraternal Insurance, p. 11. Mr. Basye quotes Article XVII of the Constitution.

should be used to defray funeral expenses, the remainder of them, if any, to be "properly and judiciously applied for the benefit of the family or heirs of the deceased." The entire benefit was to equal in dollars the number of members who had contributed, but was not to exceed $2000 on any one death; that is, each surviving member was to contribute $1 upon the death of a member, the contribution being made in advance to promote prompt settlement. Then as soon as a death occurred, another $1 was due from each remaining member in order to prepare for the next death. Such contributions are known as post-mortem assessments.

Numerous societies followed, all operating on this unscientific basis. Many of them passed out of existence, for reasons obvious to one familiar with the principles set forth and explained in previous chapters of this volume, yet thousands of dollars were paid to needy orphans and widows even by those societies which have been considered failures. The reasons usually given for the rapid growth of fraternal insurance are, (1) the numerous failures among old-line companies in the early 70's, (2) the low rates of the fraternals, (3) the disappointment on the part of holders of old-line policies when extravagant dividend estimates failed to materialize, and (4) the widely-spread impression that reserves were unnecessary and likely to be mismanaged. During the early 70's when fraternals got under way, old-line companies could appeal only to the rich and the moderately well-to-do. Fraternal insurance offered protection, cheap enough to be within the reach of the workingman. It also appealed to others who were interested in the social features.

There had been occasional instances in which the funds of legal-reserve companies had been improperly managed, and whenever a case of such mismanagement appeared in the public press, the advocates of fraternal insurance were prone to make much of it. Many persons could not understand why the old-line companies found it necessary to hold huge funds in reserve when the fraternals were managing to pay claims without reserve funds. One of the slogans of the fraternals was, "Keep your reserve in your pocket." It was thought that assessments could be depended on to provide for the payment of claims.

Early in the history of fraternalism it became evident that it was impracticable to levy an assessment upon the happening of each death. It, therefore, became the practice to collect regularly so much from each member to create a fund from which benefits might be paid. The second step consisted in grading the assessment according to age

of entry. In fact, the fraternals in this country repeated in large measure the early history of life protection in England before the necessity of providing for higher mortality rates at the older ages had yet been demonstrated by experience.

In the earlier period, many of the fraternals collected assessments which would have been inadequate to meet their obligations even if the average age of the group could have been kept low. When taken to task by men who understood the principles involved, the fraternalists were inclined to answer that the actuaries were prejudiced, or that by taking in new members and keeping the average age of the group low, they would aways be able to meet their claims. In fact, the "average-age" or "new blood" fallacy may be taken as the fundamental cause of failure of the early fraternal systems. It is, of course, obvious that if a large group of persons of different ages should be insured, the total of the natural premiums per 1000, determined on each life separately, divided by the number in the group would yield the average premium. If each person is charged this premium, then the company will be able to meet first-year claims. If new members at young ages are taken into the group each year so that the average age is maintained, no reserve funds need be held nor need the premium be increased, as has been demonstrated by our more recent experience with group insurance. With insurance, however, as the primary purpose of entering the group, and the right of a person to exercise an option as to whether he shall enter it, it is difficult to secure young members, who realize that they will be taxed to pay the death claims of a group whose average age is greater than their own. They prefer to join an organization made up for the most part of younger persons in which the cost of protection is lower. Such, in fact, has been the experience of many fraternal benefit societies. When a society is organized with young lives on the books, very low assessments for a number of years, are sufficient. Then as the society grows older it becomes increasingly difficult to secure enough new members to keep the percentage of old members low. Hence, the death rate increases, and the assessments must be correspondingly increased or the benefits reduced. Resort to either alternative causes healthy lives to withdraw and those in ill health to remain. Meanwhile new members in good health and at young ages refuse to join in any considerable numbers, preferring a younger organization. The older organization must then be either abandoned or reorganized on a more scientific basis.

The early fraternals have been severely criticized, and in some

instances condemned as a menace to the welfare of the community. Impostors may indeed have occasionally found this a fruitful field of endeavor. We must remember, however, that other organizations have not been immune from the activities of scoundrels. Well intentioned but misguided were those who advocated fraternal insurance on the unsound basis described above.

A calm and unbiased appraisal of the first 30-years' work of fraternal benefit societies leads to the conclusion that it was not altogether in vain. Thousands of dollars were paid out to widows and orphans who would otherwise have been without any protection because of the inability of their breadwinners to pay old-line premiums. Viewed as cheap, temporary protection, temporary in the sense that there were several years of it before the society in each case got into difficulties,—the system had its advantages. The chief cause of complaint against the early system is that thousands of persons who had made sacrifices for many years to keep their assessments paid up found *themselves* without protection, often late in life or when otherwise uninsurable.

It was because of ignorance of first principles on the part of managers of the early fraternal societies, their delusion concerning the possibility of maintaining a low average age, and the desire to afford protection at low cost, that assessments were not based on sound foundations nor were methods pursued which would assure the permanence of the system. They had to learn from experience the necessity of operating on an actuarially sound basis. This lesson gradually began to be borne in upon the older societies during the decade from 1891 to 1900. At the beginning of this decade the societies which had been in operation for 10 or 20 years began to feel the pressure of advancing age, hence new organizations were able to increase their memberships to appreciable numbers with the greatest of ease. The difficulty encountered by the older societies in securing new members, the necessity of considering increased assessments or reduced benefits, and the effect of adverse selection, caused the beginning of the movement for adequate rates and sound methods which was inaugurated during this decade.

It may be asked, "Why didn't the legislatures of the various states pass laws regulating fraternal benefit societies, and setting up minimum valuation standards for them as they did for the old-line companies?" The answer is not far to seek. About one-third of the voters belonged to some society. The fraternals maintained that they were the great protectors of the workingman. Their membership

also included men of influence, many of whom had joined for social or political reasons. Whenever legislation designed to control their activities was proposed, it was an easy matter to allege that it was a "plot" on the part of old-line companies to drive fraternals out of business, and then to stir up enough opposition to defeat the bill. For a legislator to support a bill in the face of such opposition and thus alienate a large portion of his constituents was virtually considered political suicide.

Nevertheless it became evident after a time that mere opposition to proposed legislation was not best for the societies. Other organizations which were purely assessment associations without the fraternal tie to bind the members together sprang up and imitated the fraternals, often offering protection at very low rates and on a thoroughly unsound basis. Meanwhile, the more prominent fraternals had combined their efforts by forming an organization known as the National Fraternal Congress. In 1890, President D. H. Shields exclaimed to an assembly of this organization, "How are we to protect ourselves from the many fraudulent organizations that are flooding the country under the garb and cloak of fraternity?" A beginning in statutory legislation and state supervision had been made in 1888 when Massachusetts passed an act defining fraternal societies and providing for a report of their membership, operation, and financial transactions. The first comprehensive plan, however, for statutory regulation was adopted by the National Fraternal Congress when its committee on legislation drafted and recommended for adoption the N. F. C. Uniform Bill in 1892. This bill defined a fraternal society as one having a lodge system, with ritualistic form of work and representative government, exempted fraternals from the provisions of the insurance laws of the state, and provided various regulations of minor importance. This bill was adopted, with modifications, in several states. A clause was later added which provided that a fraternal might "create, maintain, disburse and apply a reserve or emergency fund in accordance with its constitution and by-laws."

It was not only by the recommendation and support of favorable legislation with united opposition to unfavorable bills that the National Fraternal Congress served the common interests of the societies thus joined together. The insertion of the provision making possible an emergency fund was an advance, in that the societies were beginning to realize the importance of setting up some sort of a fund to pay claims promptly and provide for emergencies. The yellow-fever epidemic of 1878 and 1879 led the A. O. U. W. to provide for the

establishment of a "special relief" fund in 1880. The Independent Order of Foresters also made provisions for a "reserve" fund at an early date, although none of them provided for valuation on an actuarially sound basis until quite recently. Some of the early leaders made unsuccessful efforts to have assessments graded according to the age of the member. By the close of the century it had become quite general to collect regular monthly assessments instead of ordering an assessment after each death. Though the post-mortem assessment system had been abandoned, for the most part, the societies found it was necessary to increase assessments or reduce benefits when the average age and mortality rate became high. In other words, during the early years of fraternal operation the assessment rates were based on current mortality. As the societies became older and the average age and death rate increased, they began to realize that their assessment rates could not remain level. Hence when the National Fraternal Congress met in 1895, its Committee on Statistics and Good of the Orders reported that except during the inceptive period of an order there exists a gradual increase in the mortuary rate wholly due to the increasing average age of the members. In fact, the recommendations and suggestions of this Committee were so revolutionary and displayed such wisdom and foresight concerning the fraternal system that a part of their report merits reproduction. It stated:

"We believe that the existing fraternal orders can be perpetuated provided that they heed the lesson and the warning that the experience of the past so plainly gives and teaches. . . . It is indispensable to recognize the Law of Mortality as the governing factor. . . . The rate fixed for life at the age of entry is common to nearly all the fraternal orders. Our experience demonstrates that it is faulty in theory, unsound in practice and should be remedied, and this can be accomplished by increasing the rate with increasing age or by so adjusting the rates as to establish a fund that shall equalize the cost throughout life, or in other words, establish a Reserve. If with our experience we should institute a new fraternal order at the present time, one or the other of these, the law permitting, perhaps more or less modified, but in substance the same, would probably be adopted. In the proposition advanced the elements of 'Safety' and 'Equity' are separated, and first to be considered is the element of 'Safety.' Loading the rate at age of entry to minimize the cost of advancing years is the old-line plan of the reserve. The establishment of such a fund has, until within a year or two, been generally condemned

by the fraternal orders, not for the reason that it is not of itself good, but that it has been improperly administered and made a means of gravest abuse. Properly adjusted to our requirements, it would make protection certain, work wrong to no man and be in the direct line of safety. Any table based on this plan must be accurately worked out and grounded on the condition of a fixed annual amount payable by installment, of which a certain per cent, with its increment of interest, will form a reserve fund. This reserve can very safely be largely reduced from the reserve based on the standard tables."

In 1897 the Congress appointed a Committee to prepare tables of rates upon the level premium, the natural premium and the step-rate plan. As a result, this Committee reported a table of mortality to the convention of 1898 which is known as the National Fraternal Congress table. It was based on the experience of old-line companies and fraternal societies, particularly the Royal Arcanum and the Ancient Order of United Workmen. The committee stated that it was a safe basis for new business and for old business at attained ages, *when the latter is in good physical condition*. It has since been demonstrated that the table reflects the societies' experience with remarkable accuracy and that the original statement of the committee is correct. For re-organization purposes it is not so satisfactory when no medical examination is required of the older members. Adverse selection in such cases is likely to result in a higher mortality rate among those remaining than that shown by most published mortality tables.

Net annual and monthly premium rates, step-rates, and modifications were also computed on the basis of the N. F. C. table with interest at 4 per cent. The first table below shows the net annual level and net monthly level rates per 1000 at ages 21 to 60, inclusive.

The second table below shows the rates by the step-rate plan to age 61 and the level rate from that age for the remainder of life. Column 1 shows the age groups, column 2 the annual rates, column 3 the monthly rates corresponding to the annual rates, column 4 a modification of the step-rates whereby an additional premium of 15 cents per month is charged and allowed to accumulate so that the level rate from age 61 on may be reduced, column 5 a similar modification but with an accumulation of 30 cents per month which reduces still further the level rate from age 61.

Societies may vary the amount of the accumulation, the third one of the tables below having been constructed to serve as the basis for such variations; that is, this table shows the amount by which the

LEVEL RATES PER 1000

Whole-Life Protection

Age	Annual	Monthly	Age	Annual	Monthly
21	10 62	.93	41	20 93	1 83
22	10.92	.96	42	21 80	1.91
23	11.24	.98	43	22 72	1.99
24	11.57	1 01	44	23 69	2 07
25	11.92	1 04	45	24 72	2 16
26	12.28	1 07	46	25 81	2.25
27	12.67	1.11	47	26.91	2 35
28	13.08	1.14	48	28.20	2 45
29	13 51	1 18	49	29 51	2 58
30	13 96	1 22	50	30 98	2 71
31	14 43	1 26	51	32 39	2.83
32	14 94	1 31	52	33 97	2 97
33	15 47	1 35	53	36 65	3.12
34	16 03	1 40	54	37 45	3 28
35	16 62	1 45	55	39 36	3.44
36	17 24	1.51	56	41 41	3.62
37	17 90	1 57	57	43 60	3 88
38	18 60	1 63	58	45 94	4 02
39	19 34	1 69	59	48 45	4 24
40	20 11	1 76	60	51 13	4.47

STEP-RATES AND MODIFICATIONS PER 1000

Whole-Life Protection

1 Ages	2 Annual	3 Monthly	4 Monthly	5 Monthly
21–25	5.11	.45	.60	.75
26–30	5.40	.48	.63	.78
31–35	5.93	.52	.67	.82
36–40	6.71	.59	.74	.89
41–45	8.14	.72	.87	1.02
46–50	10.25	.90	1.05	1 20
51–55	13.82	1.21	1.36	1.51
56–60	19.60	1.72	1 87	2 02
61+	54.01	4.73	3.00	2.50

level rate from age 61 on will be reduced if the insured pays 1 per annum more than the annual step-rate from the time he enters until he reaches age 61. For example, if the insured enters the society at age 21 and pays 6.11 per annum instead of 5.11 (the net rate) from 21 to 25 inclusive, then 6.40 instead of the net rate of 5.40, etc., his net level annual premium from age 61 on will be reduced 11.61 below 54.01, or to 42.40 per 1000 per annum. However, if he enters at age 45, the accumulation of 1 per annum additional will only reduce the level premium 2.33, or to 51.68 from age 61. Now if the annual payment were 2 in addition to the net step-rate, the accumulation would be twice as great at age 61, hence, the reduction of the level rate below 54.01 would be twice as much. Thus an accumulation of 2 per annum in addition to the net step-rate from age 21 will reduce the level rate from age 61 by, $11.61 \times 2 = 23.22$, or to $54.01 - 23.22 = 30.79$. If it were desired to reduce the level premium from age 61 to 25 per 1000, $54.01 - 25 = 29.01$, the reduction, which divided by 11.61 yields 2.50, the extra amount per annum in addition to the net step-rate if the insured is now aged 21. The table follows:

ACCUMULATION TABLE

Showing Reduction of Annual Level Rate from Age 61 Produced by Payment of 1 per Annum Additional to Net Step-Rate

Age	Reduction	Age	Reduction
21	11 61	36	4 71
22	11.00	37	4 40
23	10 41	38	4 10
24	9 85	39	3 81
25	9 30	40	3 53
26	8 80	41	3 28
27	8 30	42	3 03
28	7 82	43	2 79
29	7.36	44	2 55
30	6 93	45	2 33
31	6 52	46	2 13
32	6 13	47	1 93
33	5 75	48	1 74
34	5.39	49	1 56
35	5.04	50	1 39

Legislation.—One might suppose that the fraternals would have immediately attempted to readjust their rates of assessment so as to

bring about a condition of actuarial solvency, now that rates on a scientific and safe basis were available, and now that the societies had learned by experience that inadequate rates must inevitably lead to disaster. The fraternals, however, did no such thing. There were too many old members in the older societies, members whose rates would have been greatly increased by such a procedure. Naturally these objected to a substantial increase in their rates. They seemed to feel that the younger members should take care of them, as was contemplated when the societies were first organized. As a result the National Fraternal Congress inaugurated what later proved to be a very unfortunate movement. It consisted in an attempt to pass legislation requiring all *new* societies that attempted to organize, and those seeking admission to additional states, to charge rates not less than the net rates on the basis of the N. F. C. table and 4 per cent. The effect of such legislation would have been to allow the older and larger societies to continue on cheap rates, while new and expanding orders would have been compelled to charge relatively higher rates. With persons who understood the principles involved, and who desired permanent protection, the newer organizations on an adequate rate basis would have obtained favor and thus had an advantage in competition. This advantage might have been secured by a proper educational campaign had the younger organizations embraced their opportunity. Instead, they looked upon the proposed requirement as unfavorable in that the prospective member was likely to choose the lower-premium benefit of one of the older societies. He was likely to look upon the older organization as one well established, not realizing that the younger might be financially secure and that the older was actuarially insolvent. Consequently, the younger societies formed the Associated Fraternities of America, dubbed the proposed legislation the "Force Bill," and proceeded to oppose the National Fraternal Congress generally on matters of legislation involving the question of adequate rates. The national convention of state insurance commissioners favored the so-called "Force Bill," but it was difficult to convince legislators of its fairness. It was difficult to maintain that new organizations should be compelled to charge adequate rates so long as old organizations might admit new members at inadequate rates.

This, though at first the chief obstacle to the N. F. C. plans, later became the basis upon which the two opposing forces united for real progress. Thus the two groups of societies, opposed to each other on the question of adequate rates for a period of 10 years, were quick

to join hands in opposition to legislation inimical, as they thought, to the welfare of fraternalism generally. When both groups began thus to feel the necessity of adopting a solvent plan of operation, committees of the National Fraternal Congress and the Associated Fraternities of America met the state insurance commissioners at their national convention, held at Mobile, Ala., in September, 1910, and framed what has since been known as the Mobile Bill. The most important features of this bill were provisions establishing standards of valuation, and compelling societies on an inadequate rate basis to improve their degrees of actuarial solvency by 5 per cent every 3 years. The bill was adopted by 13 states, but was soon found to be too drastic, and abandoned in favor of another known as the New York Conference Bill which has since become the fraternal insurance law of the land.

The New York Conference Bill was framed by representatives of the organizations of fraternal societies, and of the state insurance commissioners, the final conference being held in December, 1912. The primary purpose of the bill was to bring about gradually the actuarial solvency of all fraternal benefit societies. It provides that no new society shall be incorporated which does not require stated periodical contributions sufficient to meet the mortuary obligations contracted, when valued on the basis of the N. F. C. table and 4 per cent. Outside societies seeking admission to a state which has passed the bill must comply with Section 23, which works out in a manner similar to the requirement for new societies. In case a society fails to maintain a certain prescribed degree of solvency, new members may be admitted only at the rates of assessment required of new societies.

The valuation requirements of the New York Conference bill permit of valuation by different methods. One consists in setting down actual and contingent assets, contingent assets being the present mid-year values as of age of valuation, of future net contributions as provided in the constitution and by-laws. These are balanced against actual and contingent liabilities, contingent liabilities being the present mid-year value, as of age of valuation, of benefits promised. The ratio of actual and contingent assets to actual and contingent liabilities is the percentage of solvency. The society may proceed with ordinary net valuation, with minor variations, the ratio of reserve funds accumulated to the reserve liability showing the percentage of actuarial solvency. The first method is more generally employed by the societies, although the Wisconsin reports show both ratios.

At this point the distinction between legal solvency and actuarial

solvency should be clearly noted. So long as a society is financially able to meet current claims and expenses it is legally solvent. If the ratios by the above methods of valuation are less than 1, the society is actuarially insolvent. If a society is actuarially insolvent, its present rates of assessment are too low to pay all of the benefits it has promised, unless its experience is more favorable than that contemplated in the legal valuation requirements.

One of the provisions of the New York Conference bill is that the valuation report and an explanation of facts concerning the society's condition shall be printed and mailed to each insured member not later than June 1st of each year. There is an opportunity thus given for a member to learn of his society's prospects for paying its future claims. Wisconsin states annually the percentage of actuarial solvency of all societies operating within its boundaries. This practice in other states might benefit those fraternals operating on a sound basis, as well as old-line companies, and probably the prospective purchaser of protection, the number of persons joining unsound fraternals being fewer.

The question of most importance in the mind of an intelligent prospect considering insurance with a fraternal benefit society probably is, "What reserve funds does the society have, compared with what it should have in order to pay all of its future claims?" If the true answer is, "The society has enough," he may pass on to considerations other than that of safety. If the answer is that the society's reserve funds together with the present assessment scale are not sufficient, the prospect should make further investigation before placing the future welfare of his dependents in the hands of the particular society under consideration.

The reply to such a question as the above may be: "The society does not have sufficient reserve funds to meet all of its future claims, but that does not concern you because new members are placed on an adequate rate basis, their funds are kept separate from the general funds, and a proper reserve will be kept on your contract to assure the payment of your claim."

It is true that an actuarially insolvent society may avail itself of the provisions of section 23a, and thus segregate the funds of new members from the other funds of the society. It may also follow section 23b and evaluate on the "accumulation basis" which credits old members with what is due them as a result of the actual experience of the society. Then if a member's share of losses in any future year exceeds his credit and contributions for that year, he may be

required to pay increased or additional assessments. The effect of this is to require members to pay the current cost of protection; and for members formerly on an inadequate rate basis who have accumulated insufficient reserve funds to their credit, this cost increases as they grow older. It is a practical measure to wind up the business of inadequate rate members by requiring them to make up for the inadequacy of their rates in former years. It results in the older members going over to an adequate basis, or reduces their benefits, often does both, or it causes them to withdraw from the society. The society may then keep the funds of new members separate and thus afford them the same security as they would have in a new society. However, if the annual mortuary collections do not meet all claims after the required reserve funds have been set aside, the deficit may be met by additional assessments on all members, new and old alike.

The provisions of section 23a of the bill bringing pressure to bear on societies less than 100 per cent actuarially solvent to maintain at least as great a degree of solvency as that shown by the valuation of 1917, or thereafter, are based on the theory that if a society can now meet its claims, it will continue to be able to do so as long as its financial condition grows no worse. As the provision works out, a society that maintains the prescribed degree of solvency does so by admitting new members on an adequate rate basis, and by other measures, so that the degree of solvency may be expected to continually improve, and thus the original purpose of the bill will eventually be attained.

The New York Conference bill has been adopted with modifications by approximately three-fourths of the states. It contains many important provisions other than those governing valuation and the attainment of solvency through classification of members and segregation of funds.

Whole-family Protection Bill.—In 1916 a model bill was agreed upon by the legislative committee of the National Fraternal Congress of America and the state insurance commissioners, permitting fraternal orders to insure the children of members, or those for whose support a member is responsible. Under the terms of the bill, no certificates on juveniles may be issued, until at least 500 are ready to be issued simultaneously, which number must then be maintained as a minimum. Benefits were limited to $34 at age 2, $40 at age 3, increasing to $600 at ages 16 to 18. Rates may be based on the Standard Industrial table or the English Life Table Number Six, and 4 per cent. Reserves are to be maintained on this basis, and funds

of the juvenile department must be segregated from the other funds of the society. This bill has been adopted by many states, with the result that many young persons have been added to the ranks of fraternalism.

At the Montreal meeting of the National Fraternal Congress of America in September, 1922, a committee was appointed to consolidate the various bills and laws regulating fraternal orders. The committee did not attempt to draft a model bill, nor introduce new provisions, but confined its work to an arrangement of the several bills and laws into one consistent chapter on fraternal benefit societies. The essence of the whole-family protection bill appears in the consolidated chapter under sub-sections 3 to 8, inclusive, of section 5 (Benefits). Under Section 9 (Funds), sub-sections 3 to 6, inclusive, are added to the original New York Conference bill. Section 14 (Mergers) governs mergers and transfers and is designed to prevent the promotion of a society merely for purposes of reinsurance at a profit. Fraternalists should familiarize themselves with the fraternal insurance laws of their own state, the New York Conference bill, and such modifications of it as are found in the proposed chapter.

Present Status.—The total of fraternal insurance in force at the end of a recent year was $8,687,939,447. This is slightly less than the amount formerly in force. In fact, the amount of fraternal insurance in force has remained nearly stationary, declining slightly during the past 10 years. Rate adjustments have doubtless been the principal causes of the decline in the amount of this type of insurance. The fact that the business as a whole has been able to maintain itself so well in the face of these difficulties shows the firmness with which it is entrenched in this country.

The process of readjustment to attain actuarial solvency is a painful one for any fraternal order. Fraternalists have learned that the burden of insufficiency in the matter of rates and funds to meet reserve liabilities cannot be shifted to new members. Classes of old members may be assisted by applying to their claims the mortality salvages and excess interest earnings of new classes. New members may thus assist the old without endangering their own reserve funds, but this is not equitable nor is it popular with new members. Old members alone, to make up the deficiencies in their certificates as shown by valuation, must either be assessed at prohibitive rates, or be compelled to place liens on their certificates, the amount of the lien being deducted from the face of the certificate in case of death, and being gradually reduced by a moderately increased assessment in

case of survival. Or, old members may be allowed to transfer to an adequate-rate class by paying the level rates as of age attained at transfer. Most of those who do not thus transfer are likely to go off the books in a relatively short time. Mortality experience has not been very favorable on the re-rated members, however, because of the operation of adverse selection, as indicated above. Nevertheless, injustice to new members admitted on an adequate level rate basis is avoided if their funds are kept separate in every respect, and if given the benefit of mortality salvages and excess interest earnings. It will be only in case of abnormally heavy death losses, or gross mis-management, that extra assessments on them need be expected.

Of late years it has been demonstrated that the success of any movement to transfer the old members of a fraternal society over to an adequate rate basis depends largely upon the methods employed. The most effective method consists in employing expert salesmen to canvass each member with a view to securing his consent to the transfer. If he consents to it, the transfer is made on the basis of a preliminary term policy at the age then attained by the member, the salesman being paid a substantial proportion of the first 12 monthly premiums. In this manner enough healthy lives are transferred to off-set the adverse selection resulting from a mere change in the by-laws calling for increased assessments. There have been developed special "service companies" that employ and train salesmen for this purpose, and that serve a number of societies.

Types of Certificates.—The primary purpose of re-adjusting fraternal insurance to a solvent basis is to enable the orders to furnish whole-life protection at level rates, or at step-rates definitely determined so that the insured may know in advance what the cost of his protection is going to be. Ordinary life certificates at level rates are quite popular, although the step-rate plan is often applied for. The step-rate plan, whereby the rates remain level for a period of say 5 years, then increase and again remain level for another period, is really a sort of renewable term plan, with the added advantage that a certificate issued on this basis may be continued as a whole-life contract at a level rate after age 60 or 65. In other words, the step-rate certificate requires but a small premium during the time when a person is getting started in his life's work. The same is true of the renewable term policies of old line companies. Later on when one's earning power increases a higher premium is required, but the fraternal certificate, upon the insured's reaching age 60 or 65, may be continued at a level rate for the remainder of life. Also, one may

make an additional payment above the step-rate and thus establish an accumulation to reduce the level rate beyond 60 or 65.

The fraternal benefit certificate is a relatively brief document setting forth that the holder is a member of the society, is entitled to a specified portion of the benefit fund, and must comply with the society's constitution and laws, together with present and future amendments thereto, which are declared a part of the contract. Beneficiaries may be changed, but as a general rule only members of the certificate holder's family, or other dependents, are eligible as such. The society is not obligated to pay the stipulated sum named in the certificate unless sufficient funds are available. In other words, the society does not promise to pay a definite amount in consideration of a fixed premium, and one must study the constitution and laws of the society in order to learn the terms of his contract.

The application for a fraternal benefit certificate, however, is often quite a formidable document. In some instances, the applicant warrants the truthfulness of his statements, agrees to forfeit all of his rights in case of withdrawal or expulsion, to abide by the constitution, laws, rules, and usages of the society, to forfeit his rights if he enters a proscribed occupation or dies by suicide within 5 years, or by excessive use of intoxicants, or in violation of law. Thus it is highly important for the applicant to study not only the certificate itself and the constitution and laws of the society, but he should also read the contents of the application blank with care.

Limited-payment life certificates, sometimes including old-age disability benefits, and certificates paid up at age 65 or 70, are also issued. The old-age disability benefit may be compared to a pension, and may be paid monthly, quarterly or annually for a period of years or for the remainder of life. This benefit is, of course, a deferred temporary or whole-life annuity. A very small amount of term insurance is written, the step-rate plan being preferred to term certificates, even when the latter include the renewable or convertible features. Monthly income certificates are becoming more popular for the same reasons that monthly income policies of old-line companies are preferred in many instances. Some orders confine themselves to health and accident certificates, while others will provide for limited disability benefits in their standard certificates if the insured desires this, paying an extra premium for it. Joint-life certificates are issued, usually to husband and wife, and maternity benefits are paid by some orders. Women are admitted to many societies, and the memberships of a few are composed of women exclusively. Surrender values, con-

sisting of terminal net values less a surrender charge to overcome adverse selection, are granted in cash, paid-up extended term, and paid-up fractional insurance.

Juvenile Certificates.—The feature of the "whole-family protection" movement which has attracted most attention consists in issuing juvenile certificates on the lives of members' children. Authority for their issuance was obtained through the adoption of the whole-family protection bill, and by the inclusion of its provisions in other bills. Amounts of benefits are limited as indicated in the preceding discussion, to those sufficient for funeral expenses. This limitation of the insurance to small sums is to obviate temptation to neglect children in order to gain the insurance. Until 1917, juvenile insurance was confined to the industrial departments of old-line companies. Fraternals, however, conceived the idea of issuing certificates on such risks, because of opportunities to extend their benefits to small towns and out-lying districts which would not support industrial insurance. The order may also by this means provide protection formerly supplied only by old-line companies. Industrial insurance is expensive and necessarily confined to districts quite thickly populated. The industrial agent, also, since he writes level-premium policies, is likely to advise his clients to take old-line rather than fraternal benefits when they attain a sufficient age. It was thought that if all members of the family were protected by fraternal insurance, the juvenile member could be more readily induced to take an adult certificate on attaining the required age. Hence, when a juvenile member reaches the minimum age for admission to the adult class, he may surrender his juvenile certificate, exchanging it for an adult certificate, provided the health requirements are met and the rate of contribution is adjusted. Many have thus been added to the ranks of fraternalism and the juvenile department has become a training school for future fraternalists. It is in the juvenile department that fraternalists seem to have their greatest opportunity for future progress. If it is admitted at all that insurance should be granted on young lives, the juvenile departments of fraternals with adequate rates and complete segregation of funds should be encouraged. Protection can thus be given to thousands in outlying districts who cannot be reached by the industrial agent. As for cities, competition will not, probably, there harm industrial insurance much, and fraternals should at least be given an opportunity to demonstrate their ability to conduct juvenile insurance.

Advantages Maintained for Fraternal Insurance.—Some writers maintain that fraternal insurance's greatest heritage from the first 50 years of operation is a sound financial basis. In this matter, however, the experience of fraternal orders has contributed little to the knowledge of the actuary. Mr. Walter Basye, Editor of the Fraternal Monitor, concludes that "the chief result of the system's progress is a standardized plan of operation."[3] The plan of operation involves a form of government which fraternalists maintain is superior to that of old-line companies.

Representative Form of Government.—Fraternalists maintain that even in mutual companies operating on the legal-reserve plan policyholders have little or nothing to say about the management of the company. In a fraternal benefit society, however, members meet in local lodges, discuss policies and candidates, elect representatives to meet in supreme lodge which makes laws and elects the officers who manage the business. It is maintained that real mutuality results in efficient administration, and that since control of the society rests directly with its members, a change in the administration may be readily accomplished if any considerable dissatisfaction with the management should arise. Others maintain that the members of a fraternal society have no more to say concerning the management than have the persons insured in a mutual life insurance company. The fact is that in both organizations final authority rests with the members. The desire for wholesome regulations and legal restrictions of fraternals is shown by the restrictions concerning mergers and transfers, investments and other affairs. It does not require much time to wreck a financial institution, and a form of government which enables those interested to oust the officers responsible at the end of six months or a year cannot with safety supplant that under the state regulation which assures the stability of old-line companies.

Cost of Management.—It has often been asserted that the cost of sound protection is about the same in one organization as another. It is true that mortality costs are not likely to be very different in one large representative group than in another requiring similar medical examinations, with members exposed to similar hazards. Hence, it has been maintained that when a fraternal order adopts adequate rates it assumes the characteristics of an old-line company and consequently the cost of its insurance will be but slightly different from that of the latter. Fraternalists, however, point to the advantages of the social and fraternal features, the relief and visitation of

[3] Basye, Walter, History and Operation of Fraternal Insurance, p. 190.

the sick and the higher ideals of a fraternity. They also maintain that the cost of managing a fraternal benefit society is less than the expense of conducting a legal-reserve company. It is difficult to estimate the value of the social and fraternal features, the relief and visitation of the sick and the higher ideals. The man who is looking primarily for sound protection to his dependents, or who desires insurance for business purposes, might prefer to forego these benefits and pay only for the insurance feature. Since the fraternals include these, however, and since one must purchase certificates in several societies in order to secure any considerable amount of protection (societies limit insurance to a maximum of from $3000 to $10,000 on any one life), the expenses of social and other benefits might be looked upon by the insured as simply an additional cost of his insurance.

The Deputy System.—Fraternal orders obtain new business through what is known as the deputy system. Deputies are employed to organize lodges and obtain new members by means of solicitation. The field force is usually directed by the chief officer of the order in much the same manner as that in which the agency manager of an old-line company supervises his field force. Just as the old-line companies found, as early as the decade of the Civil War, that it is necessary to pay reasonable commissions to agents and thus to secure efficient salesmen and stimulate personal solicitation, so have the fraternals learned more recently that higher compensation to deputies is essential to systematic and effective work on the part of the field force. Fraternalists admit that old-line companies write a greater volume of business, but maintain that the larger volume is not commensurate with the agency cost above that of the fraternal deputy system. This, of course, depends on the point of view. Assuming that acquisition costs are lower under the deputy system though fewer persons are insured, those who do become members may secure such protection as is offered at a lower cost. It is as logical to suppose that if all insurable persons should voluntarily apply for an adequate amount of insurance without solicitation, both agency and deputy expenses would disappear. Since most human beings are so constituted that they will not do this, the amount which should be expended to induce them to insure is a matter of opinion. To put the question in another manner, suppose Jones is finally persuaded by a persistent life insurance agent bent on a substantial commission to pay for a $20,000 ordinary life policy. Smith, his neighbor, is induced by a low-cost deputy to take a $3500 certificate. Both die within a

short time. Which system is cheaper to the two insured persons? To the community as a whole? One might reason at great length on these questions and bring out numerous circumstances and contingencies. The principal question is, will the community as a whole be better off if its producing lives are well covered by sound insurance? If so there is only one thing to do, and that is to pay the price and bring about this coverage. If the fraternals can do this at a lower cost than old-line companies, they should by all means be encouraged to extend their activities. Whether the cost will really be less when the fraternals start writing business in amounts comparable to those of old-line companies, remains to be demonstrated.

In discussion of the cost of the management of old-line companies, fraternalists point to the high salaries paid to some of the officers, to the expenses incurred in order to maintain luxurious offices and office buildings, and to the mis-management of funds, particularly as brought out by the Armstrong investigation. As to offices and office buildings there is much to be said on both sides of the question. Some companies have undoubtedly expended too much for such purposes. Yet they may make gains for the policyholders from rentals of offices, and by increase in value of acquired property. Most of them show a small rate of return on real estate owned, in many instances perhaps too low. Concerning the high salaries paid by some companies to their chief administrative officers, and the mis-management of funds, it may be remarked that fraternal orders have as yet had very little experience in the management of large sums. On December 31, 1921, the total assets of fraternal benefit societies transacting business in the State of New York was reported at $296,-059,091.10;[4] whereas the total assets of old-line companies reporting to the New York Insurance Department surpassed this sum in 1886.[5] It is to be hoped that by the time the fraternals have accumulated funds equal to those held by old-line companies to-day, there will have been no instances of mis-management on their part. The fraternals have repeated the experiences of the companies in many ways. They have learned the necessity of charging adequate rates, and of preserving funds to meet legal reserve liabilities. They have learned that it is poor economy to do without the services of, or to fail to heed the advice of, a competent actuary. They are learning that well-paid deputies are more effective than others in securing new business. They have yet to learn how to handle the huge sums necessary to

4 New York Insurance Report, Part IV, 1922, p. xxxv.
5 New York Insurance Report, Part II, 1922, p. xvii.

successful operation on a large scale. They still make much of the fact that their officers are low-paid individuals, and maintain that such a practice results in economy of management. It is asserted that high salaries are impossible because the members do not approve of them, and the members are in control through the representative form of government which prevails. If this be true, herein may consist a fundamental difficulty. High-priced labor is often cheapest in the long run. The official who receives $100,000.00 per year may well be worthy of his hire. He need only transfer funds from one investment to another at the right time to save a loss or yield a profit, or in 15 minutes prevent a poor investment the weakness of which he is able to detect because of long experience and intimate connections with the financial world, in order to earn his salary for a whole year. Again, the $100,000.00 life insurance official may be responsible for the safety of more than 20 times as much money as the $5000 fraternal officer manages. Hence, the $100,000.00 man may render services at a lower cost per dollar managed.

Right to Assess.—The reserved right to levy assessments upon the members has been set forth as an advantage of fraternal insurance in that it constitutes an element of strength. It is of doubtful value, however, increased assessments in the past having resulted too often in heavy lapse ratios. In cases of war or epidemic when the reasons for additional assessments are clearly understood, the case is different. Furthermore, it is not a feature particularly attractive to the prospective purchaser of fraternal insurance.

One of the objections to this sort of insurance commonly urged is that the insured is never certain of the provisions of his certificate. Each certificate contains a clause to the effect that the insured is bound by the constitution and by-laws, and any amendments thereto. If a majority of the members wish to amend the laws in such a manner as to displease some certificate holder, the latter has no redress but to withdraw from the society. An insurance company, of course, cannot make restrictive changes in the provisions of the policy contract, once it is issued. Many prefer the definite contract of the old-line companies because they wish to "take insurance to be sure of something."

In conclusion, it may be said that fraternal societies operating on a sound basis have an opportunity to render valuable service. If a person is a social member or wishes to join a fraternity for social purposes, he might do well enough to take a certificate of insurance along with his membership, provided he is certain that the insurance

is on an actuarially solvent basis. If one is contemplating fraternal insurance, however, he should know that for definite, level-premium protection, the fraternal plan down to this date has proved to be inferior to the legal-reserve system as regards the security of the company and the provisions of the contract.

Coöperative or Assessment Life and Casualty Associations.—These are associations which do not conform to the legal requirements of fraternal benefit societies. They are not organized on the lodge system. Some confine themselves to issuing life insurance certificates, while others issue accident and sickness benefits. Many of them are composed of members of a labor union, or of persons engaged in a particular occupation. Many of them have in the past been forced out of business because of failure to levy adequate assessments and maintain the funds necessary to provide for increasing mortality rates. Many are still operating on an inadequate rate basis. A few, however, operate on an actuarially solvent basis. For example, the Expressmen's Mutual Benefit Association of New York showed a surplus of $150,138.42, at a recent date, after providing for a reserve liability calculated on the basis of the American Experience table of mortality with interest at 3 per cent.

APPENDICES

APPENDIX A

The Amount of 1 at Different Rates of Compound Interest

TABLE I

Units of Time	¼%	¾%	⅞%	1½%	1¾%	2%
1	1 0025	1 0075	1 0087 5	1 015	1 0175	1 02
2	1 0050 0625	1 0150 5625	1 0175 7656	1 0302 25	1 0353 0625	1 0404
3	1 0075 1877	1 0226 6917	1 0264 8036	1 0456 7838	1 0534 2411	1 0612 08
4	1 0100 3756	1 0303 3919	1 0354 6206	1 0613 6355	1 0718 5903	1 0824 3216
5	1 0125 6266	1 0380 6673	1 0445 2235	1 0772 8400	1.0906 1656	1 1040 8080
6	1.0150 9406	1 0458 5224	1 0536 6192	1 0934 4326	1 1097 0235	1 1261 6242
7	1.0176 3180	1 0536 9613	1 0628 8147	1.1098 4491	1 1291 2215	1 1486 8567
8	1.0201 7588	1 0615 9885	1 0721 8168	1 1264 9259	1 1488 8178	1 1716 5938
9	1 0227 2632	1 0695 6084	1 0815 6327	1 1433 8998	1 1689 8721	1 1950 9257
10	1 0252 8313	1 0775 8255	1.0910 2695	1.1605 4083	1.1894 4449	1 2189 9442
11	1 0278 4634	1 0856 6441	1 1005 7343	1 1779 4894	1 2102 5977	1 2433 7431
12	1 0304 1596	1 0938 0690	1 1102 0345	1 1956 1817	1 2314 3931	1 2682 4179
13	1 0329 9200	1 1020 1045	1 1199 1773	1 2135 5244	1 2529 8950	1 2936 0663
14	1 0355 7448	1 1102 7553	1 1297 1701	1 2317 5573	1 2749 1682	1 3194 7876
15	1 0381 6341	1.1186 0259	1.1396 0203	1 2502 3207	1 2972 2786	1 3458 6834
16	1 0407 5882	1 1269 9211	1 1495 7355	1 2689 8555	1.3199 2935	1 3727 8571
17	1 0433 6072	1 1354 4455	1 1596 3232	1 2880 2033	1 3430 2811	1 4002 4142
18	1 0459 6912	1 1439 6039	1 1697 7910	1 3073 4064	1 3665 3111	1 4282 4625
19	1 0485 8404	1 1525 4009	1 1800 1467	1 3269 5075	1 3904 4540	1 4568 1117
20	1 0512 0550	1 1611 8414	1.1903 3980	1 3468 5501	1 4147 7820	1 4859 4740
21	1 0538 3352	1 1698 9302	1 2007 5527	1 3670 5783	1 4395 3681	1 5156 6634
22	1 0564 6810	1 1786 6722	1 2112 6188	1 3875 6370	1 4647 2871	1 5459 7967
23	1 0591 0927	1 1875 0723	1 2218 6042	1 4083 7715	1.4903 6146	1 5768 9926
24	1 0617 5704	1 1964 1353	1 2325 5170	1 4295 0281	1 5164 4279	1 6084 3725
25	1 0644 1144	1.2053 8663	1 2433 3653	1 4509 4535	1 5429 8054	1 6406 0599
26	1 0670 7247	1 2144 2703	1 2542 1572	1.4727 0953	1 5699 8269	1 6734 1811
27	1.0697 4015	1 2235 3523	1 2651 9011	1 4948 0018	1 5974 5739	1 7068 8648
28	1 0724 1450	1 2327 1175	1 2762 6052	1 5172 2218	1 6254 1290	1 7410 2421
29	1 0750 9553	1 2419 5709	1 2874 2780	1 5399 8051	1 6538 5762	1 7758 4469
30	1 0777 8327	1 2512 7176	1 2986 9280	1 5630 8022	1 6828 0013	1 8113 6158
31	1 0804 7773	1 2606 5630	1 3100 5636	1 5865 2642	1 7122 4913	1 8475 8882
32	1.0831 7892	1 2701 1122	1 3215 1935	1 6103 2432	1 7422 1349	1 8845 4059
33	1 0858 8687	1.2796 3706	1 3330 8265	1 6344 7918	1 7727 0223	1 9222 3140
34	1.0886 0159	1 2892 3434	1 3447 4712	1 6589 9637	1 8037 2452	1 9606 7603
35	1.0913 2309	1 2989 0359	1 3565 1366	1.6838 8132	1 8352 8970	1.9998 8955
36	1.0940 5140	1.3086 4537	1 3683 8315	1 7091 3954	1 8674 0727	2 0398 8734
37	1.0967 8653	1 3184 6021	1 3803 5650	1 7347 7663	1 9000 8689	2 0806 8509
38	1 0995 2850	1 3283 4866	1 3924 3462	1 7607 9828	1 9333 3841	2 1222 9879
39	1 1022 7732	1 3383 1128	1 4046 1843	1 7872 1025	1 9671 7184	2 1647 4477
40	1.1050 3301	1.3483 4861	1 4169 0884	1.8140 1841	2 0015 9734	2 2080 3966
41	1.1077 9559	1.3584 6123	1 4293 0679	1 8412 2868	2 0366 2530	2 2522 0046
42	1 1105 6508	1.3686 4969	1 4418 1322	1 8688 4712	2 0722 6624	2 2972 4447
43	1.1133 4149	1.3789 1456	1 4544 2909	1 8968 7982	2 1085 3090	2 3431 8936
44	1 1161 2485	1.3892 5642	1 4671 5534	1 9253 3302	2.1454 3019	2 3900 5314
45	1.1189 1516	1.3996 7584	1 4799 9295	1 9542 1301	2 1829 7522	2.4378 5421
46	1.1217 1245	1.4101 7341	1 4929 4289	1 9835 2621	2 2211 7728	2 4866 1129
47	1.1245 1673	1 4207 4971	1.5060 0614	2 0132 7910	2 2600 4789	2 5363 4351
48	1.1273 2802	1.4314 0533	1 5191 8370	2 0434 7829	2 2995 9872	2 5870 7039
49	1.1301 4634	1.4421 4087	1 5324 7655	2 0741 3046	2 3398 4170	2 6388 1179
50	1.1329 7171	1.4529 5693	1 5458 8572	2.1052 4242	2.3807 8893	2 6915 8803

APPENDIX A

The Amount of 1 at Different Rates of Compound Interest

TABLE I—*Continued*

Units of Time	¼%	¾%	⅞%	1½%	1¾%	2%
51	1 1358 0414	1 4638 5411	1 5594 1222	2 1368 2106	2 4224 5274	2 7454 1979
52	1 1386 4365	1 4748 3301	1 5730 5708	2 1688 7337	2 4648 4566	2 8003 2819
53	1 1414 9026	1 4858 9426	1 5868 2133	2 2014 0647	2 5079 8046	2 8563 3475
54	1 1443 4398	1 4970 3847	1 6007 0602	2 2344 2757	2 5518 7012	2 9134 6144
55	1 1472 0484	1 5082 6626	1 6147 1219	2 2679 4398	2 5965 2785	2 9717 3067
56	1 1500 7285	1 5195 7825	1 6288 4093	2 3019 6314	2 6419 6708	3 0311 6529
57	1 1529 4804	1 5309 7509	1 6430 9328	2 3364 9259	2 6882 0151	3 0917 8859
58	1 1558 3041	1 5424 5740	1 6574 7035	2 3715 3998	2 7352 4503	3 1536 2436
59	1 1587 1998	1 5540 2583	1 6719 7322	2 4071 1308	2 7831 1182	3 2166 9685
60	1 1616 1678	1 5656 8103	1 6866 0298	2 4432 1978	2 8318 1628	3.2810 3079
61	1 1645 2082	1 5774 2363	1 7013 6076	2 4798 6807	2 8813 7306	3 3466 5140
62	1 1674 3213	1 5892 5431	1 7162 4766	2 5170 6609	2 9317 9709	3 4135 8443
63	1 1703 5071	1 6011 7372	1 7312 6483	2 5548 2208	2 9831 0354	3 4818 5612
64	1 1732 7658	1 6131 8252	1 7464 1340	2 5931 4442	3 0353 0785	3 5514 9324
65	1 1762 0977	1.6252 8139	1.7616 9452	2 6320 4158	3 0884 2574	3 6225 2311
66	1 1791 5030	1 6374 7100	1 7771 0934	2 6715 2221	3 1424 7319	3 6949 7357
67	1 1820 9817	1 6497 5203	1 7926 5905	2 7115 9504	3 1974 6647	3 7688 7304
68	1 1850 5342	1 6621 2517	1 8083 4482	2 7522 6896	3 2534 2213	3 8442 5050
69	1 1880 1605	1 6745 9111	1 8241 6783	2 7935 5300	3 3103 5702	3 9211 3551
70	1 1909 8609	1.6871 5055	1 8401 2930	2 8354 5629	3 3682 8827	3 9995 5822
71	1 1939 6356	1 6998 0418	1 8562 3043	2 8779 8814	3 4272 3331	4 0795 4939
72	1 1969 4847	1 7125 5271	1 8724 7245	2 9211 5796	3 4872 0990	4 1611 4038
73	1 1999 4084	1.7253 9685	1 8888 5658	2 9649 7533	3 5482 3607	4 2443 6318
74	1 2029 4069	1 7383 3733	1 9053 8408	3 0094 4996	3 6103 3020	4 3292 5045
75	1 2059 4804	1 7513 7486	1 9220 5619	3 0545 9171	3 6735 1098	4 4158 3546
76	1 2089 6291	1 7645 1017	1 9388 7418	3 1004 1059	3 7377 9742	4 5041 5216
77	1 2119 8532	1 7777 4400	1 9558 3933	3 1469 1674	3 8032 0888	4 5942 3521
78	1 2150 1528	1 7910 7708	1 9729 5292	3 1941 2050	3 8697 6503	4 6861 1991
79	1 2180 5282	1 8045 1015	1 9902 1626	3 2420 3230	3 9374 8592	4 7798 4231
80	1.2210 9795	1 8180 4398	2 0076 3066	3 2906 6279	4 0063 9192	4 8754 3916
81	1 2241 5070	1 8316 7931	2 0251 9742	3 3400 2273	4 0765 0378	4 9729 4794
82	1 2272 1108	1 8454 1691	2 0429 1790	3 3901 2307	4 1478 4260	5 0724 0690
83	1 2302 7910	1 8592 5753	2 0607 9343	3 4409 7492	4 2204 2984	5 1738 5504
84	1.2333 5480	1 8732 0196	2 0788 2537	3 4925 8954	4 2942 8737	5 2773 3214
85	1 2364 3819	1 8872 5098	2.0970 1510	3 5449 7838	4 3694 3740	5 3828 7878
86	1 2395 2928	1 9014 0536	2 1153 6398	3 5981 5306	4 4459 0255	5 4905 3636
87	1 2426 2811	1 9156 6590	2 1338 7341	3 6521 2535	4 5237 0584	5 6003 4708
88	1 2457 3468	1 9300 3339	2 1525 4481	3 7069 0723	4 6028 7070	5 7123 5402
89	1 2488 4901	1 9445 0865	2 1713 7957	3 7625 1084	4 6834 2093	5 8266 0110
90	1.2519 7114	1.9590 9246	2 1903 7914	3 8189 4851	4 7653 8080	5 9431 3313
91	1 2551 0106	1.9737 8565	2 2095 4496	3 8762 3273	4 8487 7496	6 0619 9579
92	1 2582 3882	1 9885 8905	2 2288 7848	3 9343 7622	4 9336 2853	6 1832 3570
93	1 2613 8441	2 0035 0346	2 2483 8117	3 9933 9187	5 0199 6703	6 3069 0042
94	1 2645 3787	2 0185 2974	2 2680 5450	4 0532 9275	5 1078 1645	6 4330 3843
95	1.2676 9922	2 0336 6871	2 2878 9998	4 1140 9214	5 1972 0324	6 5616 9920
96	1 2708 6847	2 0489 2123	2 3079 1910	4 1758 0352	5 2881 5429	6 6929 3318
97	1 2740 4564	2 0642 8814	2 3281 1340	4 2384 4057	5 3806 9699	6 8267 9184
98	1 2772 3075	2 0797 7030	2 3484 8439	4 3020 1718	5 4748 5919	7 9633 2768
99	1 2804 2383	2 0953 6858	2 3690 3363	4 3665 4744	5 5706 6923	7 1025 9423
100	1 2836 2489	2 1110 8384	2 3862 6267	4 4320 4565	5 6681 5594	7 2446 4612

APPENDIX A

The Amount of 1 at Different Rates of Compound Interest

TABLE I—*Continued*

Units of Time	2½%	3%	3½%	4%	4½%	5%
1	1 025	1 03	1 035	1 04	1 045	1 05
2	1 0506 25	1 0609	1 0712 25	1 0816	1 0920 25	1 1025
3	1 0768 9063	1 0927 27	1 1087 1788	1 1248 64	1 1411 6613	1 1576 25
4	1 1038 1289	1.1255 0881	1 1475 2300	1 1698 5856	1 1925 1860	1 2155 0625
5	1 1314 0821	1.1592 7407	1.1876 8631	1 2166 5290	1 2461 8194	1 2762 8156
6	1 1596 9342	1 1940 5230	1 2292 5533	1 2653 1902	1 3022 6012	1 3400 9564
7	1 1886 8575	1 2298 7387	1 2722 7926	1 3159 3178	1 3608 6183	1 4071 0042
8	1 2184 0290	1 2667 7008	1 3168 0904	1 3685 6905	1.4221 0061	1 4774 5544
9	1 2488 6297	1 3047 7318	1 3628 9735	1 4233 1181	1 4860 9514	1 5513 2822
10	1 2800 8454	1 3439 1638	1 4105 9876	1.4802 4428	1 5529 6942	1 6288 9463
11	1 3120 8666	1 3842 3387	1 4599 6972	1 5394 5406	1 6228 5305	1 7103 3936
12	1 3448 8882	1 4257 6089	1 5110 6866	1 6010 3222	1 6958 8143	1 7958 5633
13	1 3785 1104	1 4685 3371	1 5639 5606	1 6650 7351	1 7721 9610	1 8856 4914
14	1 4129 7382	1 5125 8972	1 6186 9452	1 7316 7645	1 8519 4492	1 9799 3160
15	1.4482 9817	1.5579 6742	1 6753 4883	1 8009 4351	1 9352 8244	2 0789 2818
16	1 4845 0562	1 6047 0644	1 7339 8604	1 8729 8125	2 0223 7015	2 1828 7459
17	1 5216 1826	1 6528 4763	1 7946 7555	1 9479 0050	2 1133 7681	2 2920 1832
18	1 5596 5872	1 7024 3306	1 8574 8920	2 0258 1652	2 2084 7877	2 4066 1923
19	1 5986 5019	1 7535 0605	1 9225 0132	2.1068 4918	2 3078 6031	2 5269 5020
20	1 6386 1644	1 8061 1123	1.9897 8886	2.1911 2314	2 4117 1402	2 6532 9771
21	1 6795 8185	1 8602 9457	2 0594 3147	2 2787 6807	2 5202 4116	2 7859 6259
22	1 7215 7140	1 9161 0341	2 1315 1158	2 3699 1879	2 6336 5201	2 9252 6072
23	1 7646 1068	1 9735 8651	2 2061 1448	2 4647 1554	2 7521 6635	3 0715 2376
24	1 8087 2595	2 0327 9411	2 2833 2849	2 5633 0416	2 8760 1383	3.2250 9994
25	1 8539 4410	2 0937 7793	2 3632 4498	2 6658 3633	3 0054 3446	3.3863 5494
26	1 9002 9270	2.1565 9127	2 4459 5856	2 7724 6978	3 1406 7901	3 5556 7269
27	1 9478 0002	2.2212 8901	2 5315 6711	2 8833 6858	3 2820 0956	3.7334 5632
28	1 9964 9502	2 2879 2768	2 6201 7196	2 9987 0332	3.4296 9998	3 9201 2914
29	2 0464 0739	2 3565 6551	2 7118 7798	3 1186 5145	3.5840 3649	4 1161 3560
30	2 0975 6758	2 4272 6247	2 8067 9370	3 2433 9751	3.7453 1813	4 3219 4238
31	2 1500 0677	2 5000 8035	2 9050 3148	3 3731 3341	3 9138 5745	4 5380 3949
32	2 2037 5694	2 5750 8276	3 0067 0759	3 5080 5875	4 0899 8104	4 7649 4147
33	2 2588 5086	2 6523 3524	3 1119 4235	3 6483 8110	4 2740 3018	5 0031 8854
34	2 3153 2213	2 7319 0530	3 2208 6033	3 7943 1634	4 4663 6154	5 2533 4797
35	2 3732 0519	2.8138 6245	3 3335 9045	3.9460 8899	4 6673 4781	5 5160 1537
36	2 4325 3532	2 8982 7833	3 4502 6611	4 1039 3255	4 8773 7846	5 7918 1614
37	2 4933 4870	2 9852 2668	3 5710 2543	4.2680 8986	5 0968 6049	6 0814 0694
38	2 5556 8242	3 0747 8348	3 6960 1132	4 4388 1345	5 3262 1921	6 3854 7729
39	2 6195 7448	3.1670 2698	3 8253 7171	4 6163 6599	5 5658 9908	6 7047 5115
40	2.6850 6384	3 2620 3779	3.9592 5972	4 8010 2063	5 8163 6454	7 0399 8871
41	2 7521 9043	3 3598 9893	4 0978 3381	4 9930 6145	6 0781 0094	7 3919 8815
42	2 8209 9520	3 4606 9589	4 2412 5799	5 1927 8391	6 3516 1548	7 7615 8756
43	2 8915 2008	3 5645 1677	4 3897 0202	5 4004 9527	6 6374 3818	8 1496 6693
44	2 9638 0808	3 6714 5227	4 5433 4160	5 6165 1508	6 9361 2290	8.5571 5028
45	3 0379 0328	3 7815 9584	4 7023 5855	5 8411 7568	7 2482 4843	8 9850 0779
46	3 1138 5086	3.8950 4372	4 8669 4110	6 0748 2271	7 5744 1961	9 4342 5818
47	3 1916 9713	4 0118 9503	5 0372 8404	6 3178 1562	7 9152 6849	9 9059 7109
48	3 2714 8956	4 1322 5188	5.2135 8898	6 5705 2824	8 2714 5557	10 4012 6965
49	3 3532 7680	4 2562 1944	5 3960 6459	6 8333 4937	8 6436 7107	10 9213 3313
50	3 4371 0872	4 3839 0602	5 5849 2686	7 1066 8335	9 0326 3627	11 4673 9979

APPENDIX A

The Amount of 1 at Different Rates of Compound Interest

TABLE I—*Continued*

Units of Time	2½%	3%	3½%	4%	4½%	5%
51	3 5230 3644	4 5154 2320	5 7803 9930	7 3909 5068	9 4391 0490	12 0407 6978
52	3 6111 1235	4 6508 8590	5 9827 1327	7 6865 8871	9 8638 6463	12 6428 0826
53	3 7013 9016	4 7904 1247	6 1921 0824	7 9940 5226	10 3077 3853	13 2749 4868
54	3 7939 2491	4 9341 2485	6 4088 3202	8 3138 1435	10 7715 8677	13 9386 9611
55	3 8887 7303	5 0821 4859	6 6331 4114	8 6463 6692	11 2563 0817	14 6356 3092
56	3 9859 9236	5 2346 1305	6 8653 0108	8 9922 2160	11 7628 4204	15 3674 1246
57	4 0856 4217	5 3916 5144	7 1055 8662	9 3519 1046	12 2921 6993	16 1357 8309
58	4 1877 8322	5 5534 0098	7 3542 8215	9 7259 8688	12 8453 1758	16 9425 7224
59	4 2924 7780	5 7200 0301	7 6116 8203	10 1150 2635	13 4233 5687	17 7897 0085
60	4 3997 8975	5 8916 0310	7 8780 9090	10 5196 2741	14 0274 0793	18 6791 8589
61	4.5097 8449	6 0683 5120	8 1538 2408	10 9404 1250	14 6586 4120	19 6131 4519
62	4 6225 2910	6 2504 0173	8 4392 0793	11.3780 2900	15 3182 8014	20 5938 0245
63	4 7380 9233	6.4379 1379	8 7345 8020	11 8331 5016	16 0076 0275	21 6234 9257
64	4 8565 4464	6 6310 5120	9.0402 9051	12.3064 7617	16 7279 4487	22 7046 6720
65	4 9779 5826	6.8299 8273	9 3567 0068	12 7987 3522	17 4807 0239	23 8399 0056
66	5 1024 0721	7 0348 8222	9 6841 8520	13 3106 8463	18 2673 3400	25 0318 9559
67	5 2299 6739	7 2459 2868	10 0231 3168	13.8431 1201	19 0893 6403	26 2834 9037
68	5 3607 1658	7 4633 0654	10 3739 4129	14 3968 3649	19 9483 8541	27 5976 6488
69	5 4947 3449	7 6872 0574	10 7370 2924	14 9727 0995	20 8460 6276	28 9775 4813
70	5 6321 0286	7.9178 2191	11 1128 2526	15 5716 1835	21 7841 3558	30 4264 2554
71	5 7729 0543	8 1553 5657	11 5017 7414	16 1944 8308	22 7644 2168	31 9477 4681
72	5 9172 2806	8 4000 1727	11 9043 3624	16 8422 6241	23 7888 2066	33 5451 3415
73	6 0651 5876	8 6520 1778	12 3209 8801	17 5159 5290	24 8593 1759	35 2223 9086
74	6 2167 8773	8 9115 7832	12.7522 2259	18 2165 9102	25 9779 8688	36 9835 1040
75	6 3722 0743	9 1789 2567	13 1985 5038	18 9452 5466	27 1469 9629	38 8326 8592
76	6 5315 1261	9 4542 9344	13 6604 9964	19 7030 6485	28 3686 1112	40 7743 2022
77	6 6948 0043	9 7379 2224	14 1386 1713	20 4911 8744	29 6451 9862	42 8130 3623
78	6 8621 7044	10 0300 5991	14 6334 6873	21 3108 3494	30 9792 3256	44 9536 8804
79	7 0337 2470	10 3309 6171	15 1456 4013	22 1632 6834	32 3732 9802	47 2013 7244
80	7 2095 6782	10 6408 9056	15 6757 3754	23 0497 9907	33.8300 9643	49 5614 4107
81	7 3898 0701	10 9601 1727	16 2243 8835	23 9717 9103	35.3524 5077	52 0395 1312
82	7 5745 5219	11 2889 2079	16 7922 4195	24 9306 6267	36 9433 1106	54 6414 8878
83	7 7639 1599	11 6275 8842	17 3799 7041	25 9278 8918	38 6057 6006	57 3735 6322
84	7 9580 1389	11 9764 1607	17 9882 6938	26 9650 0475	40 3430 1926	60 2422 4138
85	8 1569 6424	12 3357 0855	18 6178 5881	28 0436 0494	42 1584 5513	63 2543 5344
86	8 3608 8834	12 7057 7981	19 2694 8387	29.1653 4914	44 0555 8561	66.4170 7112
87	8 5699 1055	13 0869 5320	19 9439 1580	30 3319 6310	46.0380 8696	69 7379 2467
88	8 7841 5832	13.4795 6180	20 6419 5285	31 5452 4163	48 1098 0087	73 2248 2091
89	9 0037 6228	13 8839 4865	21 3644 2120	32.8070 5129	50 2747 4191	76 8860 6195
90	9 2288 5633	14.3004 6711	22 1121 7595	34 1193 3334	52 5371 0530	80 7303 6505
91	9 4595 7774	14 7294 8112	22 8861 0210	35 4841 0668	54 9012 7503	84 7668 8330
92	9 6960 6718	15 1713 6556	23 6871 1568	36 9034 7094	57 3718 3241	89 0052 2747
93	9 9384 6886	15 6265 0652	24 5161 6473	38 3796 0978	59.9535 6487	93.4554 8884
94	10 1869 3058	16 0953 0172	25 3742 3049	39 9147 9417	62 6514 7529	98.1282 6328
95	10 4416 0385	16 5781 6077	26 2623 2856	41 5113 8594	65 4707 9168	103 0346 7645
96	10 7026 4395	17 0755 0559	27 1815 1006	43 1718 4138	68 4169 7730	108 1864 1027
97	10 9702 1004	17 5877 7076	28 1328 6291	44 8987 1503	71 4957 4128	113 5957 3078
98	11 2444 6530	18 1154 0388	29 1175 1311	46.6946 6363	74 7130 4964	119 2755 1732
99	11 5255 7693	18 6588 6600	30 1366 2607	48 5624 5018	78 0751 3687	125.2392 9319
100	11 8137 1635	19 2186 3198	31 1914 0798	50 5049 4818	81 5885 1803	131 5012 5785

APPENDIX A

The Present Value of 1 at Different Rates of Compound interest

TABLE II

Units of Time	¼%	¾%	⅞%	1½%	1¾%	2%
1	0 9975 0623	0 9925 5583	0.9913 2590	0 9852 2167	0 9828 0098	0 9803 9216
2	0 9950 1869	0 9851 6708	0.9827 2704	0 9706 6175	0 9658 9777	0 9611 6878
3	0 9925 3734	0 9778 3333	0 9742 0276	0 9563 1699	0 9492 8528	0 9423 2233
4	0 9900 6219	0 9705 5417	0 9657 5243	0 9421 8423	0 9329 5851	0 9238 4543
5	0 9875 9321	0 9633 2920	0 9573 7539	0 9282 6033	0 9169 1254	0 9057 3081
6	0 9851 3038	0.9561 5802	0 9490 7102	0 9145 4219	0 9011 4254	0 8879 7138
7	0 9826 7370	0 9490 4022	0 9408 3868	0 9010 2679	0 8856 4378	0 8705 6018
8	0 9802 2314	0 9419 7540	0 9326 7775	0 8877 1112	0 8704 1157	0 8534 9037
9	0 9777 7869	0 9349 6318	0 9245 8761	0 8745 9224	0 8554 4135	0 8367 5527
10	0 9753 4034	0 9280 0315	0 9165 6765	0 8616 6723	0 8407 2860	0 8203 4830
11	0 9729 0807	0 9210 9494	0 9086 1724	0 8489 3323	0 8262 6889	0 8042 6304
12	0 9704 8187	0.9142 3815	0 9007 3581	0 8363 8742	0 8120 5788	0 7884 9318
13	0 9680 6171	0 9074 3241	0 8929 2273	0 8240 2702	0 7980 9128	0 7730 3253
14	0 9656 4759	0 9006 7733	0 8851 7743	0 8118 4928	0 7843 6490	0 7578 7502
15	0 9632 3949	0 8939 7254	0 8774 9931	0 7998 5150	0 7708 7459	0 7430 1473
16	0 9608 3740	0 8873 1766	0 8698 8779	0 7880 3104	0 7576 1631	0 7284 4581
17	0 9584 4130	0 8807 1231	0 8623 4230	0 7763 8526	0 7445 8605	0 7141 6256
18	0 9560 5117	0 8741 5614	0 8548 6225	0 7649 1159	0 7317 7990	0 7001 5937
19	0 9536 6700	0 8676 4878	0 8474 4709	0 7536 0747	0 7191 9401	0 6864 3076
20	0.9512 8878	0 8611 8985	0 8400 9624	0 7424 7042	0 7068 2458	0 6729 7133
21	0 9489 1649	0 8547 7901	0 8328 0917	0 7314 9795	0 6946 6789	0 6597 7582
22	0 9465 5011	0 8484 1589	0 8255 8530	0 7206 8763	0 6827 2028	0 6468 3904
23	0 9441 8964	0 8421 0014	0 8184 2409	0 7100 3708	0 6709 7817	0 6341 5592
24	0 9418 3505	0 8358 3140	0 8113 2499	0 6995 4392	0 6594 3800	0 6217 2149
25	0 9394 8634	0 8296 0933	0 8042 8748	0 6892 0583	0 6480 9632	0 6095 3087
26	0 9371 4348	0.8234 3358	0 7973 1101	0 6790 2052	0 6369 4970	0 5975 7928
27	0 9348 0646	0 8173 0380	0 7903 9505	0 6689 8574	0 6259 9479	0 5858 6204
28	0 9324 7527	0 8112 1966	0 7835 3908	0 6590 9925	0 6152 2829	0 5743 7455
29	0 9301 4990	0 8051 8080	0 7767 4258	0 6493 5887	0 6046 4697	0 5631 1231
30	0 9278 3032	0 7991 8690	0 7700 0504	0.6397 6243	0 5942 4764	0 5520 7089
31	0 9255 1653	0 7932 3762	0 7633 2594	0 6303 0781	0 5840 2716	0.5412 4597
32	0 9232 0851	0 7873 3262	0 7567 0477	0 6209 9292	0 5739 8247	0 5306 3330
33	0 9209 0624	0 7814 7158	0 7501 4104	0 6118 1568	0 5641 1053	0 5202 2873
34	0 9186 0972	0 7756 5418	0 7436 3424	0 6027 7407	0 5544 0839	0 5100 2817
35	0 9163 1892	0 7698 8008	0 7371 8388	0 5938 6608	0 5448 7311	0 5000 2761
36	0 9140 3384	0 7641 4896	0 7307 8947	0 5850 8974	0 5355 0183	0 4902 2315
37	0 9117 5445	0 7584 6051	0 7244 5053	0 5764 4309	0 5262 9172	0 4806 1093
38	0 9094 8075	0 7528 1440	0 7181 6657	0 5679 2423	0 5172 4002	0 4711 8719
39	0 9072 1272	0 7472 1032	0 7119 3712	0 5595 3126	0 5083 4400	0 4619 4822
40	0.9049 5034	0 7416 4796	0 7057 6171	0 5512 6232	0 4996 0098	0 4528 9042
41	0 9026 9361	0 7361 2701	0 6996 3986	0 5431 1559	0 4910 0834	0 4440 1021
42	0.9004 4250	0 7306 4716	0 6935 7111	0 5350 8925	0 4825 6348	0 4353 0413
43	0.8981 9701	0 7252 0809	0 6875 5500	0 5271 8153	0 4742 6386	0 4267 6875
44	0.8959 5712	0 7198 0952	0 6815 9108	0 5193 9067	0 4661 0699	0 4184 0074
45	0 8937 2281	0 7144 5114	0 6756 7889	0 5117 1494	0 4580 9040	0 4101 9680
46	0.8914 9407	0.7091 3264	0 6698 1798	0 5041 5265	0.4502 1170	0 4021 5373
47	0 8892 7090	0 7038 5374	0 6640 0792	0 4967 0212	0 4424 6850	0 3942 6836
48	0 8870 5326	0 6986 1414	0 6582 4824	0 4893 6170	0 4348 5848	0 3865 3761
49	0.8848 4116	0 6934 1353	0 6525 3853	0.4821 2975	0 4273 7934	0 3789 5844
50	0.8826 3457	0 6882 5165	0 6468 7835	0 4750 0468	0.4200 2883	0 3715 2788

APPENDIX A

The Present Value of 1 at Different Rates of Compound Interest

TABLE II—*Continued*

Units of Time	¼%	¾%	⅞%	1½%	1¾%	2%
51	0 8804 3349	0 6831 2819	0 6412 6726	0 4679 8491	0 4128 0475	0 3642 4302
52	0 8782 3790	0 6780 4286	0 6357 0484	0 4610 6887	0 4057 0492	0 3571 0100
53	0 8760 4778	0 6729 9540	0 6301 9067	0 4542 5505	0 3987 2719	0 3500 9902
54	0 8738 6312	0 6679 8551	0 6247 2433	0 4475 4192	0 3918 6947	0 3432 3433
55	0 8716 8391	0 6630 1291	0 6193 0541	0 4409 2800	0 3851 2970	0 3365 0425
56	0 8695 1013	0 6580 7733	0 6139 3349	0 4344 1182	0 3785 0585	0 3299 0613
57	0 8673 4178	0 6531 7849	0 6086 0817	0 4279 9194	0 3719 9592	0 3234 3738
58	0 8651 7883	0 6483 1612	0 6033 2904	0 4216 6694	0 3655 9796	0 3170 9547
59	0 8630 2128	0 6434 8995	0 5980 9571	0 4154 3541	0 3593 1003	0 3108 7791
60	0 8608 6911	0 6386 9970	0 5929 0776	0 4092 9597	0 3531 3025	0 3047 8227
61	0 8587 2230	0 6339 4511	0 5877 6482	0 4032 4726	0 3470 5676	0 2988 0614
62	0 8565 8085	0 6292 2592	0 5826 6649	0 3972 8794	0 3410 8772	0 2929 4720
63	0 8544 4474	0 6245 4185	0 5776 1238	0 3914 1669	0 3352 2135	0 2872 0314
64	0.8523 1395	0 6198 9266	0 5726 0211	0 3856 3221	0 3294 5587	0 2815 7170
65	0 8501 8848	0 6152 7807	0 5676 3530	0 3799 3321	0 3237 8956	0 2760 5069
66	0 8480 6831	0.6106 9784	0 5627 1158	0 3743 1843	0 3182 2069	0 2706 3793
67	0 8459 5343	0 6061 5170	0 5578 3056	0 3687 8663	0 3127 4761	0 2653 3130
68	0 8438 4382	0 6016 3940	0 5529 9188	0 3633 3658	0 3073 6866	0 2601 2873
69	0 8417 3947	0 5971 6070	0 5481 9517	0 3579 6708	0 3020 8222	0 2550 2817
70	0 8396 4037	0.5927 1533	0.5434 4007	0 3526 7692	0 2968 8670	0.2500 2761
71	0 8375 4650	0 5883 0306	0 5387 2622	0 3474 6495	0 2917 8054	0 2451 2511
72	0 8354 5786	0 5839 2363	0 5340 5325	0 3423 3000	0 2867 6221	0 2403 1874
73	0 8333 7442	0 5795 7681	0 5294 2082	0 3372 7093	0 2818 3018	0 2356 0661
74	0 8312 9618	0 5752 6234	0 5248 2857	0 3322 8663	0 2769 8298	0 2309 8687
75	0 8292 2312	0.5709 7999	0.5202 7615	0.3273 7599	0.2722 1914	0 2264 5771
76	0 8271 5523	0 5667 2952	0 5157 6322	0 3225 3793	0 2675 3724	0.2220 1737
77	0 8250 9250	0 5625 1069	0 5112 8944	0 3177 7136	0 2629 3586	0 2176 6408
78	0 8230 3491	0 5583 2326	0 5068 5447	0 3130 7523	0 2584 1362	0 2133 9616
79	0 8209 8246	0 5541 6701	0 5024 5796	0 3084 4850	0 2539 6916	0 2092 1192
80	0 8189 3512	0 5500 4170	0 4980 9959	0 3038 9015	0 2496 0114	0 2051 0973
81	0 8168 9289	0 5459 4710	0 4937 7902	0 2993 9916	0 2453 0825	0 2010 8797
82	0 8148 5575	0 5418 8297	0 4894 9593	0 2949 7454	0 2410 8919	0 1971 4507
83	0 8128 2369	0 5378 4911	0 4852 4999	0 2906 1531	0.2369 4269	0 1932 7948
84	0 8107 9670	0.5338 4527	0 4810 4089	0 2863 2050	0 2328 6751	0 1894 8968
85	0 8087 7476	0 5298 7123	0 4768 6829	0 2820 8917	0 2288 6242	0 1857 7420
86	0 8067 5787	0 5259 2678	0 4727 3188	0 2779 2036	0 2249 2621	0 1821 3157
87	0 8047 4600	0 5220 1169	0 4686 3136	0 2738 1316	0 2210 5770	0 1785 6036
88	0 8027 3915	0 5181 2575	0 4645 6640	0 2697 6666	0 2172 5572	0 1750 5918
89	0 8007 3731	0 5142 6873	0 4605 3671	0 2657 7997	0 2135 1914	0 1716 2665
90	0 7987 4046	0 5104 4043	0 4565 4197	0 2618 5218	0 2098 4682	0 1682 6142
91	0 7967 4859	0 5066 4063	0 4525 8187	0.2579 8245	0 2062 3766	0 1649 6217
92	0 7947 6168	0 5028 6911	0 4486 5613	0 2541 6990	0 2026 9057	0 1617 2762
93	0 7927 7973	0.4991 2567	0 4447 6444	0 2504 1369	0.1992 0450	0 1585 5649
94	0 7908 0273	0 4954 1009	0 4409 0651	0 2467 1300	0 1957 7837	0.1554 4754
95	0.7888 3065	0.4917 2217	0.4370 8204	0 2430 6699	0 1924 1118	0 1523 9955
96	0 7868 6349	0 4880 6171	0 4332 9075	0 2394 7487	0.1891 0190	0 1494 1132
97	0 7849 0124	0 4844 2850	0 4295 3234	0 2359 3583	0 1858 4953	0 1464 8169
98	0 7829 4388	0 4808 2233	0 4258 0654	0 2324 4909	0 1826 5310	0 1436 0950
99	0 7809 9140	0 4772 4301	0 4221 1305	0 2290 1389	0.1795 1165	0 1407 9363
100	0.7790 4379	0 4736 9033	0 4184 5159	0 2256 2944	0 1764 2422	0 1380 3297

APPENDIX A

The Present Value of 1 at Different Rates of Compound Interest

TABLE II—*Continued*

Units of Time	2½%	3%	3½%	4%	4½%	5%
1	0 9756 0976	0 9708 7379	0 9661 8357	0 9615 3846	0 9569 3780	0 9523 8095
2	0 9518 1440	0 9425 9591	0 9335 1070	0.9245 5621	0 9157 2995	0 9070 2948
3	0 9285 9941	0 9151 4166	0 9019 4271	0 8889 9636	0 8762 9660	0 8638 3760
4	0 9059 5064	0 8884 8705	0 8714 4223	0 8548 0419	0 8385 6134	0 8227 0247
5	0 8838 5429	0.8626 0878	0 8419 7317	0 8219 2711	0 8024 5105	0 7835 2617
6	0 8622 9687	0 8374 8426	0.8135 0064	0 7903 1453	0 7678 9574	0 7462 1540
7	0 8412 6524	0 8130 9151	0.7859 9096	0 7599 1781	0 7348 2846	0 7106 8133
8	0 8207 4657	0 7894 0923	0.7594 1156	0 7306 9021	0 7031 8513	0 6768 3936
9	0 8007 2836	0 7664 1673	0 7337 3097	0 7025 8674	0 6729 0443	0 6446 0892
10	0 7811 9840	0 7440 9391	0 7089 1881	0 6755 6417	0.6439 2768	0.6139 1325
11	0 7621 4478	0 7224 2128	0 6849 4571	0 6495 8093	0 6161 9874	0 5846 7929
12	0 7435 5589	0 7013 7988	0 6617 8330	0 6245 9705	0 5896 6386	0 5568 3742
13	0 7254 2038	0 6809 5134	0 6394 0415	0 6005 7409	0 5642 7164	0 5303 2135
14	0 7077 2720	0 6611 1781	0 6177 8179	0 5774 7508	0 5399 7286	0 5050 6795
15	0 6904 6556	0 6418 6195	0.5968 9062	0 5552 6450	0.5167 2044	0 4810 1710
16	0 6736 2493	0 6231 6694	0.5767 0591	0 5339 0818	0.4944 6932	0 4581 1152
17	0 6571 9506	0 6050 1645	0 5572 0378	0 5133 7325	0 4731 7639	0 4362 9669
18	0 6411 6591	0 5873 9461	0 5383 6114	0 4936 2812	0 4528 0037	0 4155 2065
19	0 6255 2772	0 5702 8603	0.5201 5569	0 4746 4242	0 4333 0179	0 3957 3396
20	0 6102 7094	0 5536 7575	0.5025 6588	0 4563 8695	0 4146 4286	0 3768 8948
21	0 5953 8629	0 5375 4928	0.4855 7090	0 4388 3360	0.3967 8743	0 3589 4236
22	0 5808 6467	0 5218 9250	0 4691 5063	0 4219 5539	0.3797 0089	0 3418 4987
23	0 5666 9724	0 5066 9175	0 4532 8563	0 4057 2633	0 3633 5013	0.3255 7131
24	0 5528 7535	0 4919 3374	0.4379 5713	0 3901 2147	0 3477 0347	0.3100 6791
25	0 5393 9059	0 4776 0557	0.4231 4699	0 3751 1680	0.3327 3060	0 2953 0277
26	0 5262 3472	0 4636 9473	0 4088 3767	0.3606 8923	0.3184 0248	0 2812 4073
27	0 5133 9973	0 4501 8906	0.3950 1224	0 3468 1657	0 3046 9137	0 2678 4832
28	0 5008 7778	0 4370 7675	0 3816 5434	0 3334 7747	0 2915 7069	0 2550 9364
29	0 4886 6125	0.4243 4636	0.3687 4815	0 3206 5141	0 2790 1502	0 2429 4632
30	0 4767 4269	0 4119 8676	0 3562 7841	0 3083 1867	0.2670 0002	0 2313 7745
31	0 4651 1481	0 3999 8715	0.3442 3035	0 2964 6026	0 2555 0241	0 2203 5947
32	0 4537 7055	0 3883 3703	0.3325 8971	0 2850 5794	0 2444 9991	0 2098 6617
33	0 4427 0298	0 3770 2625	0.3213 4271	0 2740 9417	0 2339 7121	0 1998 7254
34	0 4319 0534	0 3660 4490	0 3104 7605	0 2635 5209	0 2238 9589	0 1903 5480
35	0 4213 7107	0 3553 8340	0.2999 7686	0.2534 1547	0 2142 5444	0 1812 9029
36	0 4110 9372	0 3450 3243	0.2898 3272	0 2436 6872	0 2050 2817	0 1726 5741
37	0 4010 6705	0 3349 8294	0.2800 3161	0 2342 9685	0.1961 9921	0 1644 3563
38	0 3912 8492	0 3252 2615	0.2705 6194	0 2252 8543	0 1877 5044	0 1566 0536
39	0 3817 4139	0.3157 5355	0.2614 1250	0.2166 2061	0 1796 6549	0 1491 4797
40	0 3724 3062	0 3065 5684	0 2525 7247	0 2082 8904	0 1719 2870	0 1420 4568
41	0 3633 4695	0.2976 2800	0 2440 3137	0 2002 7793	0 1645 2507	0 1352 8160
42	0 3544 8483	0 2889 5922	0 2357 7910	0 1925 7493	0 1574 4026	0 1288 3962
43	0 3458 3886	0 2805 4294	0 2278 0590	0 1851 6820	0 1506 6054	0 1227 0440
44	0.3374 0376	0.2723 7178	0 2201 0231	0 1780 4635	0 1441 7276	0 1168 6133
45	0.3291 7440	0 2644 3862	0 2126 5924	0 1711 9841	0 1379 6437	0.1112 9651
46	0 3211 4576	0 2567 3653	0 2054 6787	0 1646 1386	0 1320 2332	0.1059 9668
47	0 3133 1294	0 2492 5876	0 1985 1968	0 1582 8256	0 1263 3810	0 1009 4921
48	0.3056 7116	0 2419 9880	0 1918 0645	0 1521 9476	0 1208 9771	0 0961 4211
49	0.2982 1576	0 2349 5029	0 1853 2024	0 1463 4112	0 1156 9158	0 0915 6391
50	0.2909 4221	0 2281 0708	0.1790 5337	0 1407 1262	0.1107 0965	0 0872 0373

APPENDIX A

The Present Value of 1 at Different Rates of Compound Interest

TABLE II—*Continued*

Units of Time	2½%	3%	3½%	4%	4½%	5%
51	0 2838 4606	0 2214 6318	0 1729 9843	0 1353 0059	0.1059 4225	0 0830 5117
52	0 2769 2298	0.2150 1280	0 1671 4824	0 1300 9672	0 1013 8014	0 0790 9635
53	0 2701 6876	0 2087 5029	0 1614 9589	0 1250 9300	0 0970 1449	0 0753 2986
54	0 2635 7928	0 2026 7019	0 1560 3467	0 1202 8173	0 0928 3683	0 0717 4272
55	0 2571 5052	0 1967 6717	0 1507 5814	0 1156 5551	0 0888 3907	0 0683 2640
56	0 2508 7855	0 1910 3609	0 1456 6004	0 1112 0722	0.0850 1347	0 0650 7276
57	0 2447 5956	0 1854 7193	0 1407 3433	0 1069 3002	0.0813 5260	0 0619 7406
58	0 2387 8982	0 1800 6984	0 1359 7520	0 1028 1733	0 0778 4938	0 0590 2291
59	0 2329 6568	0 1748 2508	0 1313 7701	0 0988 6282	0 0744 9701	0 0562 1230
60	0 2272 8359	0.1697 3309	0 1269 3431	0 0950 6040	0 0712 8901	0 0535 3552
61	0 2217 4009	0.1647 8941	0 1226 4184	0 0914 0423	0 0682 1915	0 0509 8621
62	0 2163 3179	0 1599 8972	0 1184 9453	0 0878 8868	0.0652 8148	0 0485 5830
63	0 2110 5541	0.1553 2982	0 1144 8747	0 0845 0835	0 0624 7032	0 0462 4600
64	0 2059 0771	0 1508 0565	0 1106 1591	0 0812 5803	0.0597 8021	0 0440 4381
65	0 2008 8557	0 1464 1325	0 1068 7528	0 0781 3272	0 0572 0594	0 0419 4648
66	0 1959 8593	0 1421 4879	0 1032 6114	0 0751 2762	0 0547 4253	0 0399 4903
67	0 1912 0578	0 1380 0853	0 0997 6922	0 0722 3809	0 0523 8519	0 0380 4670
68	0 1865 4223	0 1339 8887	0 0963 9538	0 0694 5970	0 0501 2937	0 0362 3495
69	0 1819 9241	0 1300 8628	0 0931 3563	0 0667 8818	0 0479 7069	0 0345 0948
70	0 1775 5358	0 1262 9736	0 0899 8612	0 0642 1940	0 0459 0497	0 0328 6617
71	0 1732 2300	0.1226 1880	0 0869 4311	0 0617 4942	0 0439 2820	0 0313 0111
72	0 1689 9805	0 1190 4737	0 0840 0300	0 0593 7445	0 0420 3655	0 0298 1058
73	0.1648 7615	0 1155 7998	0 0811 6232	0 0570 9081	0 0402 2637	0 0283 9103
74	0 1608 5478	0 1122 1357	0 0784 1770	0 0548 9501	0 0384 9413	0.0270 3908
75	0 1569 3149	0 1089 4521	0 0757 6590	0 0527 8367	0 0368 3649	0 0257 5150
76	0 1531 0389	0 1057 7205	0 0732 0376	0 0507 5353	0 0352 5023	0 0245 2524
77	0 1493 6965	0 1026 9131	0 0707 2827	0 0488 0147	0 0337 3228	0 0233 5737
78	0 1457 2649	0 0997 0030	0 0683 3650	0 0469 2449	0 0322 7969	0 0222 4512
79	0 1421 7218	0 0967 9641	0 0660 2560	0 0451 1970	0 0308 8965	0 0211 8582
80	0 1387 0457	0 0939 7710	0 0637 9285	0 0433 8433	0.0295 5948	0 0201 7698
81	0 1353 2153	0 0912 3990	0 0616 3561	0 0417 1570	0 0282 8658	0 0192 1617
82	0 1320 2101	0 0885 8243	0 0595 5131	0 0401 1125	0 0270 6850	0 0183 0111
83	0 1288 0098	0 0860 0236	0 0575 3750	0 0385 6851	0 0259 0287	0 0174 2963
84	0 1256 5949	0 0834 9743	0 0555 9178	0 0370 8510	0 0247 8744	0 0165 9965
85	0 1225 9463	0 0810 6547	0 0537 1187	0 0356 5875	0 0237 2003	0 0158 0919
86	0 1196 0452	0 0787 0434	0 0518 9553	0 0342 8726	0.0226 9860	0 0150 5637
87	0 1166 8733	0 0764 1198	0 0501 4060	0 0329 6852	0 0217 2115	0 0143 3940
88	0 1138 4130	0 0741 8639	0 0484 4503	0 0317 0050	0 0207 8579	0 0136 5657
89	0 1110 6468	0 0720 2562	0 0468 0679	0 0304 8125	0 0198 9070	0 0130 0626
90	0 1083 5579	0 0699 2779	0 0452 2395	0 0293 0890	0 0190 3417	0 0123 8691
91	0 1057 1296	0.0678 9105	0 0436 9464	0 0281 8163	0 0182 1451	0 0117 9706
92	0.1031 3460	0 0659 1364	0.0422 1704	0 0270 9772	0 0174 3016	0 0112 3530
93	0 1006 1912	0 0639 9383	0 0407 8941	0 0260 5550	0 0166 7958	0 0107 0028
94	0 0981 6500	0 0621 2993	0 0394 1006	0.0250 5337	0 0159 6132	0 0101 9074
95	0 0957 7073	0 0603 2032	0 0380 7735	0 0240 8978	0 0152 7399	0 0097 0547
96	0 0934 3486	0 0585 6342	0 0367 8971	0 0231 6325	0 0146 1626	0 0092 4331
97	0 0911 5596	0.0568 5769	0 0355 4562	0 0222 7235	0 0139 8685	0 0088 0315
98	0 0889 3264	0 0552 0164	0 0343 4359	0 0214 1572	0 0133 8454	0 0083 8395
99	0 0867 6355	0 0535 9383	0 0331 8221	0 0205 9204	0 0128 0817	0 0079 8471
100	0 0846 4737	0.0520 3284	0 0320 6011	0 0198 0004	0 0122 5663	0 0076 0449

APPENDIX A

THE AMOUNT OF 1 PER ANNUM AT DIFFERENT RATES OF COMPOUND INTEREST

TABLE III

Units of Time	¼%	¾%	⅞%	1½%	1¾%	2%
1	1 0025	1 0075	1 0087 5	1 015	1 0175	1.02
2	2 0075 0625	2 0225 5625	2 0263 2656	2 0452 25	2 0528 0625	2 0604
3	3 0150 2502	3 0452 2542	3 0528 0692	3 0909 0338	3 1062 3036	3 1216 08
4	4 0250 6258	4 0755 6461	4.0882 6898	4 1522 6693	4 1780 8939	4 2040 4016
5	5.0376 2523	5 1136 3135	5.1327 9133	5 2295 5093	5 2687 0596	5 3081 2096
6	6 0527 1930	6 1594 8358	6 1864 5326	6 3229 9419	6 3784 0831	6 4342 8338
7	7 0703 5110	7 2131 7971	7.2493 3472	7 4328 3911	7 5075 3045	7 5829 6905
8	8 0905 2697	8 2747 7856	8 3215 1640	8 5593 3169	8 6564 1224	8 7546 2843
9	9 1132 5329	9 3443 3940	9 4030 7967	9 7027 2167	9 8253 9945	9 9497 2100
10	10 1385 3642	10 4219 2194	10 4941 0662	10 8632 6249	11 0148 4394	11 1687 1542
11	11 1663 8277	11 5075 8636	11 5946 8005	12 0412 1143	12 2251 0371	12 4120 8973
12	12 1967 9872	12 6013 9325	12 7048 8350	13 2368 2960	13 4565 4303	13 6803 3152
13	13 2297 9072	13 7034 0370	13 8248 0123	14 4503 8205	14 7095 3253	14 9739 3815
14	14 2653 6520	14 8136 7923	14 9545 1824	15 6821 3778	15 9844 4935	16 2934 1692
15	15 3035 2861	15 9322 8183	16 0941 2028	16 9323 6984	17 2816 7721	17 6392 8525
16	16 3442 8743	17 0592 7394	17 2436 9383	18 2013 5539	18 6016 0656	19 0120 7096
17	17 3876 4815	18 1947 1849	18 4033 2615	19 4893 7572	19 9446 3468	20 4123 1238
18	18 4336 1727	19 3386 7888	19 5731 0526	20 7967 1636	21 3111 6578	21 8405 5863
19	19 4822 0131	20 4912 1897	20 7531 1993	22 1236 6710	22 7016 1119	23 2973 6980
20	20 5334 0682	21 6524 0312	21.9434 5973	23.4705 2211	24 1163 8938	24 7833 1719
21	21 5872 4033	22 8222 9614	23.1442 1500	24 8375 7994	25 5559 2620	26 2989 8354
22	22 6437 0843	24 0009 6336	24 3554 7688	26 2251 4364	27 0206 5490	27 8449 6321
23	23 7028 1770	25 1884 7059	25 5773 3730	27 6335 2080	28 5110 1637	29 4218 6247
24	24 7645 7475	26 3848 8412	26 8098 8900	29 0630 2361	30 0274 5915	31 0302 9972
25	25 8289 8619	27 5902 7075	28 0532 2553	30 5139 6896	31 5704 3969	32 6709 0572
26	26 8960 5865	28 8046 9778	29 3074 4126	31 9866 7850	33.1404 2238	34 3443 2383
27	27 9657 9880	30 0282 3301	30 5726 3137	33 4814 7867	34 7378 7977	36 0512 1031
28	29 0382 1330	31 2609 4476	31 8488 9189	34 9987 0085	36.3632 9267	37 7922 3451
29	30 1133 0883	32 5029 0184	33 1363 1970	36 5386 8137	38 0171 5029	39 5680 7921
30	31 1910 9210	33 7541 7361	34.4350 1249	38 1017 6159	39 6999 5042	41 3794 4079
31	32 2715 6983	35 0148 2991	35 7450 6885	39 6882 8801	41 4121 9955	43 2270 2961
32	33 3547 4876	36 2849 4113	37 0665 8820	41 2986 1233	43 1544 1305	45 1115 7020
33	34 4406 3563	37 5645 7819	38 3996 7085	42 9330 9152	44 9271 1527	47 0338 0160
34	35 5292 3722	38 8538 1253	39 7444 1797	44 5920 8789	46 7308 3979	48 9944 7763
35	36 6205 6031	40 1527 1612	41 1009 3163	46 2759 6921	48 5661 2949	50 9943 6719
36	37 7146 1171	41 4613 6149	42.4693 1478	47 9851 0874	50 4335 3675	53 0342 5453
37	38 8113 9824	42 7798 2170	43 8496 7128	49 7198 8538	52 3336 2365	55 1149 3962
38	39 9109 2673	44 1081 7037	45.2421 0591	51 4806 8366	54 2669 6206	57.2372 3841
39	41 0132 0405	45 4464 8164	46 6467 2433	53 2678 9391	56 2341 3390	59 4019 8318
40	42 1182 3706	46 7948 3026	48 0636 3317	55 0819 1232	58 2357 3124	61 6100 2284
41	43 2260 3265	48 1532 9148	49 4929 3996	56 9231 4100	60 2723 5654	63 8622 2330
42	44 3365 9774	49 5219 4117	50 9347 5319	58 7919 8812	62 3446 2278	66 1594 6777
43	45 4499 3923	50 9008 5573	52 3891 8228	60 6888 6794	64 4531 5367	68 5026 5712
44	46 5660 6408	52 2901 1215	53 8563 3762	62 6142 0096	66 5985 8386	70 8927 1027
45	47 6849 7924	53 6897 8799	55 3363 3058	64 5684 1398	68 7815 5908	73.3305 6447
46	48 8066 9169	55 0999 6140	56 8292 7347	66 5519 4018	71 0027 3637	75 8171 7576
47	49 9312 0842	56 5207 1111	58 3352 7961	68 5652 1929	73 2627 8425	78 3535 1927
48	51 0585 3644	57 9521 1644	59 8544 6331	70 6086 9758	75 5623 8298	80 9405 8966
49	52 1886 8278	59 3942 5732	61.3869 3986	72 6828 2804	77 9022 2468	83 5794 0145
50	53.3216 5449	60 8472 1424	62 9328 2559	74.7880 7046	80 2830 1361	86 2709 8948

APPENDIX A

The Amount of 1 per Annum at Different Rates of Compound Interest

TABLE III—*Continued*

Units of Time	¼%	¾%	⅞%	1½%	1¾%	2%
51	54 4574 5862	62 3110 6835	64 4922 3781	76 9248 9152	82 7054 6635	89 0164 0927
52	55 5961 0227	63 7859 0136	66 0652 9489	79 0937 6489	85 1703 1201	91 8167 3746
53	56 7375 9252	65 2717 9562	67 6521 1622	81 2951 7136	87 6782 9247	94 6730 7221
54	57 8819 3650	66 7688 3409	69 2528 2224	83 5295 9893	90 2301 6259	97 5865 3365
55	59.0291 4135	68 2771 0035	70.8675 3443	85 7975 4292	92 8266 9043	100 5582 6432
56	60 1792 1420	69 7966 7860	72 4963 7536	88 0995 0606	95 4686 5752	103 5894 2961
57	61 3321 6223	71 3276 5369	74 1394 6864	90 4359 9865	98 1568 5902	106 6812 1820
58	62 4879 9264	72 8701 1109	75 7969 3900	92 8075 3863	100 8921 0405	109 8348 4257
59	63.6467 1262	74,4241 3693	77 4689 1221	95 2146 5171	103 6752 1588	113 0515 3942
60	64 8083 2940	75 9898 1795	79 1555 1519	97 6578 7149	106 5070 3215	116 3325 7021
61	65 9728 5023	77 5672 4159	80 8568 7595	100 1377 3956	109 3884 0522	119 6792 2161
62	67 1402 8235	79 1564 9590	82 5731 2362	102.6548 0565	112 3202 0231	123 0928 0604
63	68 3106 3306	80 7576 6962	84 3043 8845	105 2096 2774	115 3033 0585	126 5746 6216
64	69 4839 0964	82 3708 5214	86 0508 0185	107.8027 7215	118 3386 1370	130 1261 5541
65	70 6601 1942	83.9961 3353	87 8124 9636	110 4348 1374	121 4270 3944	133.7486 7852
66	71 8392 6971	85 6336 0453	89 5896 0571	113 1063 3594	124 5695 1263	137 4436 5209
67	73 0213 6789	87 2833 5657	91 3822 6476	115 8179 3098	127 7669 7910	141 2125 2513
68	74 2064 2131	88 9454 8174	93 1906 0957	118 5701 9995	131 0204 0124	145 0567 7563
69	75 3944 3736	90 6200 7285	95 0147 7741	121 3637 5295	134 3307 5826	148 9779 1114
70	76 5854 2345	92 3072 2340	96 8549 0671	124.1992 0924	137.6990 4653	152 9774 6937
71	77 7793 8701	94 0070 2758	98 7111 3714	127 0771 9738	141 1262 7984	157 0570 1875
72	78 9763 3548	95 7195 8028	100 5836 0959	129 9983 5534	144 6134 8974	161 2181 5913
73	80 1762 7632	97 4449 7714	102 4724 6618	132 9633 3067	148 1617 2581	165 4625 2231
74	81 3792 1701	99.1833 1446	104 3778 5025	135 9727 8063	151 7720 5601	169 7917 7276
75	82 5851 6505	100 9346 8932	106 2999 0644	139 0273 7234	155 4455 6699	174 2076 0821
76	83 7941 2797	102 6991 9949	108 2387 8063	142 1277 8292	159 1833 6441	178 7117 6038
77	85 0061 1329	104 4769 4349	110 1946 1996	145 2746 9967	162 9865 7329	183 3059 9558
78	86 2211 2857	106 2680 2056	112 1675 7288	148 4688 2016	166.8563 3832	187 9921 1549
79	87 4391 8139	108 0725 3072	114.1577 8914	151 7108 5247	170 7938 2424	192 7719 5780
80	88 6602 7934	109.8905 7470	116 1654 1980	155 0015 1525	174 8002 1617	197 6473 9696
81	89 8844 3004	111 7222 5401	118 1906 1722	158 3415 3798	178 8767 1995	202 6203 4490
82	91 1116 4112	113 5676 7091	120 2335 3512	161 7316 6105	183 0245 6255	207 6927 5180
83	92 3419 2022	115 4269 2845	122 2943 2855	165 1726 3597	187 2449 9239	212 8666 0683
84	93 5752 7502	117 3001 3041	124 3731 5393	168 6652 2551	191 5392 7976	218 1439 3807
85	94 8117 1321	119 1873 8139	126.4701 6903	172 2102 0389	195 9087 1716	223 5268 1775
86	96 0512 4249	121 0887 8675	128 5855 3301	175 8083 5695	200 3546 1971	229 0173 5411
87	97 2938 7060	123 0044 5265	130 7194 0642	179 4604 8230	204.8783 2555	234 6177 0119
88	98 5396 0527	124 9344 8604	132 8719 5123	183 1673 8954	209 4811 9625	240 3300 5521
89	99 7884 5429	126 8789 9469	135 0433 3080	186 9299 0038	214.1646 1718	246 1566 5632
90	101.0404 2542	128 8380 8715	137 2337 0994	190 7488 4889	218 9299 9798	252 0997 8944
91	102 2955 2649	130.8118 7280	139 4432 5491	194 6250 8162	223 7787 7295	258 1617 8523
92	103 5537 6530	132 8004 6185	141 6721 3339	198 5594 5784	228.7124 0148	264 3450 2004
93	104 8151 4972	134 8039 6531	143 9205 1455	202 5528 4971	233 7323 6850	270 6519 2135
94	106 0796 8759	136 8224 9505	146 1885 6906	206.6061 4246	238 8401 8495	277 0849 5978
95	107.3473 8681	138 8561 6377	148 4764 6903	210 7202 3459	244 0373 8819	283 6466 5898
96	108 6182 5528	140 9050 8499	150 7843 8813	214 8960 3811	249 3255 4248	290 3395 9216
97	109 8923 0091	142 9693 7313	153 1125 0153	219 1344 7868	254 7062 3947	297 1663 8400
98	111 1695 3167	145 0491 4343	155 4609 8592	223 4364 9586	260 1810 9866	304 1297 1168
99	112 4499 5550	147 1445 1201	157 8300 1955	227 8030 4330	265 7517 6789	311 2323 0591
100	113 7335 8038	149 2555 9585	160.2162 8222	232 2350 8895	271 4199 2383	318 4769 5203

APPENDIX A

THE AMOUNT OF 1 PER ANNUM AT DIFFERENT RATES OF COMBINED INTEREST

TABLE III—*Continued*

Units of Time	2½%	3%	3½%	4%	4½%	5%
1	1 025	1 03	1 035	1 04	1 045	1 05
2	2 0756 25	2 0909	2 1062 25	2.1216	2 1370 25	2 1525
3	3 1525 1563	3 1836 27	3 2149 4288	3 2464 64	3 2781 9113	3 3101 25
4	4 2563 2852	4 3091 3581	4 3624 6588	4 4163 2256	4 4707 0973	4 5256 3125
5	5.3877 3673	5 4684 0988	5 5501 5218	5 6329 7546	5 7168 9166	5 8019 1281
6	6 5474 3015	6 6624 6218	6 7794 0751	6 8982 9448	7 0191 5179	7.1420 0845
7	7 7361 1590	7 8923 3605	8 0516 8677	8 2142 2626	8 3800 1362	8 5491 0888
8	8 9545 1880	9.1591 0613	9.3684 9581	9 5827 9531	9 8021 1423	10 0265 6432
9	10 2033 8177	10 4638 7931	10 7313 9316	11 0061 0712	11 2882 0937	11 5778 9254
10	11 4834 6631	11 8077 9569	12 1419 9192	12 4863 5141	12 8411 7879	13 2067 8716
11	12 7955 5297	13 1920 2956	13 6019 6164	14 0258 0546	14 4640 3184	14 9171 2652
12	14 1404 4179	14 6177 9045	15 1130 3030	15 6268 3768	16 1599 1327	16 7129 8285
13	15 5189 5284	16 0863 2416	16 6769 8636	17 2919 1119	17 9321 0937	18 5986 3199
14	16 9319 2666	17 5989 1389	18 2956 8088	19 0235 8764	19 7840 5429	20 5785 6359
15	18 3802 2483	19 1568 8130	19 9710 2971	20.8245 3114	21 7193 3673	22 6574 9177
16	19 8647 3045	20 7615 8774	21 7050 1575	22 6975 1239	23 7417 0689	24 8403 6636
17	21 3863 4871	22 4144 3537	23 4996 9130	24 6454 1288	25 8550 8370	27 1323 8467
18	22 9460 0743	24 1168 6844	25 3571 8050	26 6712 2940	28 0635 6246	29 5390 0391
19	24.5446 5761	25 8703 7449	27 2796 8181	28 7780 7858	30 3714 2277	32.0659 5410
20	26 1832 7405	27 6764 8572	29 2694 7068	30.9692 0172	32 7831 3680	34.7192 5181
21	27 8628 5590	29 5367 8030	31 3289 0215	33 2479 6979	35.3033 7795	37 5052 1440
22	29 5844 2730	31 4528 8370	33 4604 1373	35 6178 8858	37 9370 2996	40 4304 7512
23	31 3490 3798	33 4264 7022	35 6665 2821	38 0826 0412	40 6891 9631	43 5019 9887
24	33 1577 6393	35 4592 6432	37 9498 5669	40 6459 0829	43 5652 1015	46 7270 9882
25	35 0117 0803	37 5530 4225	40 3131 0168	43.3117 4462	46 5706 4460	50 1134 5376
26	36 9120 0073	39 7096 3352	42.7590 6024	46 0842 1440	49 7113 2361	53 6691 2645
27	38 8598 0075	41 9309 2252	45.2906 2734	48 9675 8298	52.9933 3317	57 4025 8277
28	40 8562 9577	44 2188 5020	47 9107 9930	51 9662 8630	56 4230 3316	61 3227 1191
29	42.9027 0316	46 5754 1571	50 6226 7728	55 0849 3775	60 0070 6966	65 4388 4750
30	45 0002 7074	49 0026 7818	53 4294 7098	58 3283 3526	63 7523 8779	69 7607 8988
31	47 1502 7751	51 5027 5852	56 3345 0247	61 7014 6867	67.6662 4524	74 2988 2937
32	49 3540 3445	54 0778 4128	59 3412 1005	65 2095 2742	71.7562 2628	79 0637 7084
33	51 6128 8531	56 7301 7652	62.4531 5240	68 8579 0851	76 0302 5646	84 0669 5938
34	53 9282 0744	59 4620 8181	65 6740 1274	72 6522 2486	80.4966 1800	89 3203 0735
35	56.3014 1263	62 2759 4427	69 0076 0318	76 5983 1385	85.1639 6581	94 8363 2272
36	58 7339 4794	65 1742 2259	72 4578 6930	80 7022 4640	90 0413 4427	100 6281 3886
37	61 2272 9664	68 1594 4927	76 0288 9472	84 9703 3626	95 1382 0476	106 7095 4580
38	63 7829 7906	71 2342 3275	79 7249 0604	89 4091 4971	100.4644 2398	113 0950 2309
39	66 4025 5354	74 4012 5973	83 5502 7775	94 0255 1570	106.0303 2306	119 7997 7424
40	69 0876 1737	77.6632 9753	87 5095 3747	98 8265 3633	111 8466 8760	126 8397 6295
41	71.8398 0781	81 0231 9645	91 6073 7128	103 8195 9778	117 9247 8854	134 2317 5110
42	74 6608 0300	84 4838 9234	95 8486 2928	109 0123 8169	124 2764 0402	141 9933 3866
43	77 5523 2308	88 0484 0911	100 2383 3130	114 4128 7696	130 9138 4220	150 1430 0559
44	80 5161 3116	91 7198 6139	104 7816 7290	120 0293 9204	137 8499 6510	158 7001 5587
45	83.5540 3443	95.5014 5723	109.4840 3145	125 8705 6772	145 0982 1353	167.6851 6366
46	86 6678 8530	99.3965 0095	114.3509 7255	131.9453 9043	152 6726 3314	177.1194 2185
47	89 8595 8243	103 4083 9598	119 3882 5659	138 2632 0604	160.5879 0163	187 0253 9294
48	93.1310 7199	107 5406 4785	124 6018 4557	144 8337 3429	168 8593 5720	197 4266 6259
49	96.4843 4879	111 7968 6729	129 9979 1016	151 6670 8366	177.5030 2828	208 3479 9572
50	99 9214 5751	116 1807 7331	135.5828 3702	158 7737 6700	186.5356 6455	219 8153 9550

APPENDIX A

The Amount of 1 per Annum at Different Rates of Compound Interest

TABLE III—*Continued*

Units of Time	2½%	3%	3½%	4%	4½%	5%
51	103 4444 9395	120 6961 9651	141 3632 3631	166 1647 1768	195 9747 6946	231 8561 6528
52	107 0556 0629	125 3470 8240	147 3459 4958	173 8513 0639	205 8386 3408	244 4989 7354
53	110 7569 9645	130 1374 9488	153 5380 5782	181 8453 5865	216 1463 7262	257 7739 2222
54	114 5509 2136	135 0716 1972	159 9468 8984	190 1591 7299	226 9179 5938	271 7126 1833
55	118 4396 9440	140 1537 6831	166 5800 3099	198 8055 3991	238 1742 6756	286 3482 4924
56	122 4256 8676	145 3883 8136	173 4453 3207	207 7977 6151	249 9371 0960	301 7156 6171
57	126 5113 2893	150 7800 3280	180 5509 1869	217 1496 7197	262 2292 7953	317 8514 4479
58	130 6991 1215	156 3334 3379	187 9052 0085	226 8756 5885	275 0745 9711	334 7940 1703
59	134 9915 8995	162 0534 3680	195 5168 8288	236 9906 8520	288 4979 5398	352 5837 1788
60	139 3913 7970	167.9450 3991	203 3949 7378	247.5103 1261	302 5253 6190	371.2629 0378
61	143 9011 6419	174 0133 9110	211 5487 9786	258 4507 2511	317 1840 0319	390 8760 4897
62	148 5236 9330	180 2637 9284	219 9880 0579	269 8287 5412	332 5022 8333	411 4698 5141
63	153 2617 8563	186 7017 0662	228 7225 8599	281 6619 0428	348 5098 8608	433 0933 4398
64	158 1183 3027	193.3327 5782	237.7628 7650	293 9683 8045	365 2378 3096	455 7980 1118
65	163 0962 8853	200.1627 4055	247.1195 7718	306 7671 1567	382 7185 3335	479 6379 1174
66	168 1986 9574	207 1976 2277	256 8037 6238	320 0778 0030	400 9858 6735	504 6698 0733
67	173 4236 6314	214 4435 5145	266 8268 9406	333 9209 1231	420 0752 3138	530 9532 9770
68	178 7893 7971	221 9068 5800	277.2008 3535	348 3177 4880	440 0236 1679	558 5509 6258
69	184 2841 1421	229.5940 6374	287 9378 6459	363 2904 5876	460 8696 7955	587 5285 1071
70	189 9162 1706	237 5118 8565	299.0506 8985	378 8620 7711	482 6538 1513	617.9549 3625
71	195 6891 2249	245 6672 4222	310.5524 6400	395 0565 6019	505 4182 3681	649 9026 8306
72	201 6063 5055	254 0672 5949	322 4568 0024	411 8988 2260	529 2070 5747	683 4478 1721
73	207 6715 0931	262 7192 7727	334 7777 8824	429 4147 7550	554 0663 7505	718 6702 0807
74	213 8882 9705	271 6308 5559	347 5300 1083	447 6313 6652	580 0443 6193	755 6537 1848
75	220.2605 0447	280 8097 8126	360 7285 6121	466 5766 2118	607.1913 5822	794 4864 0440
76	226 7920 1709	290 2640 7469	374 3890 6085	486 2796 8603	635 5599 6934	835 2607 2462
77	233 4868 1751	300 0019 9693	388 5276 7798	506 7708 7347	665 2051 6796	878 0737 6085
78	240 3489 8795	310 0320 5684	403 1611 4671	528 0817 0841	696 1844 0052	923 0274 4889
79	247 3827 1265	320 3630 1855	418 3067 8685	550 2449 7675	728 5576 9854	970 2288 2131
80	254 5922 8047	331 0039 0910	433 9825 2439	573.2947 7582	762 3877 9497	1019 7902 6240
81	261 9820 8748	341 9640 2638	450 2069 1274	597 2665 6685	797 7402 4575	1071 8297 7552
82	269 5566 3966	353 2529 4717	466 9991 5469	622 1972 2952	834 6835 5680	1126 4712 6430
83	277 3205 5566	364 8805 3558	484 3791 2510	648 1251 1870	873 2893 1686	1183 8448 2752
84	285 2785 6955	376 8569 5165	502 3673 9448	675 0901 2345	913 6323 3612	1244 0870 6889
85	293 4355 3379	389 1926 6020	520 9852 5329	703 1337 2839	955 7907 9125	1307 3414 2234
86	301 7964 2213	401 8984 4001	540 2547 3715	732 2990 7753	999 8463 7685	1373 7584 9345
87	310 3663 3268	414 9853 9321	560 1986 5295	762 6310 4063	1045 8844 6381	1443 4964 1812
88	319 1504 9100	428 4649 5500	580 8406 0581	794 1762 8225	1093 9942 6468	1516 7212 3903
89	328 1542 5328	442 3489 0365	602 2050 2701	826 9833 3354	1144 2690 0659	1593 6073 0098
90	337 3831 0961	456 6493 7076	624 3172 0295	861 1026 6688	1196 8061 1189	1674 3376 6603
91	346 8426 8735	471 3788 5189	647 2033 0506	896 5867 7356	1251 7073 8692	1759 1045 4933
92	356.5387 5453	486 5502 1744	670 8904 2073	933 4902 4450	1309 0792 1933	1848 1097 7680
93	366 4772 2339	502 1767 2397	695 4065 8546	971 8698 5428	1369 0327 8420	1941 5652 6564
94	376 6641 5398	518 2720 2569	720.7808 1595	1011 7846 4845	1431 6842 5949	2039 6935 2892
95	387 1057 5783	534 8501 8645	747 0431 4451	1053 2960 3439	1497 1550 5117	2142 7282 0537
96	397 8084 0177	551 9256 9205	774 2246 5457	1096 4678 7577	1565 5720 2847	2250 9146 1564
97	408 7786 1182	569 5134 6281	802 3575 1748	1141 3665 9080	1637 0677 6976	2364 5103 4642
98	420 0230 7711	587 6288 6669	831 4750 3059	1188 0612 5443	1711 7808 1939	2483 7858 6374
99	431 5486 5404	606 2877 3270	861 6116 5666	1236 6237 0461	1789 8559 5627	2609 0251 5693
100	443 3623 7039	625 5063 6468	892 8030 6464	1287.1286 5279	1871 4444 7430	2740 5264 1477

APPENDIX A

The Present Value of 1 per Annum at Different Rates of Compound Interest

TABLE IV

Units of Time	¼%	¾%	⅞%	1½%	1¾%	2%
1	0 9975 0623	0 9925 5583	0 9913 2590	0 9852 2167	0 9828 0098	0 9803 9216
2	1 9925 2492	1 9777 2291	1 9740 5294	1 9558 8342	1 9486 9875	1 9415 6094
3	2 9850 6227	2 9555 5624	2 9482 5570	2 9122 0042	2 8979 8403	2 8838 8327
4	3 9751 2446	3 9261 1041	3 9140 0813	3 8543 8465	3 8309 4254	3 8077 2870
5	4 9627 1766	4 8894 3961	4 8713 8352	4.7826 4497	4 7478 5508	4 7134 5951
6	5 9478 4804	5 8455 9763	5 8204 5454	5.6971 8717	5 6489 9762	5 6014 3089
7	6 9305 2174	6 7946 3785	6 7612 9323	6 5982 1396	6 5346 4139	6 4719 9107
8	7 9107 4487	7 7366 1325	7 6939 7098	7.4859 2508	7 4050 5297	7 3254 8144
9	8 8885 2357	8 6715 7642	8 6185 5859	8 3605 1732	8 2604 9432	8.1622 3671
10	9.8638 6391	9 5995 7958	9.5351 2624	9.2221 8455	9 1012 2291	8 9825 8501
11	10 8367 7198	10.5206 7452	10 4437 4348	10.0711 1779	9.9274 9181	9 7868 4805
12	11 8072 5384	11 4349 1267	11 3444 7929	10.9075 0521	10 7395 4969	10.5753 4122
13	12 7753 1555	12 3423 4508	12 2374 0202	11 7315 3222	11.5376 4097	11 3483 7375
14	13 7409 6314	13 2430 2242	13 1225 7945	12 5433 8150	12 3220 0587	12.1062 4877
15	14 7042 0264	14 1369 9495	14.0000 7876	13 3422 3301	13 0928 8046	12 8492 6350
16	15 6650 4004	15 0243 1261	14 8699 6656	14.1312 6405	13 8504 9677	13.5777 0931
17	16 6234 8133	15 9050 2492	15.7323 0885	14 9076 4931	14 5950 8282	14.2918 7188
18	17 5795 3250	16 7791 8107	16 5871 7111	15 6725 6089	15 3268 6272	14 9920 3125
19	18 5331 9950	17.6468 2984	17.4346 1820	16 4261 6837	16 0460 5673	15 6784 6201
20	19 4844 8828	18 5080 1969	18 2747 1445	17.1686 3879	16 7528 8130	16 3514 3334
21	20 4334 0477	19 3627 9870	19 1075 2361	17.9001 3673	17 4475 4919	17 0112 0916
22	21 3799 5488	20 2112 1459	19 9331 0891	18 6208 2437	18 1302 6948	17 6580 4820
23	22 3241 4452	21 0533 1473	20 7515 3400	19 3308 6145	18 8012 4764	18 2922 0412
24	23 2659 7957	21 8891 4614	21 5628 5799	20 0304 0537	19 4606 8565	18 9139 2560
25	24.2054 6591	22 7187 5547	22 3671 4547	20 7196 1120	20.1087 8196	19 5234 5647
26	25 1426 0939	23 5421 8905	23 1644 5647	21 3986 3172	20 7457 3166	20 1210 3576
27	26 0774 1585	24 3594 9286	23 9548 5152	22 0676 1746	21 3717 2644	20 7068 9780
28	27 0098 9112	25 1707 1251	24 7383 9060	22.7267 1671	21 9869 5474	21 2812 7236
29	27 9400 4102	25 9758 9331	25 5151 3319	23 3760 7558	22.5916 0171	21 8443 8466
30	28 8678 7134	26.7750 8021	26 2851 3823	24.0158 3801	23.1858 4934	22.3964 5555
31	29 7933 8787	27 5683 1783	27.0484 6417	24.6461 4582	23 7698 7650	22 9377 0152
32	30 7165 9638	28 3556 5045	27.8051 6894	25.2671 3874	24 3438 5897	23 4683 3482
33	31 6375 0262	29 1371 2203	28 5553 0998	25.8789 5442	24 9079 6951	23 9885 6355
34	32 5561 1234	29 9127 7621	29 2989 4422	26.4817 2849	25 4623 7789	24 4985 9172
35	33 4724 3126	30.6826 5629	30.0361 2809	27.0755 9458	26 0072 5100	24.9986 1933
36	34 3864 6510	31 4468 0525	30.7669 1757	27 6606 8431	26 5427 5283	25 4888 4248
37	35 2982 1955	32 2052 6576	31.4913 6810	28.2371 2740	27 0690 4455	25 9694 5341
38	36 2077 0030	32 9580 8016	32.2095 3467	28 8050 5163	27 5862 8457	26 4406 4060
39	37 1149 1302	33 7052 9048	32 9214 7179	29 3645 8288	28 0946 2857	26 9025 8883
40	38 0198 6336	34 4469 3844	33 6272 3350	29.9158 4520	28 5942 2955	27.3554 7924
41	38 9225 5697	35.1830 6545	34.3268 7335	30 4589 6079	29 0852 3789	27.7994 8945
42	39 8229 9947	35 9137 1260	35 0204 4446	30.9940 5004	29 5678 0136	28.2347 9358
43	40 7211 9648	36.6389 2070	35 7079 9947	31 5212 3157	30 0420 6522	28.6615 6233
44	41 6171 5359	37 3587 3022	36 3895 9055	32 0406 2223	30 5081 7221	29 0799 6307
45	42.5108 7640	38 0731 8136	37 0652 6944	32 5523 3718	30.9662 6261	29.4901 5987
46	43 4023 7047	38.7823 1401	37 7350 8743	33.0564 8983	31.4164 7431	29 8923 1360
47	44 2916 4137	39.4861 6774	38 3990 9535	33 5531 9195	31 8589 4281	30 2865 8196
48	45 1786 9463	40.1847 8189	39 0573 4359	34 0425 5365	32 2938 0129	30.6731 1957
49	46.0635 3580	40.8781 9542	39 7098 8212	34 5246 8339	32 7211 8063	31.0520 7801
50	46 9461 7037	41.5664 4707	40.3567 6047	34 9996 8807	33.1412 0946	31.4236 0589

APPENDIX A

THE PRESENT VALUE OF 1 PER ANNUM AT DIFFERENT RATES OF COMPOUND INTEREST

TABLE IV—*Continued*

Units of Time	¼%	¾%	⅞%	1½%	1¾%	2%
51	47 8266 0386	42.2495 7525	40 9980 2772	35 4676 7298	33 5540 1421	31.7878 4892
52	48.7048 4176	42 9276 1812	41 6337 3256	35 9287 4185	33 9597 1913	32 1449 4992
53	49 5808 8953	43 6006 1351	42.2639 2324	36 3829 9690	34 3584 4632	32.4950 4894
54	50 4547 5265	44 2685 9902	42.8886 4757	36 8305 3882	34 7503 1579	32 8382 8327
55	51.3264 3656	44 9316 1193	43.5079 5298	37 2714 6681	35 1354 4550	33 1747 8752
56	52 1959 4667	45 5896 8926	44 1218 8647	37 7058 7863	35 5139 5135	33 5046 9365
57	53.0632 8847	46 2428 6776	44.7304 9465	38 1338 7058	35 8859 4727	33 8281 3103
58	53 9284 6730	46 8911 8388	45 3338 2369	38 5555 3751	36 2515 4523	34 1452 2650
59	54.7914 8858	47 5346 7382	45 9319 1939	38 9709 7292	36 6108 5526	34 4561 0441
60	55.6523 5769	48 1733 7352	46.5248 2716	39 3802 6889	36.9639 8552	34 7608 8668
61	56 5110 7999	48 8073 1863	47 1125 9198	39 7835 1614	37 3110 4228	35 0596 9282
62	57 3676 6083	49 4365 4455	47 6952 5847	40 1808 0408	37 6521 3000	35 3526 4002
63	58 2221 0557	50 0610 8640	48 2728 7085	40 5722 2077	37 9873 5135	35 6398 4316
64	59 0744 1952	50 6809 7906	48 8454 7296	40 9578 5298	38 3168 0723	35 9214 1486
65	59 9246 0800	51 2962 5713	49 4131 0826	41 3377 8618	38.6405 9678	36 1974 6555
66	60.7726 7631	51 9069 5497	49 9758 1984	41 7121 0461	38 9588 1748	36 4681 0348
67	61 6186 2974	52 5131 0667	50 5336 5040	42 0808 9125	39 2715 6509	36 7334 3478
68	62 4624 7355	53 1147 4607	51 0866 4228	42 4442 2783	39 5789 3375	36 9935 6351
69	63 3042 1302	53 7119 0677	51 6348 3745	42 8021 9490	39 8810 1597	37 2485 9168
70	64 1438 5339	54 3046 2210	52 1782 7752	43 1548 7183	40 1779 0267	37 4986 1929
71	64 9813 9989	54 8929 2516	52 7170 0374	43 5023 3678	40 4696 8321	37.7437 4441
72	65 8168 5774	55 4768 4880	53 2510 5699	43 8446 6677	40 7564 4542	37 9840 6314
73	66 6502 3216	56 0564 2561	53 7804 7781	44 1819 3771	41 0382 7560	38.2196 6975
74	67 4815 2834	56 6316 8795	54 3053 0638	44 5142 2434	41 3152 5857	38 4506 5662
75	68 3107 5146	57 2026 6794	54.8255 8253	44 8416 0034	41 5874 7771	38 6771 1433
76	69 1379 0670	57 7693 9746	55 3413 4575	45 1641 3826	41 8550 1495	38 8991 3170
77	69 9629 9920	58 3319 0815	55.8526 3520	45 4819 0962	42 1179 5081	39.1167 9578
78	70 7860 3411	58 8902 3141	56 3594 8966	45 7949 8485	42 3763 6443	39 3301 9194
79	71 6070 1657	59 4443 9842	56 8619 4762	46 1034 3335	42 6303 3359	39.5394 0386
80	72 4259 5169	59 9944 4012	57 3600 4721	46 4073 2349	42 8799 3474	39 7445 1359
81	73 2428 4458	60 5403 8722	57 8538 2623	46 7067 2265	43 1252 4298	39 9456 0156
82	74 0577 0033	61 0822 7019	58.3433 2216	47 0016 9720	43.3663 3217	40 1427 4663
83	74 8705 2402	61 6201 1930	58 8285 7215	47 2923 1251	43 6032 7486	40 3360 2611
84	75 6813 2072	62 1539 6456	59 3096 1304	47 5786 3301	43 8361 4237	40 5255 1579
85	76 4900 9548	62 6838 3579	59 7864 8133	47.8607 2218	44.0650 0479	40 7112 8999
86	77 2968 5335	63 2097 6257	60.2592 1321	48 1386 4254	44 2899 3099	40.8934 2156
87	78 1015 9935	63 7317 7427	60 7278 4457	48 4124 5571	44 5109 8869	41 0719 8192
88	78 9043 3850	64 2499 0002	61.1924 1097	48 6822 2237	44 7282 4441	41 2470 4110
89	79 7050 7581	64 7641 6875	61 6529 4768	48 9480 0234	44 9417 6355	41 4186 6774
90	80 5038 1627	65 2746 0918	62 1094 8965	49 2098 5452	45 1516 1037	41 5869 2916
91	81 3005 6486	65 7812 4981	62 5620 7152	49 4678 3696	45 3578 4803	41 7518 9133
92	82 0953 2654	66 2841 1892	63 0107 2765	49 7220 0686	45 5605 3860	41 9136 1895
93	82.8881 0628	66 7832 4458	63 4554 9210	49 9724 2055	45 7597 4310	42 0721 7545
94	83.6789 0900	67 2786 5467	63 8963 9861	50 2191 3355	45 9555 2147	42 2276 2299
95	84 4677 3966	67 7703 7685	64 3334 8065	50 4622 0054	46 1479 3265	42 3800 2254
96	85 2546 0315	68 2584 3856	64.7667 7140	50 7016 7541	46 3370 3455	42 5294 3386
97	86.0395 0439	68 7428 6705	65 1963 0375	50.9376 1124	46 5228 8408	42.6759 1555
98	86.8224 4827	69.2236 8938	65 6221 1028	51 1700 6034	46 7055 3718	42 8195 2505
99	87.6034 3967	69 7009 3239	66 0442 2333	51 3990 7422	46 8850 4882	42 9603 1867
100	88 3824 8346	70 1746 2272	66.4626 7492	51 6247 0367	47 0614 7304	43 0983 5164

APPENDIX A

The Present Value of 1 per Annum at Different Rates of Compound Interest

TABLE IV—*Continued*

Units of Time	2½%	3%	3½%	4%	4½%	5%
1	0 9756 0976	0 9708 7379	0 9661 8357	0 9615 3846	0 9569 3780	0 9523 8095
2	1 9274 2415	1 9134 6970	1 8996 9428	1 8860 9467	1 8726 6775	1 8594 1043
3	2 8560 2356	2 8286 1135	2 8016 3698	2 7750 9103	2 7489 6435	2 7232 4803
4	3 7619 7421	3 7170 9840	3 6730 7921	3 6298 9522	3.5875 2570	3 5459 5050
5	4 6458 2850	4.5797 0719	4 5150 5238	4 4518 2233	4 3899 7674	4 3294 7667
6	5 5081 2536	5 4171 9144	5 3285 5302	5 2421 3686	5.1578 7248	5 0756 9206
7	6 3493 9060	6 2302 8296	6 1145 4398	6 0020 5467	5.8927 0094	5 7863 7340
8	7 1701 3717	7 0196 9219	6 8739 5554	6 7327 4487	6.5958 8607	6.4632 1276
9	7 9708 6553	7 7861 0892	7 6076 8651	7.4353 3161	7 2687 9050	7 1078 2168
10	8 7520 6393	8 5302 0284	8 3166 0532	8 1108 9578	7.9127 1818	7 7217 3493
11	9 5142 0871	9 2526 2411	9 0015 5104	8 7604 7671	8 5289 1692	8 3064 1422
12	10 2577 6460	9 9540 0399	9 6633 3433	9 3850 7376	9.1185 8078	8 8632 5164
13	10 9831 8497	10 6349 5533	10 3027 3849	9 9856 4785	9 6828 5242	9 3935 7299
14	11 6909 1217	11 2960 7314	10 9205 2028	10 5631 2293	10 2228 2528	9 8986 4094
15	12 3813 7773	11 9379 3509	11 5174 1090	11 1183 8743	10.7395 4573	10.3796 5804
16	13 0550 0266	12 5611 0203	12 0941 1681	11 6522 9561	11 2340 1505	10 8377 6956
17	13 7121 9772	13 1661 1847	12 6513 2059	12 1656 6885	11 7071 9143	11 2740 6625
18	14 3533 6363	13 7535 1308	13 1896 8173	12 6592 9697	12 1599 9180	11 6895 8690
19	14 9788 9134	14 3237 9911	13 7098 3742	13 1339 3940	12 5932 9359	12 0853 2086
20	15 5891 6229	14 8774 7486	14 2124 0330	13 5903 2634	13 0079 3645	12 4622 1034
21	16 1845 4857	15 4150 2414	14 6979 7420	14 0291 5995	13.4047 2388	12 8211 5271
22	16 7654 1324	15 9369 1664	15 1671 2484	14 4511 1533	13 7844 2476	13 1630 0258
23	17 3321 1048	16 4436 0839	15 6204 1047	14 8568 4167	14 1477 7489	13 4885 7388
24	17 8849 8583	16 9355 4212	16 0583 6760	15 2469 6314	14 4954 7837	13 7986 4179
25	18 4243 7642	17 4131 4769	16 4815 1459	15 6220 7994	14.8282 0896	14 0939 4457
26	18 9506 1114	17 8768 4242	16 8903 5226	15 9827 6918	15 1466 1145	14 3751 8530
27	19 4640 1087	18 3270 3147	17 2853 6451	16 3295 8575	15.4513 0282	14 6430 3362
28	19 9648 8866	18 7641 0823	17 6670 1885	16 6630 6322	15.7428 7351	14 8981 2726
29	20 4535 4991	19 1884 5459	18 0357 6700	16 9837 1463	16 0218 8853	15 1410 7358
30	20 9302 9259	19 6004 4135	18 3920 4541	17 2920 3330	16.2888 8854	15 3724 5103
31	21 3954 0741	20 0004 2849	18 7362 7576	17 5884 9356	16 5443 9095	15 5928 1050
32	21 8491 7796	20 3887 6553	19 0688 6547	17 8735 5150	16 7888 9086	15 8026 7667
33	22 2918 8094	20 7657 9178	19 3902 0818	18 1476 4567	17 0228 6207	16 0025 4921
34	22 7237 8628	21 1318 3668	19 7006 8423	18 4111 9776	17 2467 5796	16 1929 0401
35	23 1451 5734	21 4872 2007	20 0006 6110	18 6646 1323	17.4610 1240	16 3741 9429
36	23.5562 5107	21 8322 5250	20 2904 9381	18 9082 8195	17 6660 4058	16 5468 5171
37	23 9573 1812	22 1672 3544	20 5705 2542	19 1425 7880	17 8622 3979	16 7112 8734
38	24 3486 0304	22 4924 6159	20 8410 8736	19 3678 6423	18 0499 9023	16 8678 9271
39	24 7303 4443	22 8082 1513	21 1024 9987	19 5844 8484	18 2296 5572	17 0170 4067
40	25.1027 7505	23 1147 7197	21 3550 7234	19 7927 7388	18.4015 8442	17 1590 8635
41	25 4661 2200	23 4123 9997	21 5991 0371	19 9930 5181	18 5661 0949	17 2943 6796
42	25 8206 0683	23 7013 5920	21 8348 8281	20 1856 2674	18 7235 4975	17.4232 0758
43	26 1664 4569	23 9819 0213	22 0626 8870	20 3707 9494	18 8742 1029	17 5459 1198
44	26 5038 4945	24 2542 7392	22 2827 9102	20 5488 4129	19 0183 8305	17 6627 7331
45	26 8330 2386	24 5187 1254	22 4954 5026	20 7200 3970	19 1563 4742	17.7740 6982
46	27 1541 6962	24 7754 4907	22 7009 1813	20 8846 5356	19 2883 7074	17 8800 6650
47	27 4674 8255	25 0247 0783	22 8994 3780	21 0429 3612	19 4147 0884	17 9810 1571
48	27 7731 5371	25 2667 0664	23 0912 4425	21 1951 3088	19 5356 0654	18 0771 5782
49	28 0713 6947	25 5016 5693	23 2765 6450	21 3414 7200	19 6512 9813	18 1687 2173
50	28.3623 1168	25.7297 6401	23 4556 1787	21 4821 8462	19 7620 0778	18 2559 2546

APPENDIX A

THE PRESENT VALUE OF 1 PER ANNUM AT DIFFERENT RATES OF COMPOUND INTEREST

TABLE IV—*Continued*

Units of Time	2½%	3%	3½%	4%	4½%	5%
51	28 6461 5774	25 9512 2719	23 6286 1630	21 6174 8521	19 8679 5003	18 3389 7663
52	28 9230 8072	26 1662 3919	23 7957 6454	21 7475 8193	19 9693 3017	18 4180 7298
53	29 1932 4948	26 3749 9028	23 9572 6043	21 8726 7493	20 0663 4466	18 4934 0284
54	29 4568 2876	26 5776 6047	24 1132 9510	21 9929 5667	20 1591 8149	18 5651 4556
55	29 7139 7928	26 7744 2764	24 2640 5323	22 1086 1218	20 2480 2057	18 6334 7196
56	29 9648 5784	26 9654 6373	24 4097 1327	22 2198 1940	20 3330 3404	18 6985 4473
57	30 2096 1740	27 1509 3566	24 5504 4760	22 3267 4943	20 4143 8664	18 7605 1879
58	30 4484 0722	27 3310 0549	24 6864 2281	22 4295 6676	20 4922 3602	18 8195 4170
59	30 6813 7290	27 5058 3058	24 8177 9981	22 5284 2957	20 5667 3303	18 8757 5400
60	30 9086 5649	27 6755 6367	24 9447 3412	22 6234 8997	20 6380 2204	18 9292 8952
61	31 1303 9657	27 8403 5307	25 0673 7596	22 7148 9421	20 7062 4118	18 9802 7574
62	31 3467 2836	28 0003 4279	25 1858 7049	22 8027 8289	20 7715 2266	19 0288 3404
63	31 5577 8377	28 1556 7261	25 3003 5796	22 8872 9124	20 8339 9298	19 0750 8003
64	31 7636 9148	28 3064 7826	25 4109 7388	22 9685 4927	20 8937 7319	19 1191 2384
65	31 9645 7705	28 4528 9152	25 5178 4916	23 0466 8199	20 9509 7913	19 1610 7033
66	32 1605 6298	28 5950 4031	25 6211 1030	23 1218 0961	21 0057 2165	19 2010 1936
67	32 3517 6876	28 7330 4884	25 7208 7951	23 1940 4770	21 0581 0684	19 2390 6606
68	32 5383 1099	28 8670 3771	25 8172 7489	23 2635 0740	21 1082 3621	19 2753 0101
69	32 7203 0340	28 9971 2399	25 9104 1052	23 3302 9558	21 1562 0690	19 3098 1048
70	32 8978 5698	29 1234 2135	26 0003 9664	23 3945 1498	21 2021 1187	19 3426 7665
71	33 0710 7998	29 2460 4015	26.0873 3975	23 4562 6440	21 2460 4007	19 3739 7776
72	33 2400 7803	29 3650 8752	26 1713 4275	23 5156 3885	21 2880 7662	19 4037 8834
73	33 4049 5417	29 4806 6750	26 2525 0508	23 5727 2966	21 3283 0298	19 4321 7937
74	33 5658 0895	29 5928 8106	26 3309 2278	23 6276 2468	21 3667 9711	19 4592 1845
75	33 7227 4044	29.7018 2628	26 4066 8868	23 6804 0834	21 4036 3360	19 4849 6995
76	33 8758 4433	29 8075 9833	26 4798 9244	23 7311 6187	21 4388 8383	19 5095 9519
77	34 0252 1398	29 9102 8964	26 5506 2072	23 7799 6333	21 4726 1611	19 5328 5257
78	34 1709 4047	30 0099 8994	26 6189 5721	23 8268 8782	21 5048 9579	19 5550 9768
79	34 3131 1265	30 1067 8635	26 6849 8281	23 8720 0752	21 5357 8545	19 5762 8351
80	34 4518 1722	30 2007 6345	26 7487 7567	23 9153 9185	21 5653 4493	19 5964 6048
81	34 5871 3875	30 2920 0335	26 8104 1127	23 9571 0754	21 5936 3151	19 6156 7665
82	34 7191 5976	30 3805 8577	26 8699 6258	23 9972 1879	21 6207 0001	19 6339 7776
83	34 8479 6074	30 4665 8813	26 9275 0008	24 0357 8730	21 6466 0288	19 6514 0739
84	34 9736 2023	30 5500 8556	26 9830 9186	24 0728 7240	21 6713 9032	19 6680 0704
85	35 0962 1486	30 6311 5103	27 0368 0373	24 1085 3116	21 6951 1035	19 6838 1623
86	35 2158 1938	30 7098 5537	27 0886 9926	24 1428 1842	21 7178 0895	19 6988 7260
87	35 3325 0671	30 7862 6735	27 1388 3986	24 1757 8694	21 7395 3009	19 7132 1200
88	35 4463 4801	30 8604 5374	27 1872 8489	24 2074 8745	21 7603 1588	19.7268 6857
89	35 5574 1269	30 9324 7936	27 2340 9168	24 2379 6870	21 7802 0658	19 7398 7483
90	35 6657 6848	31 0024 0714	27 2793 1564	24 2672 7759	21 7992 4075	19 7522 6174
91	35 7714 8144	31 0702 9820	27 3230 1028	24 2954 5923	21 8171 5526	19 7640 5880
92	35 8746 1604	31 1362 1184	27 3652 2732	24 3225 5695	21 8348 8542	19 7752 9410
93	35 9752 3516	31 2002 0567	27 4060 1673	24 3486 1245	21 8515 6499	19 7859 9438
94	36 0734 0016	31 2623 3560	27 4454 2680	24 3736 6582	21 8675 2631	19 7961 8512
95	36 1691 7089	31 3226 5592	27 4835 0415	24 3977 5559	21 8828 0030	19 8058 9059
96	36 2626 0574	31 3812 1934	27 5202 9387	24 4209 1884	21 8974 1655	19 8151 3390
97	36 3537 6170	31 4380 7703	27 5558 3948	24 4431 9119	21 9114 0340	19 8239 3705
98	36 4426 9434	31 4932 7867	27 5901 8308	24 4646 0692	21 9247 8794	19 8323 2100
99	36 5294 5790	31 5468 7250	27 6233 6529	24 4851 9896	21 9375 9612	19 8403 0571
100	36 6141 0526	31 5989 0534	27 6554 2540	24 5049 9900	21 9498 5274	19 8479 1020

APPENDIX B

American Experience Table of Mortality: Derived Tables

Age	Number Living	Number Dying	Complete Expectation of Life	Present Value, Life Annuity of 1 American Experience, 3%	Net Single Premiums. American Experience, 3%	Net Annual Premiums American Experience, 3%
10	100,000	749	48.72	23.3430	290.98	
11	99,251	746	48.08	23.2248	294.43	
12	98,505	743	47.45	23.1026	297.98	
13	97,762	740	46.80	22.9765	301.65	
14	97,022	737	46.16	22.8463	305.45	
15	96,285	735	45.50	22.7119	309.36	
16	95,550	732	44.85	22.5731	313.40	
17	94,818	729	44.19	22.4298	317.58	
18	94,089	727	43.53	22.2817	321.89	
19	93,362	725	42.87	22.1289	326.34	
20	92,637	723	42.20	21.9711	330.94	14.41
21	91,914	722	41.53	21.8083	335.68	14.72
22	91,192	721	40.85	21.6404	340.57	15.04
23	90,471	720	40.17	21.4672	345.61	15.38
24	89,751	719	39.49	21.2886	350.82	15.74
25	89,032	718	38.81	21.1044	356.18	16.11
26	88,314	718	38.12	20.9142	361.72	16.51
27	87,596	718	37.43	20.7182	367.43	16.92
28	86,878	718	36.73	20.5161	373.32	17.35
29	86,160	719	36.03	20.3077	379.39	17.81
30	85,441	720	35.33	20.0930	385.64	18.28
31	84,721	721	34.63	19.8716	392.09	18.79
32	84,000	723	33.92	19.6434	398.73	19.32
33	83,277	726	33.21	19.4084	405.58	19.87
34	82,551	729	32.50	19.1665	412.63	20.46
35	81,822	732	31.78	18.9174	419.88	21.08
36	81,090	737	31.07	18.6608	427.36	21.74
37	80,353	742	30.35	18.3969	435.04	22.43
38	79,611	749	29.62	18.1254	442.95	23.16
39	78,862	756	28.90	17.8465	451.07	23.93
40	78,106	765	28.18	17.5598	459.42	24.75
41	77,341	774	27.45	17.2655	468.00	25.62
42	76,567	785	26.72	16.9632	476.80	26.54
43	75,782	797	26.00	16.6531	485.83	27.52
44	74,985	812	25.27	16.3350	495.10	28.56
45	74,173	828	24.54	16.0093	504.59	29.67
46	73,345	848	23.81	15.6757	514.30	30.84
47	72,497	870	23.08	15.3348	524.23	32.09
48	71,627	896	22.36	14.9867	534.37	33.43
49	70,731	927	21.63	14.6319	544.70	34.85
50	69,804	962	20.91	14.2710	555.22	36.36
51	68,842	1001	20.20	13.9045	565.89	37.97
52	67,841	1044	19.49	13.5329	576.71	39.68
53	66,797	1091	18.79	13.1568	587.67	41.51
54	65,706	1143	18.09	12.7765	598.74	43.46

APPENDIX B

American Experience Table, etc.—*Continued*

Age	Number Living	Number Dying	Complete Expectation of Life	Present Value, Life Annuity of 1 American Experience, 3%	Net Single Premiums American Experience, 3%	Net Annual Premiums American Experience, 3%
55	64,563	1199	17 40	12 3928	609 92	45 54
56	63,364	1260	16 72	12 0061	621 18	47.76
57	62,104	1325	16 05	11 6172	632 51	50.13
58	60,779	1394	15 39	11 2265	643 89	52.66
59	59,385	1468	14 74	10 8348	655 30	55.37
60	57,917	1546	14 10	10 4427	666 72	58 27
61	56,371	1628	13 47	10 0509	678 13	61 36
62	54,743	1713	12 86	9 6603	689 50	64 68
63	53,030	1800	12 26	9 2716	700 83	68 23
64	51,230	1889	11 67	8 8852	712 08	72.04
65	49,341	1980	11 10	8 5022	723 24	76.11
66	47,361	2070	10 54	8 1233	734 27	80.48
67	45,291	2158	10 00	7 7495	745 16	85.17
68	43,133	2243	9 47	7 3813	755 88	90.19
69	40,890	2321	8 97	7 0198	766.41	95.57
70	38,569	2391	8.48	6 6655	776 73	101.33
71	36,178	2448	8 00	6 3192	786 82	
72	33,730	2487	7 55	5 9811	796 67	
73	31,243	2505	7 11	5 6509	806 28	
74	28,738	2501	6 68	5 3278	815.69	
75	26,237	2476	6 27	5 0108	824 93	
76	23,761	2431	5 88	4 6989	834 01	
77	21,330	2369	5 49	4 3915	842 97	
78	18,961	2291	5 11	4 0883	851 80	
79	16,670	2196	4 74	3 7897	860 49	
80	14,474	2091	4 39	3 4956	869 06	
81	12,383	1964	4 05	3 2085	877 42	
82	10,419	1816	3 71	2 9277	885 60	
83	8,603	1648	3 39	2 6521	893 63	
84	6,955	1470	3 08	2 3789	901 59	
85	5,485	1292	2 77	2 1069	909 51	
86	4,193	1114	2 47	1 8388	917 32	
87	3,079	933	2 18	1 5793	924 88	
88	2,146	744	1 91	1 3338	932 02	
89	1,402	555	1 66	1 1029	938 75	
90	847	385	1 42	0 8804	945 23	
91	462	246	1.19	0 6625	951.58	
92	216	137	.98	0 4594	957 49	
93	79	58	80	0 2939	962 31	
94	21	18	.64	0 1387	966 83	
95	3	3	.50	0 0000	970 87	

APPENDIX C

Commutation Columns—American Experience

Three Per Cent

Age	D_x	N_x	S_x	M_x	R_x
10	74,409 3	1,736,936 2	35,170,109	21,651 75	734,215 97
11	71,701 0	1,665,235 3	33,433,173	21,110 66	712,564 22
12	69,089 4	1,596,145 9	31,767,938	20,587 43	691,453 57
13	66,571 2	1,529,574 7	30,171,792	20,081 48	670,866 14
14	64,143 0	1,465,431 7	28,642,217	19,592 25	650,784 66
15	61,801 7	1,403,630 1	27,176,785	19,119 20	631,192 41
16	59,543 6	1,344,086 5	25,773,155	18,661 17	612,073 21
17	57,366.4	1,286,720 0	24,429,069	18,218 30	593,412 04
18	55,267 4	1,231,452 6	23,142,349	17,790 09	575,193 74
19	53,243.0	1,178,209 6	21,910,896	17,375 49	557,403 65
20	51,290 9	1,126,918 8	20,732,687	16,974 08	540,028 16
21	49,408 3	1,077,510 4	19,605,768	16,585 43	523,054 09
22	47,592 4	1,029,918 0	18,528,257	16,208 62	506,468 66
23	45,840 9	984,077 1	17,498,339	15,843 30	490,260 04
24	44,151.6	939,925 6	16,514,262	15,489 11	474,416 74
25	42,522 2	897,403 4	15,574,337	15,145 71	458,927 63
26	40,950 8	856,452 6	14,676,933	14,812 77	443,781 93
27	39,434 8	817,017 9	13,820,480	14,489 54	428,969 15
28	37,972 4	779,045 5	13,003,463	14,175 72	414,479 62
29	36,561 7	742,483 8	12,224,417	13,871 04	400,303 90
30	35,200 6	707,283 3	11,481,933	13,574 82	386,432 86
31	33,887 3	673,396 0	10,774,650	13,286 83	372,858 05
32	32,620 3	640,775 7	10,101,254	13,006 84	359,571 22
33	31,397 6	609,378 0	9,460,479	12,734 25	346,564 38
34	30,217 4	579,160 7	8,851,100	12,468 50	333,830 14
35	29,078 2	550,082 5	8,271,940	12,209 42	321,361 64
36	27,978 7	522,103 8	7,721,857	11,956 86	309,152 22
37	26,916 9	495,186 9	7,199,753	11,709.98	297,195 36
38	25,891 6	469,295 3	6,704,567	11,468 66	285,485 38
39	24,901 0	444,394 4	6,235,271	11,232 16	274,016 72
40	23,943 9	420,450 5	5,790,877	11,000 40	262,784 56
41	23,018 8	397,431 6	5,370,426	10,772 72	251,784 16
42	22,124 7	375,306 9	4,972,995	10,549 06	241,011 44
43	21,260 1	354,046 8	4,597,688	10,328 84	230,462 38
44	20,423 8	333,623 0	4,243,641	10,111 76	220,133 54
45	19,614 2	314,008 8	3,910,018	9,897 032	210,021 787
46	18,830 3	295,178 4	3,597,009	9,684 454	200,124 755
47	18,070 5	277,107 9	3,300,831	9,473 083	190,440 301
48	17,333 6	259,774 3	3,023,723	9,262 544	180,967 218
49	16,618 3	243,156 0	2,763,949	9,052 028	171,704 674
50	15,922 8	227,233 2	2,520,793	8,840 573	162,652 646
51	15,246 0	211,987 3	2,293,559	8,627 525	153,812 073
52	14,586 7	197,400 6	2,081,572	8,412 298	145,184 548
53	13,943 9	183,456 7	1,884,172	8,194 362	136,772 250
54	13,316 6	170,140 0	1,700,715	7,973 249	128,577 888

APPENDIX C

COMMUTATION COLUMNS—AMERICAN EXPERIENCE—*Continued*

Age	D_x	N_x	S_x	M_x	R_x
55	12,703 88	157,436 2	1,530,575	7,748 344	120,604 639
56	12,104 81	145,331 3	1,373,139	7,519 292	112,856 295
57	11,518.55	133,812 8	1,227,807	7,285 597	105,337 003
58	10,944.47	122,868 3	1,093,995	7,047 005	98,051 405
59	10,381 99	112,486 3	971,126 3	6,803 299	91,004 401
60	9,830 432	102,655 9	858,640 0	6,554 130	84,201 103
61	9,289 343	93,366 56	755,984 0	6,299 366	77,646 972
62	8,758 318	84,608 24	662,617 5	6,038 903	71,347 606
63	8,237 140	76,371 10	578,009 3	5,772 823	65,308 704
64	7,725 774	68,645 33	501,638 2	5,501 373	59,535 881
65	7,224 176	61,421 15	432,992 9	5,224 798	54,034 509
66	6,732 309	54,688 84	371,571 7	4,943 343	48,809 711
67	6,250 543	48,438 30	316,882 9	4,657 666	43,866 368
68	5,779 342	42,658 96	268,444 6	4,368 518	39,208 702
69	5,319 228	37,339 73	225,785 6	4,076 734	34,840 184
70	4,871 163	32,468 57	188,445 9	3,783 598	30,763 450
71	4,436 103	28,032 46	155,977 3	3,490 417	26,979 852
72	4,015 468	24,017 00	127,944 8	3,198 989	23,489 436
73	3,611 065	20,405 93	103,927 8	2,911 541	20,290 447
74	3,224 793	17,181 14	83,521 92	2,630 446	17,378 906
75	2,858 395	14,322 74	66,340 78	2,357 974	14,748 460
76	2,513 249	11,809 49	52,018 04	2,096 083	12,390 486
77	2,190 406	9,619 086	40,208 55	1,846 440	10,294 403
78	1,890 417	7,728 668	30,589 46	1,610 250	8,447 963
79	1,613.596	6,115 072	22,860 79	1,388 489	6,837 713
80	1,360 224	4,754 848	16,745 72	1,182 116	5,449 224
81	1,129 824	3,625 024	11,990 87	991 3330	4,267 108
82	922 9402	2,702 084	8,365 850	817 3571	3,275 775
83	739 8781	1,962 206	5,663 766	661 1768	2,458 418
84	580 7245	1,381 481	3,701 561	523 5730	1,797 241
85	444 6441	936 8368	2,320 080	404 4068	1,273 668
86	330 0073	606 8295	1,383 243	302 7208	869 2616
87	235 2725	371 5570	776 4136	217 5978	566 5408
88	159 2040	212 3530	404 8566	148.3819	348 9430
89	100 9799	111 3731	192 5037	94 79488	200 5610
90	59 22884	52 14422	81 13062	55 98496	105 7661
91	31 36567	20 77855	28 98640	29 84690	49 78119
92	14 23735	6 54120	8 20785	13 63215	19 93429
93	5 05551	1 48569	1 66665	4 86499	6 30214
94	1 30473	.18096	.18096	1 26146	1 43715
95	.18096	.00000	.00000	.17569	.17569

APPENDIX D

Select Mortality Table

$[x]$	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	$l_{[x]+4}$	l_{x+5}
20	91,690	91,331	90,866	90,328	89,716	89,032
21	90,968	90,611	90,145	89,607	88,996	88,314
22	90,246	89,890	89,425	88,887	88,277	87,596
23	89,528	89,172	88,707	88,170	87,561	86,878
24	88,808	88,452	87,989	87,452	86,842	86,160
25	88,089	87,734	87,271	86,734	86,125	85,441
26	87,370	87,014	86,551	86,014	85,404	84,721
27	86,650	86,296	85,832	85,294	84,684	84,000
28	85,933	85,578	85,114	84,577	83,965	83,277
29	85,212	84,857	84,392	83,853	83,240	82,551
30	84,491	84,136	83,670	83,130	82,514	81,822
31	83,768	83,412	82,947	82,404	81,786	81,090
32	83,041	82,684	82,215	81,671	81,051	80,353
33	82,317	81,958	81,487	80,941	80,316	79,611
34	81,585	81,225	80,753	80,203	79,574	78,862
35	80,850	80,488	80,013	79,458	78,822	78,106
36	80,111	79,746	79,268	78,708	78,067	77,341
37	79,364	78,997	78,515	77,951	77,302	76,567
38	78,612	78,242	77,755	77,184	76,528	75,782
39	77,852	77,478	76,984	76,406	75,741	74,985
40	77,081	76,704	76,206	75,621	74,945	74,173
41	76,301	75,919	75,414	74,819	74,131	73,345
42	75,509	75,122	74,609	74,003	73,301	72,497
43	74,707	74,314	73,790	73,173	72,454	71,627
44	73,884	73,484	72,951	72,319	71,581	70,731
45	73,045	72,637	72,092	71,443	70,684	69,804
46	72,186	71,768	71,208	70,541	69,754	68,842
47	71,302	70,875	70,298	69,607	68,791	67,841
48	70,390	69,950	69,354	68,637	67,789	66,797
49	69,445	68,991	68,372	67,627	66,742	65,706
50	68,462	67,991	67,349	66,572	65,648	64,563
51	67,440	66,950	66,281	65,470	64,502	63,364
52	66,374	65,864	65,164	64,315	63,299	62,104
53	65,257	64,725	63,993	63,102	62,035	60,779
54	64,088	63,532	62,765	61,830	60,709	59,385
55	62,861	62,277	61,472	60,490	59,310	57,917
56	61,572	60,959	60,113	59,080	57,839	56,371
57	60,213	59,570	58,683	57,594	56,287	54,743
58	58,783	58,109	57,174	56,030	54,655	53,030
59	57,277	56,569	55,588	54,384	52,937	51,230
60	55,692	54,948	53,916	52,651	51,133	49,341
61	54,022	53,241	52,159	50,831	49,238	47,361
62	52,266	51,448	50,313	48,922	47,253	45,291
63	50,420	49,564	48,376	46,920	45,177	43,133
64	48,484	47,590	46,349	44,830	43,015	40,890
65	46,458	45,526	44,232	42,652	40,767	38,569

INDEX

Q

R

Y